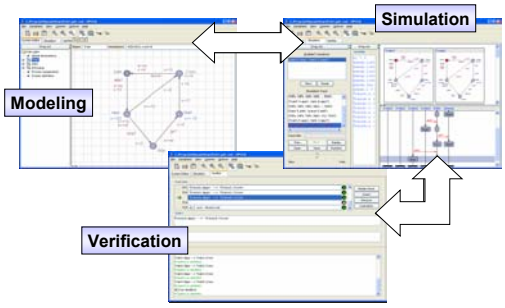
UPPAAL tutorial

- What's inside UPPAAL
- The UPPAAL input languages

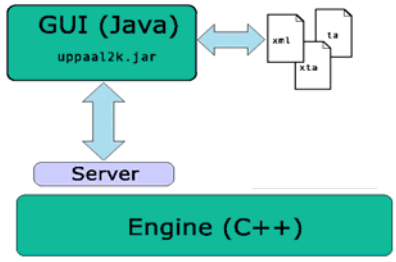
UPPAAL tool

- Developed jointly by Uppsala & Aalborg University
- >>20,000 downloads since 1995

UPPAAL Tool



Architecture of UPPAAL



Linux, Windows, Solaris, MacOS

What's inside UPPAAL

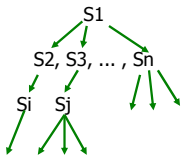
OUTLINE

- Data Structures
  - DBM's (Difference Bounds Matrices)
  - Canonical and Minimal Constraints
- Algorithms
  - Reachability analysis
  - Liveness checking
- Verification Options

All Operations on Zones

(needed for verification)

- Transformation
  - Conjunction
  - Post condition (delay)
  - Reset
- Consistency Checking
  - Inclusion
  - Emptiness

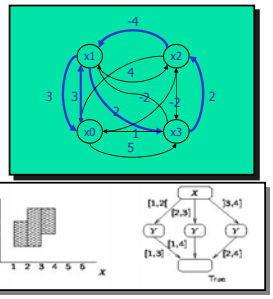


Zones = Conjuctive constraints

- A zone Z is a conjunctive formula:
  g_1 & g_2 & ... & g_n
  where g_i may be $x_i \sim b_i$ or $x_i - x_j \sim b_{ij}$
- Use a zero-clock x_0 (constant 0), we have
  $\{x_i - x_j \sim b_{ij} \mid \sim$ is $<$ or $\leq$, $i,j \leq n\}$
- This can be represented as a MATRIX, DBM
  (Difference Bound Matrices)

Datastructures for Zones in UPPAAL

- Difference Bounded Matrices
  [Bellman58, Dill89]
- Minimal Constraint Form
  [RTSS97]
- Clock Difference Diagrams
  [CAV99]



Canonical Datastructures for Zones

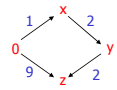
Difference Bounded Matrices Bellman 1958, Dill 1989

Inclusion

Z1

x<=1
y-x<=2
z-y<=2
z<=9

Graph

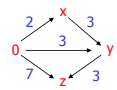


? ⊆ ?

Z2

x<=2
y-x<=3
y<=3
z-y<=3
z<=7

Graph



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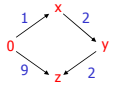
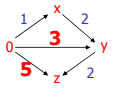


Shortest Path Closure



? ⊆ ?

Z1 ⊆ Z2 !

Z2

x<=2
y-x<=3
y<=3
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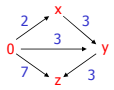
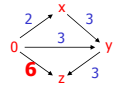
z<=7

