Schedulability Analysis of Timed Systems

with contributions from

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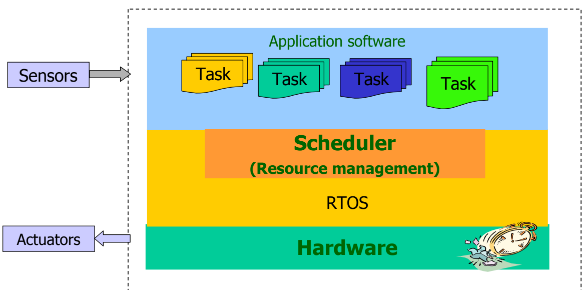
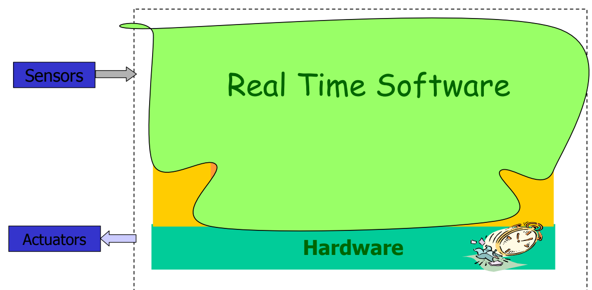
Paul Pettersson and Anders Wall

PROBLEM SETTING

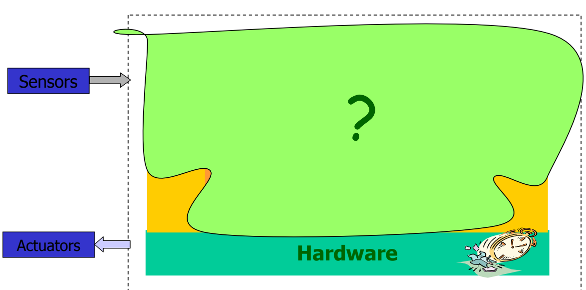
Real Time Systems



Real Time Systems



Real Time Systems

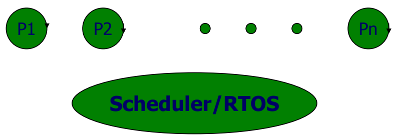


"Who is Who" in Timed Systems

- Real Time Scheduling [RTSS …]
  - Task models, Schedulability analysis
  - Real time operating systems
- Automata/logic-based methods [CAV, TACAS …]
  - (Timed) Automata, Petri Nets, Process Algebras …
  - Modelling, Model checking …
- (Real-Time) Programming Languages […]
  - Esterel, Signal, Lustre, Ada …
- … …

”Classic” Real Time Scheduling

Periodic tasks



well-developed techniques e.g. Rate-Monotonic Scheduling

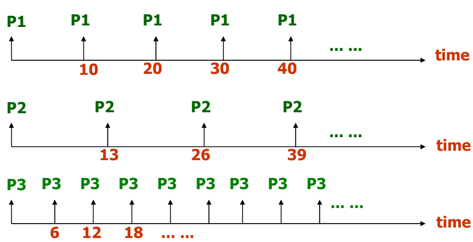
Rate-Monotonic Scheduling

- P1...Pn arrive at fixed rates
- Fixed Prioirity Order: higher frequency => higher priority
- Always run the task with highest priority (FPS)
- Schedulability can be checked by solving (or UB test):

$$Ri = Bi + Ci + \sum_{j \in HP(i)} \lceil Ri/Tj \rceil * Cj$$

If Ri <= Di for all tasks Pi, the system is schedulable

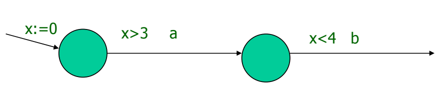
Arrival Rates (of Tasks): Periodic



In ”real life”

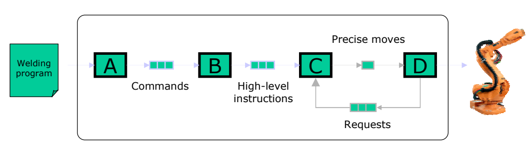
- tasks may share many resources (not only CPU time)
- tasks may have complex control stuctures and interactions
- tasks may not be that ”regular” (often non-periodic)