Graph



Shortest Path Closure



Canonical Datastructures for Zones

Difference Bounded Matrices Bellman 1958, Dill 1989

Emptiness

Z

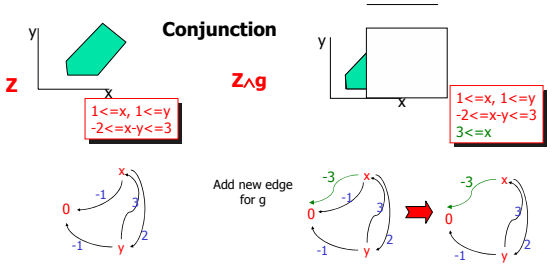
x<=1
y>=5
y-x<=3

Graph

Negative Cycle
iff
empty solution set

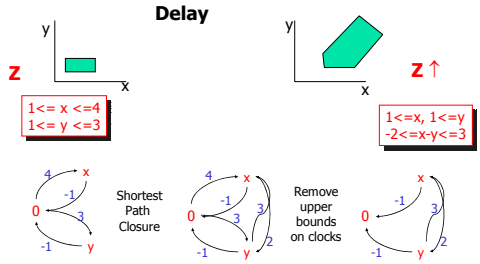
Canonical Datastructures for Zones

Difference Bounded Matrices



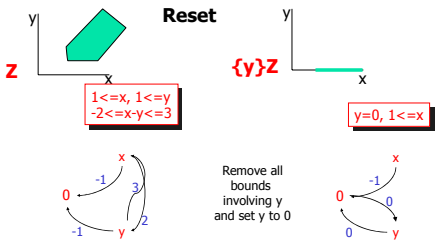
Canonical Dastructures for Zones

Difference Bounded Matrices



Canonical Datastructures for Zones

Difference Bounded Matrices



COMPLEXITY

- Computing the shortest path closure, the cannonical form of a zone: $O(n^3)$ [Dijkstra's alg.]
- Run-time complexity, mostly in O(n)
  (when we keep all zones in cannonical form)

Datastructures for Zones in UPPAAL

- Difference Bounded Matrices
  [Bellman58, Dill89]
- Minimal Constraint Form
  [RTSS97]
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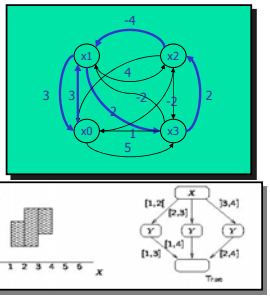
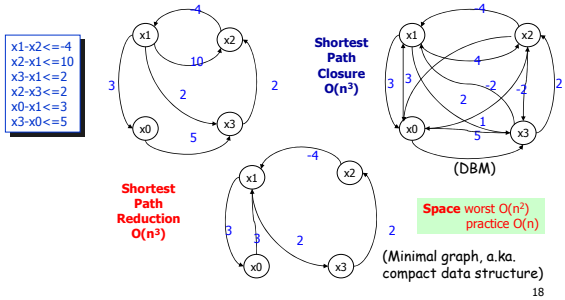
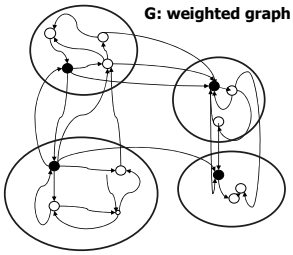


Minimal Graph



Graph Reduction Algorithm

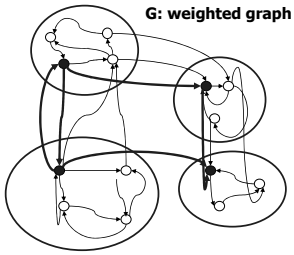
G: weighted graph



1. Equivalence classes based on 0-cycles.

Graph Reduction Algorithm

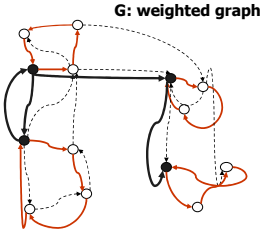
G: weighted graph



1. Equivalence classes based on 0-cycles.
2. Graph based on representatives.
   Safe to remove redundant edges

Graph Reduction Algorithm

G: weighted graph



1. Equivalence classes based on 0-cycles.
2. Graph based on representatives.
   Safe to remove redundant edges
3. **Shortest Path Reduction**
   =
   One cycle pr. class
   +
   Removal of redundant edges between classes

Datastructures for Zones in UPPAAL

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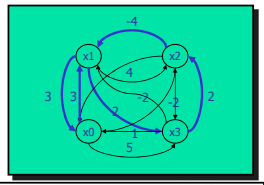
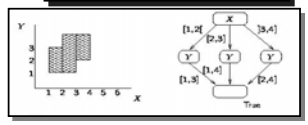
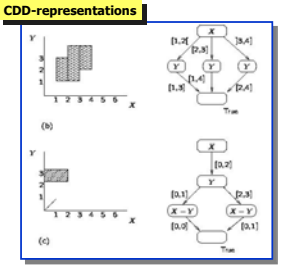




Other Symbolic Datastructures

- NDD's Maler et. al.
- CDD's UPPAAL/CAV99
- DDD's Møller, Lichtenberg
- Polyhedra HyTech
- .....