Task Arrival Patterns: Timed Traces



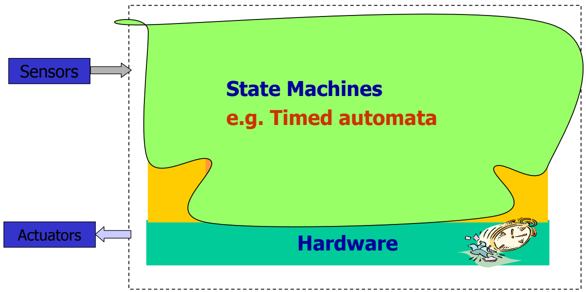
(3.3, a) (3.4, b),
(6.5, a),
(3.6, a) (3.9, b),
(3.14, a) (3.14159, b)
... ...

The ABB Robot Controller

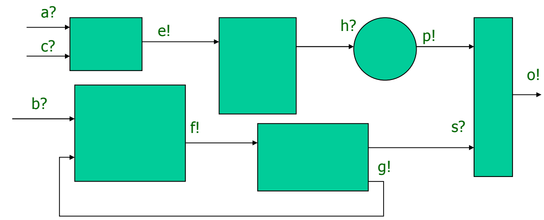


- ABB robot controller (2 500 000 loc)
- Real time tasks A,B,C,D
- Read inputs from channels write output to channels
- Task priority order D>C>B>A (FPS)
- Buffer overflow/underflow, WCRT

Automata-Based Approaches

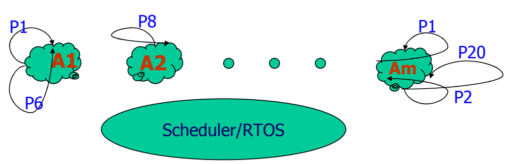


Networks of Real-Time Components



Automata-based Approaches

A controller = a set of timed automata accepting events trigger tasks Pi's



How to schedule tasks/automata? Worst-case response times?

Problems to solve

- Schedulability analysis: check
  (A1 || A2 || ... || An || Scheduler) satisfies K
  - A scheduler is given e.g. FPS, RMS, EDF etc
  - K is a requirement specifying e.g. safety
- Schedule synthesis: find X such that
  (A1 || A2 || ... || An || X) satisfies K

OUTLINE

- A Model for Timed Systems [1998]
  - Timed automata with tasks
- Schedulability and Decidability [TACAS 02,04]
  - Timed automata with bounded subtraction
- More Efficient Algoritms [TCS 06, IC 07]
  - Schedulability analysis using 2 clocks
  - (similar to Rate-Monotonic Scheduling)

Implemented in the TIMES tool

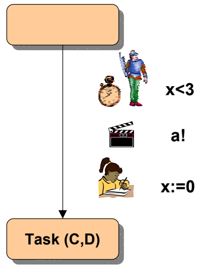
- TIMES Demo
- Current/Ongoing Work

The MODEL

(Timed Automata with Tasks)

Timed Automata with Tasks

- Events
  - Discrete Transitions
- Timing constraints
  - Clocks / Guards / Resets
  - Complex arrival rates
- Tasks
  - Asynchronous execution
  - WCET, Deadline
  - Scheduling policy
  - Precedence constraints
  - Resource constraints



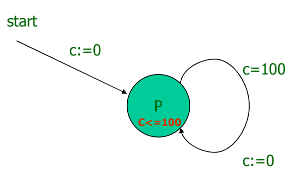
Tasks = Executable Programs (e.g. C, Java)

- Task parameters:
  - C: WCET
  - D: Relative Deadline
  - (other parameters for schedulling e.g. Priority)
- Task Interface:

```
Task P
{
v1 := F1(v1...vn)
...
vn := Fn(v1...vn)
}
```

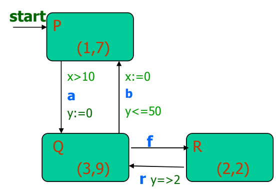
(a set of variables updated)

Example: periodic tasks



c: clock
P: task

Timed Automata with Tasks (Example)