CDD-representations



Inside the UPPAAL tool

- Data Structures
  - DBM's (Difference Bounds Matrices)
  - Canonical and Minimal Constraints
- Algorithms
  - Reachability analysis
  - Liveness checking
- Verification Options

Timed CTL in UPPAAL

EF p | AG p | EG p | AF p | p - -> q

P ::= A.l | g_c | g_d | not p | p or p | p and p | p imply p

Process Location (a location in automaton A)

Clock constraint

predicate over data variables

p leads to q
denotes
AG (p imply AF q)

Timed CTL in UPPAAL

EF p | AG p | EG p | AF p | p - -> q

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Process Location (a location in automaton A)

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SAFETY PROPERTIES

SAFETY Properties

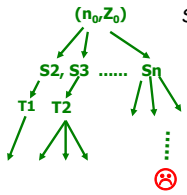
in UPPAAL

F ::= EF P | AG P

Reachability

Invariant = ¬ EF ¬ P
Thus, **AG P** is also checked by reachability analysis!

We have a search problem



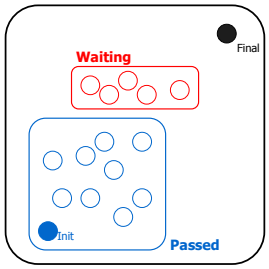
Symbolic state

Symbolic transitions

Reachable?
EF ☹

Forward Reachability

Init -> Final ?



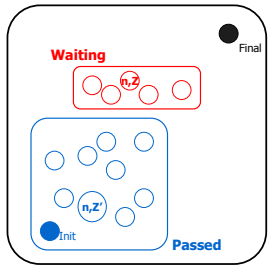
INITIAL **Passed** := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ⊇ Z (n,Z') in **Passed** **then** **STOP**
- **else** /explore/ add { (m,U) : (n,Z) => (m,U) } to **Waiting**;
  Add (n,Z) to **Passed**

UNTIL **Waiting** = Ø
or
Final is in **Waiting**

Forward Reachability

Init -> Final ?



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- **else** (explore) add { (m,U) : (n,Z) => (m,U) } to **Waiting**;
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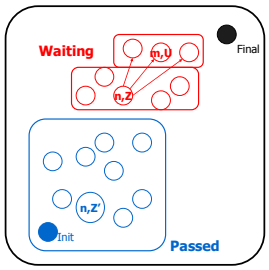
Forward Reachability

Init -> Final ?



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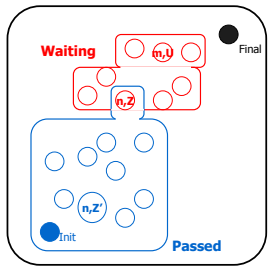

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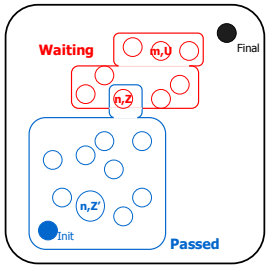

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UNTIL **Waiting** = Ø
or
Final is in **Waiting**

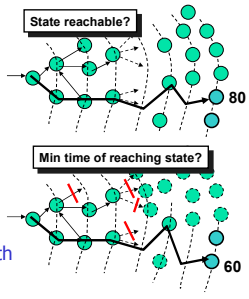
Further question

Can we find the path with shortest delay, leading to P ?
(i.e. a state satisfying P)

OBSERVATION:
Many scheduling problems can be phrased naturally as reachability problems for timed automata.