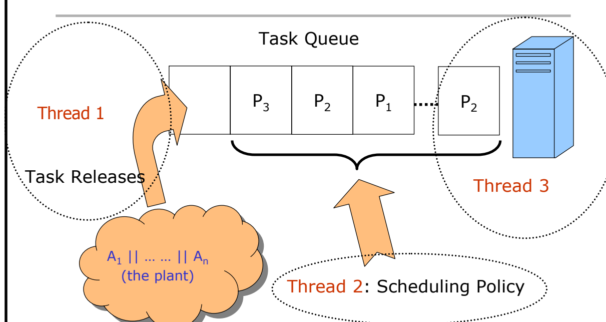
- Processor 1 (event handler)
  - Initially, P in the queue
  - Run-to-Completion/Stablization
    - Whenever a available and x>10, Q is put in the queue
    - Then
      - Whenever b available and y<=50, P is put in the queue
      - Whenever f available, R is put in the queue.
- Processor 2 (task handler)
  - Schedule and Compute tasks in the queue

P	P	Q	Q	Q	R	...	...	...

Timed Automata withTasks

- Assume a set of tasks Pr
- A timed automaton with tasks is a tuple: $<N,n_0,T,M>$
  - $<N,n_0,T>$ is a standard timed automaton
    - N is a set of nodes
    - n_0 is the initial node
    - $T \subseteq N \times (B(C) \times Act \times 2^C) \times N$ is the set of 'edges'
      - C is a set of clocks
      - Act is a set of actions
      - B(C) is the set of clock constraints e.g. X <10 etc
  - $M: N \rightarrow 2^{Pr}$ is a mapping which assigns each node a set of tasks

The Execution Platform

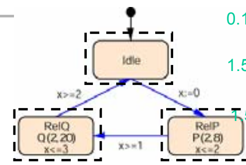


States/Configurations of automata

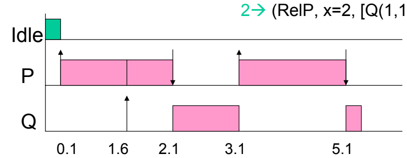
A **state** is a triple: (m, u, q)

- m — Location (node)
- u — clock assignment (valuation)
- q — task queue

Run of TAT



(Idle, x=0, [])
0.1→ (Idle, x=0.1, [])
→ (RelP, x=0, [P(2,8)])
1.5→ (RelP, x=1.5, [P(0.5,6.5)])
→ (RelQ, x=1.5, [P(0.5,6.5),Q(2,20)])
1.5→ (RelQ, x=3, [Q(1,18.5)])
→ (Idle, x=3, [Q(1,18.5)])
→ (RelP, x=0, [P(2,8),Q(1,18.5)])
2→ (RelP, x=2, [Q(1,16.5)])



Sch and Run

- **Sch** is a function sorting task queues according to a given scheduler e.g FPS,EDF,FIFO etc

  Example: EDF [P(2, 10), Q(4, 7)] = [Q(4, 7), P(2, 10)]

- **Run** is a function corresponding to running the first task of the queue for a given amount of time.

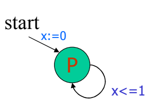
  Examples: Run(0.5, [Q(4, 7), P(2, 10)]) = [Q(3.5, 6.5), P(2, 9.5)]
  Run(5, [Q(4, 7), P(2, 10)]) = [P(1, 5)]

Semantics (as transition systems)

- States: <m,u,q>
  - m is a location
  - u is a clock assignment (valuation)
  - q is a queue of tasks (ready to run)
- Transitions:
  1. (m,u,q) −a→ (n, r(u), Sch[M(n)::q]) if (m) −g a r→ (n) & g(u)
  2. (m,u,q) −d→ (m, u+d, Run(d,q)) where d is a real

OBS: q is growing (by actions) and shrinking (by delays)

"Zenoness" = Non-Schedulability



P=(2,3)

Zeno: ∞ many P's may arrive within 1 time unit !

P(2,3)	P(2,3)	P(2,3)	P(2,3)	P(2,3)	...					..

But after 2 copies, the queue will be non-schedulable

SCHEDULABILITY

Schedulability of automata

a state is a triple: (m, u, q)

- m: location
- u: clock assignment
- q: task queue

•A state is schedulable if q is schedulable

•An automaton is schedulable if all reachable states are

Schedulability of Automata

Assume a scheduler Sch:

- A state (m,u,q) is schedulable with Sch if
  - $Sch(q)= [P_1(c_1,d_1)P_2(c_2,d_2)\ldots P_n(c_n,d_n)]$ and
  - $(c_1+\ldots+c_i)<=d_i$ for all $i<=n$ (i.e. all deadlines met)
- An automaton is schedulable with Sch if all its reachable states are schedulable
- An automaton is schedulable with a class of scheduling policies if it is schedulable with every Sch in the class.