Verification vs. Optimization

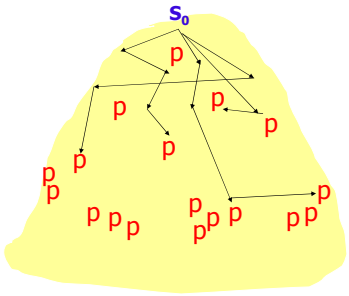
- Verification Algorithms:
  - Checks a logical property of the entire state-space of a model.
  - Efficient Blind search.
- Optimization Algorithms:
  - Finds (near) optimal solutions.
  - Uses techniques to avoid non-optimal parts of the state-space (e.g. Branch and Bound).
- Goal: solve opt. problems with verification.



OPTIMAL REACHABILITY

The maximal and minimal delay problem

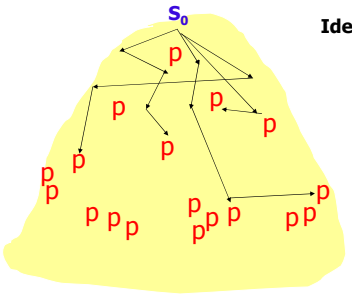
Find the trace leading to P with min delay



There may be a lot of pathes leading to P

Which one with the shortest delay?

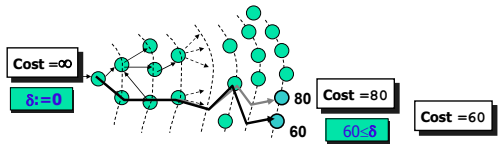
Find the trace leading to P with min delay



Idea: delay as "**Cost**" to reach a state, thus **cost** increases with time at rate 1

An Simple Algorithm for minimal-cost reachability

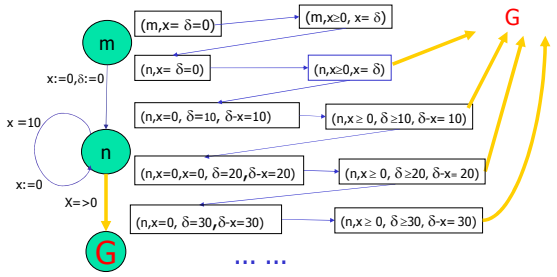
- State-Space Exploration + Use of global variable `Cost` and global clock **δ**
- Update `Cost` whenever goal state with **min(C) < `Cost`** is found:



- Terminates when entire state-space is explored.

Problem: The search may never terminate!

Example (min delay to reach G)



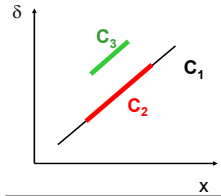
The minimal **delay** = 0 but the search may never terminate!
Problem: How to **symbolically** represent the zone **C.**

Priced-Zone

- Cost = minimal total time
- **C** can be represented as the zone Z^{δ}, where:
  - Z^{δ} original (ordinary) DBM plus…
  - δ clock keeping track of the cost/time.
- Delay, Reset, Conjunction etc. on Z are the standard DBM-operations
- Delay-Cost is incremented by Delay-operation on Z^{δ}.

Priced-Zone

- Cost = min total time
- **C** can be represented as the zone Z^{δ}, where:
  - Z^{δ} is the original zone Z extended with the global clock δ keeping track of the cost/time.
  - Delay, Reset, Conjunction etc. on C are the standard DBM-operations
- But inclusion-checking will be different



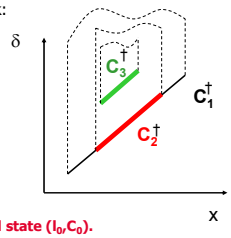
Then: $C_3 \sqsubseteq C_2 \sqsubseteq C_1$
But: $C_3 \not\subseteq C_2 \subseteq C_1$

Solution: $()^{\dagger}$-widening operation

- $()^{\dagger}$ removes upper bound on the δ–clock:

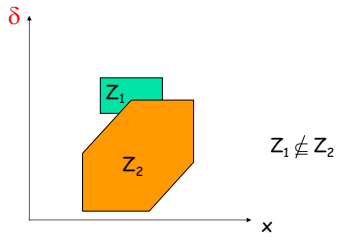
$C_3 \sqsubseteq C_2 \sqsubseteq C_1$

$C_3^{\dagger} \subseteq C_2^{\dagger} \subseteq C_1^{\dagger}$



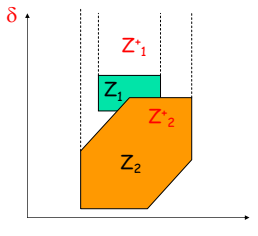
- In the Algorithm:
  - Delay(C†) = (Delay(C†))†
  - Reset(x,C†) = (Reset(x,C†))†
  - $C_1^{\dagger} \wedge g = (C_1^{\dagger} \wedge g)^{\dagger}$
  - **It is suffices to apply $()^{\dagger}$ to the initial state (l_0,C_0).**

Example (widening for Min)



$Z_1 \not\subseteq Z_2$

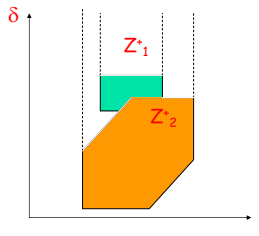
Example (widening for Min)



Z⁺= Widen(Z)

$Z_1 \not\subseteq Z_2$

Example (widening for Min)



Z⁺= Widen(Z)

$Z'_1 \subseteq Z'_2$!

$Z_1 \sqsubseteq Z_2$

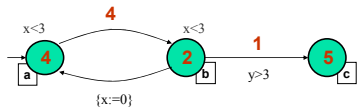
An Algorithm (Min)

```
Cost:=∞, Pass := {}, Wait := {(l0,C0)}
while Wait ≠ {} do
   select (l,C) from Wait
   if (l,C) ⊨ P and Min(C)<Cost then Cost:= Min(C)
   if (l,C) ⊑ (l,C') for some (l,C') in Pass then skip
       otherwise add (l,C) to Pass
       and forall (m,C') such that (l,C) → (m,C'):
         add (m,C') to Wait
Return Cost
```

One-step reachability relation

Output: `Cost` = the min cost of a found trace satisfying `P`.

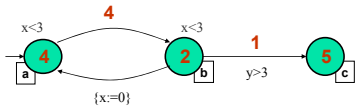
Further reading: Priced Timed Automata [Larsen et al]



- Timed Automata + Costs on transitions and locations.
- Uniformly Priced = Same cost in all locations (edges may have different costs).

- Cost of performing transition: Transition cost.
- Cost of performing delay **d**: (**d** x location cost).

Priced Timed Automata



Trace:

(**a**,x=y=0) —4→ (**b**,x=y=0) —ε(2.5), 2.5 x 2→ (**b**,x=y=2.5) —0→ (**a**,x=0,y=2.5)

Cost of Execution Trace:

Sum of costs: **4 + 5 + 0 = 9**

Problem: Finding the minimum cost of reaching c !

Inside the UPPAAL tool

- Data Structures
  - DBM's (Difference Bounds Matrices)
  - Canonical and Minimal Constraints
- Algorithms
  - Reachability analysis
  - Liveness checking
- Verification Options

Timed CTL in UPPAAL

EF p | AG p | EG p | AF p | p - -> q

P ::= A.l | g_c | g_d | not p | p or p | p and p | p imply p

Process Location (a location in automaton A)

Clock constraint

predicate over data variables

LIVENESS PROPERTIES

SAFETY PROPERTIES

p leads to q *denotes* **AG (p imply AF q)**

LIVENESS Properties

in UPPAAL

F ::= EG p | AF p | p - -> q

Possibly always P is equivalent to (¬ AF ¬ P)

Eventually P is equivalent to (¬ EG ¬ P)

P **leads to** Q is equivalent to AG (P imply AF Q)

Algorithm for checking AF P

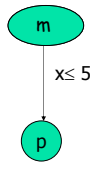
Eventually P

Bouajjani, Tripakis, Yovine'97
On-the-fly symbolic model checking of TCTL

Question

AF P

"P will be true for sure in future"

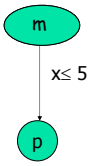


?? Does this automaton satisfy AF P

Note that

AF P

"P will be true for sure in future"

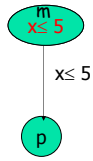


NO !!!! there is a path:
(m, x=0) →(m,x=1)→(m,2) … (m,x=k) …
Idling forever in location m

Note that

AF P

"P will be true for sure in future"