DECIDABILITY

Schedulability Analysis (Non-preemptive scheduling)

FACT [1998]

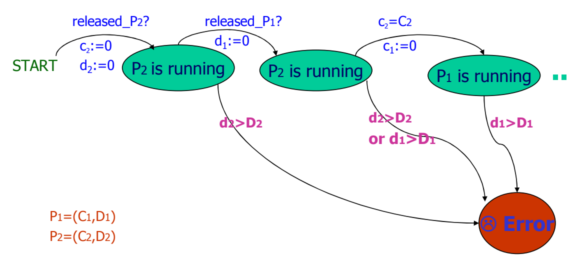
For Non-preemptive schedulers, the schedulability of an automaton can be checked by reachability analysis on ordinary timed automata.

Proof ideas (1):
Size of schedulable queues is bounded

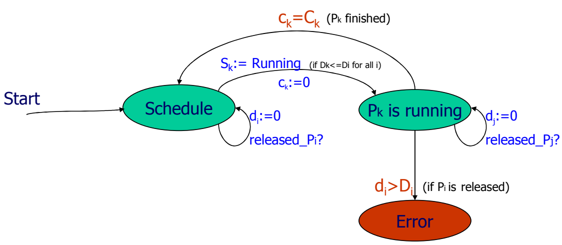
- The maximal number of instances of P_i in a schedulable queue is bounded by $M_i = \lceil D_i/C_i \rceil$
- The maximal size of schedulable queues is bounded by $M_1 + M_2+\ldots+M_n$
- To code the queue/scheduler, for each task instance, use 2 clocks:
  - c_i remembers the computing time
  - d_i remembers the deadline

• • •	(c_i,d_i)	• • •

Proof ideas (2):
The scheduler as an automaton



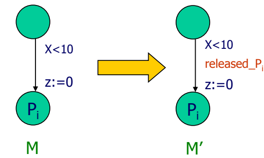
The scheduler automaton



SCHEDULER

Proof Ideas (3)

- Modify the original automaton M: adding 'release!' to inform the scheduler



- Check reactability of the error state for
  M' || SCHEDULER

How about preemptive scheduling?

- We may try the same ideas
  - Use clocks to remember computing times and deadlines
- BUT a running task may be stopped to run a more 'urgent' task
  - Thus we need stop-watches to remember "accumulated computing times"
- Then the schedulability problem is undecidable ?
- This is wrong !!

Decidability Result [TACAS 2002]

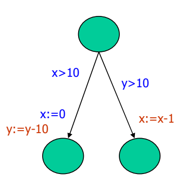
FACT

For Preemptive schedulers, the schedulability of an automaton can be checked by reachability analysis on Bounded Substraction Timed Automata (BSA).

NOTE
- Reachability for BSA is decidable
- Preemptive EDF is optimal; thus the general schedulability checking problem is decidable.

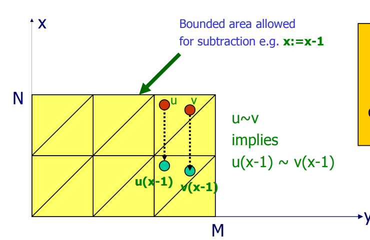
Timed automata with subtraction
i.e. Subtraction Automata, [McManis and Varaiya, CAV94]

- Subtraction automata are timed automata extended with subtraction on clocks
- That is, in addition to reset x:=0, it is also allowed to update a clock x with x:= x-n where n is a natural number



Bounded Subtraction Automata

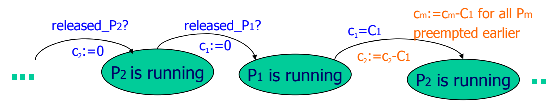
- A subtraction automaton is bounded if its clocks are non-negative and bounded with a maximal constant (or subtraction is only allowed in the bounded zone).



FACT:
Location Reachability checking is decidable!

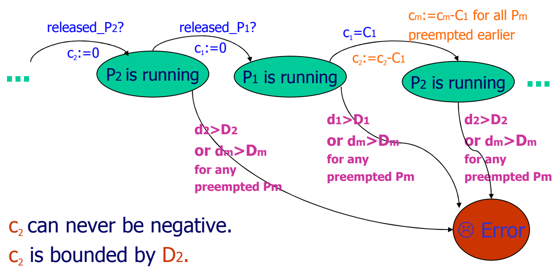
Schedulability Checking as a reachability problem for Bounded Subtraction Automata

Proof ideas (no stop but subtraction :-)



- Model the scheduler as a subtraction automaton
  - Do not stop the computing clock c_2 when a new task P_1 is released
  - Let c_2 for P_2 (preempted) run until the task P_1 (with higher priority) finishes, then perform $c_2 := c_2 - C_1$ (note: C_1 is the computing time for P_1).

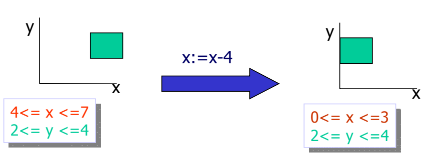
Proof ideas (clocks are bounded):



- c_2 can never be negative.
- c_2 is bounded by D_2.

END of proof

Schedulability analysis using DBM's



Subtraction on Clocks, added to DBM-library (UPPAAL)

Complexity

#clocks (needed)
= 2 x #instances (maximal number of schedulable task instances)
= 2 x $\sum_i \lceil Di/Ci \rceil$

This is a huge number in the worst case
But the run-time complexity is not so bad!

It works anyway !!!

- #active tasks in the queue is normally small, and the run-time complexity is only related to #active clocks
- If Too many active tasks in the queue (i.e. Too many active clocks), the check will stop sooner and report ”non-schedulable”
- AND the analysis can be done symbolically!

WE CAN DO BETTER ! [TACAS 03, TCS 06]

For fixed priority scheduling strategies (FPS),
we need only 2 clocks (and ordinary timed automata)!

The 2-CLOCK ENCODING

(for fixed-priority scheduling strategies)

Main Idea

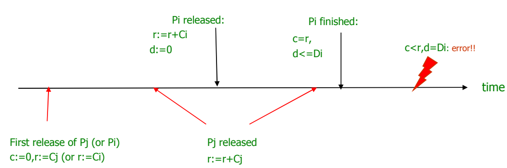
- Check the schedulability of tasks one by one according to priority order (highest priority first)
- This is similar to response time analysis in RMS

To code the queue/scheduler, we need:

- 1 integer variable for Pi:
  - r denotes the response time as in RMS
    (the total computing time needed before Pi finishes)
- 2 clocks for Pi:
  - c remembers the accumulated computing time
    (so much has been computed so far)
  - d remembers the ”deadline”

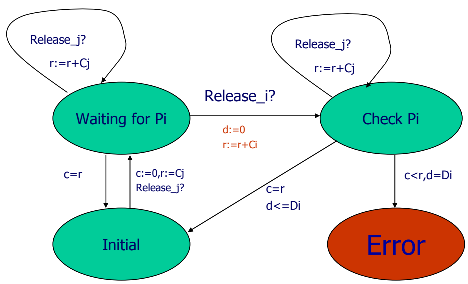
Intuition of the encoding: $Ri = Ci + \Sigma_{pri(Pj)>pri(Pi)} Cj$

– Assume: priority(Pj) > priority(Pi) and Pi is analyzed



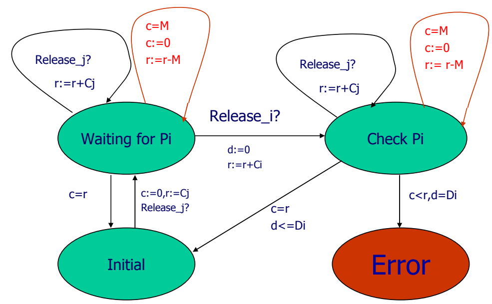
When Pi finishes, r = Ri

The "FPS scheduler": analyzing Pi



Note that it is not clear that c and r are bounded !