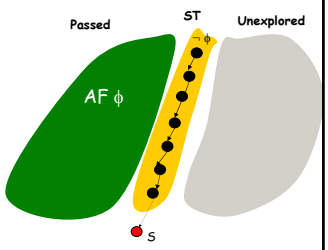


This automaton satisfies AF P

Liveness Algorithm

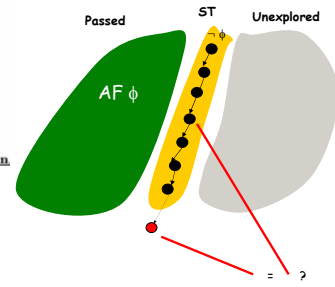
Bouajjani, Tripakis, Yovine, 97

```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S,ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
        exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
          Search(delay(S', ¬φ))
        od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



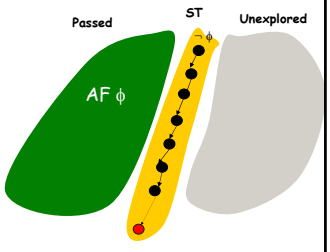
Liveness Algorithm

```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S,ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
        exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
          Search(delay(S', ¬φ))
        od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



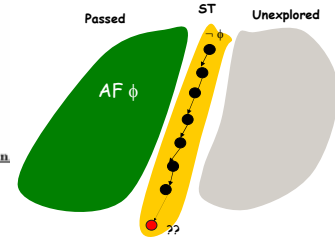
Liveness Algorithm

```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S,ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
        exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
          Search(delay(S', ¬φ))
        od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



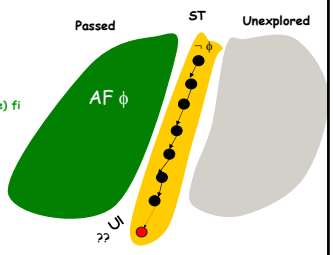
Liveness Algorithm

```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S,ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
        exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
          Search(delay(S', ¬φ))
        od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



Liveness Algorithm

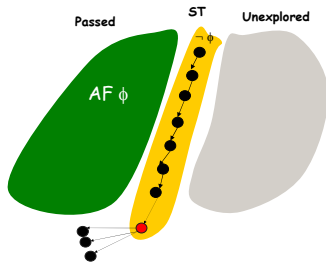
```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡   if empty(S) then exit(true) fi
  if loop(S, ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
                    exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
           Search(delay(S', ¬φ))
         od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



61

Liveness Algorithm

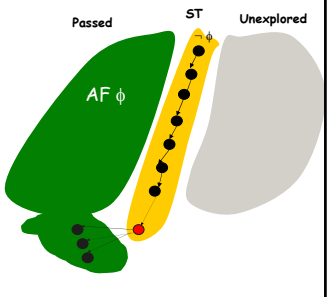
```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S, ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
                    exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
           Search(delay(S', ¬φ))
         od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



62

Liveness Algorithm

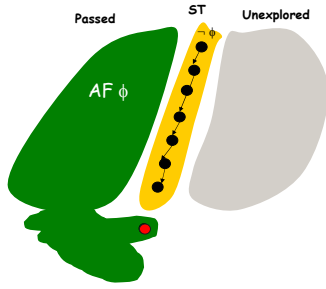
```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S, ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
                    exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
           Search(delay(S', ¬φ))
         od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



63

Liveness Algorithm

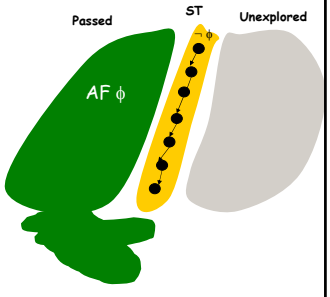
```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S, ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
                    exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
           Search(delay(S', ¬φ))
         od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



64

Liveness Algorithm

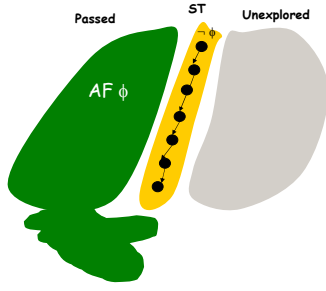
```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S, ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
                    exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
           Search(delay(S', ¬φ))
         od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



65

Liveness Algorithm