The "FPS scheduler": analyzing Pi
(we need the boundedness)



OBS: r-c is the only interesting info, so M can be any integer! Let M=Ci

c and r are bounded

- c is bounded by M
- r is bounded by r_{max} + Ci
  - Where r_{max} is the maximal value of r from previous analysis for all tasks Pj with higher priority

So the scheduler is a standard TA **END**

SUMMARY: Decidability

- For Non-preemptive schedulers, the problem can be solved using standard TA.
- For preemptive schedulers, the problem can be solved using BSA (Bounded Substraction Automata).
- For fixed-priority schedulers, the problem can be solved using TA with only 2 extra clocks – similar to the classic RMA technique (Rate-Monotonic Analysis).

Undecidability

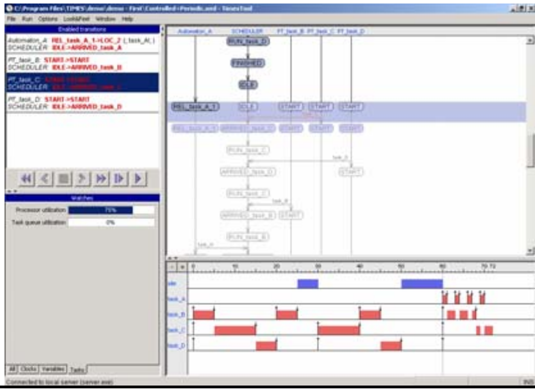
Unfortunately, the problem will be undecidable if the following conditions hold together:

1. Preemptive scheduling
2. Interval computation times
3. Feedback i.e. the finishing time of tasks may influence the release times of new tasks.

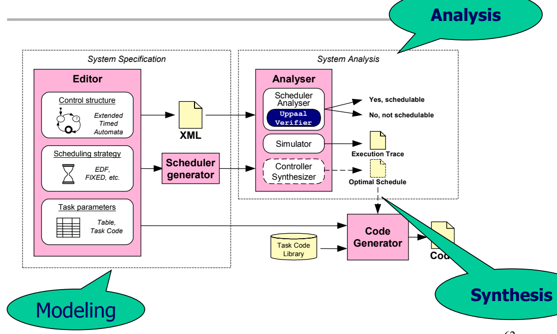
Conclusions/Remarks

- Unification of model-checking, real time scheduling, and synchronous programming: a unified model for timed systems (can express complex temporal and resource constraints).
- The first decidability result (and efficient algorithms) for preemptive scheduling in dense time models:
  - The analysis is symbolic (using DBM's in the UPPAAL tool)
- Implementation: **TIMES**

TIMES demo

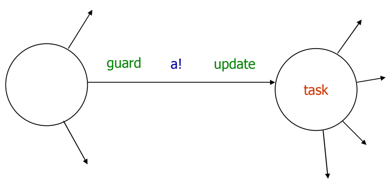


An Overview of TIMES



The INPUT LANGUAGE is very much like "guarded commands"

OBS: guard and update may contain data variables (integer, array)



- guard, update: "synchronous" computation which takes "no time" [CONCUR'04]
  - we adopt the synchronous hypothesis
- task: "asynchronous" computation whch takes time

Tasks = Executable Programs (e.g. C, Java)

- Task Type
  - Synchronous or Asynchronous
  - Non-Periodic (triggered by events) or Periodic
- Task parameters: C, D etc
  - C: Computing time and D: Relative Deadline
  - other parameters for schedulling e.g. priority, period
- Task Interface (variables updated 'atomically')
  - $X_i := F_i(X_1 \ldots X_n)$
- Tasks may have shared variables
  - with automata
  - with other tasks (priority ceiling protocols)
- Tasks with precedence constraints

Functionality/Features of TIMES

- GUI
  - modeling: automata with asynchronous tasks
  - editing, task library, visualization etc
- Simulation
  - symbolic execution as MSC's and Gant Charts
- Verification
  - all you do with UPPAAL
  - Schedulability analysis
- Code Generation

Code Generation in TIMES

- Run Time Systems
  - Event Handler, OS interrupt processing system or Polling
  - Task scheduler, generated from task parameters
- Application Tasks = threads (or processes)
  - Already there! (written in C)
  - Current version of TIMES support LegoOS

TIMES Demo


system.xml - TimesTool
System
Tasks
Task parameter
Start
Times