```
proc Eventually(S0, φ) ≡
  ST := ∅
  Passed := ∅
  Search(delay(S0, ¬φ))
  exit(true)
end
proc Search(S) ≡
  if loop(S, ST) then exit(false) fi
  S := S ∧ ¬φ
  push(ST, S)
  if unbounded(S) ∨ deadlocked(S) then
                    exit(false) fi
  if ∀S' ∈ Passed : S ⋢ S'
    then foreach S' : S ⇒ S' do
           Search(delay(S', ¬φ))
         od
  fi
  Passed := Passed ∪ {pop(ST)}
end
```



66

Question: Time bound synthesis

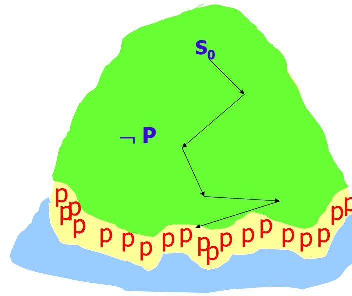
AF P "P will be true eventually"
But no time bound is given.

Assume AF P is satisfied by an automaton A.
Can we calculate the Max time bound?

OBS: we know how to calculate the Min !

Assume AF P is satisfied

Find the trace leading to P with the max delay



Almost the same algorithm as for synthesizing Min

We need to explore the Green part

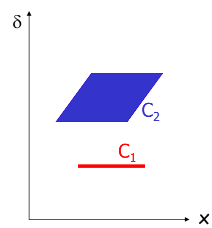
An Algorithm (Max)

```
Cost:=0, Pass := {}, Wait := {(l0,C0)}
while Wait ≠ {} do
   select (l,C) from Wait
   if (l,C) ⊨ P and Max(C)>Cost then Cost:= Max(C)
   else if forall (l,C') in Pass: C ⋢ C' then
      add (l,C) to Pass
      forall (m,C') such that (l,C) ⇝ (m,C'):
         add (m,C') to Wait
Return Cost
```

One-step reachability relation

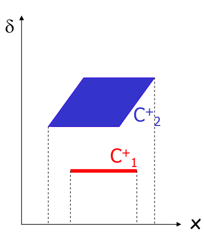
Output: Cost = the min cost of a found trace satisfying P.
BUT: ⊑ is defined on zones where the lower bound of "cost" is removed

Zone-Widening operation for Max



$C_1 \nsubseteq C_2$

Zone-Widening operation for Max



$C_1 \nsubseteq C_2$

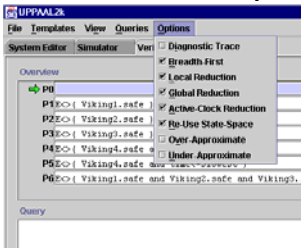
$C^+_1 \subseteq C^+_2$

$C_1 \sqsubseteq C_2$!

Inside the UPPAAL tool

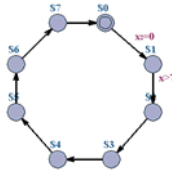
- Data Structures
  - DBM's (Difference Bounds Matrices)
  - Canonical and Minimal Constraints
- Algorithms
  - Reachability analysis
  - Liveness checking
  - Termination
- Verification Options

Verification Options



- Diagnostic Trace
- Breadth-First
- Depth-First
- Local Reduction
- Active-Clock Reduction
- Global Reduction
- Re-Use State-Space
- Over-Approximation
- Under-Approximation

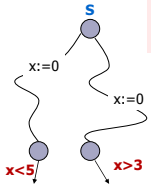
Inactive (passive) Clock Reduction



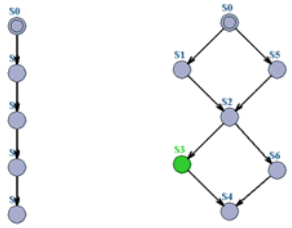
x is only ***active*** in location **S1**

Definition

x is ***inactive*** at **S** if on all path from **S**, x is always reset before being tested.



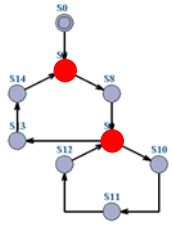
Global Reduction
(When to store symbolic state)



However, **Passed** list useful for efficiency

No Cycles: **Passed** list not needed for *termination*

Global Reduction [RTSS97]
(When to store symbolic state)

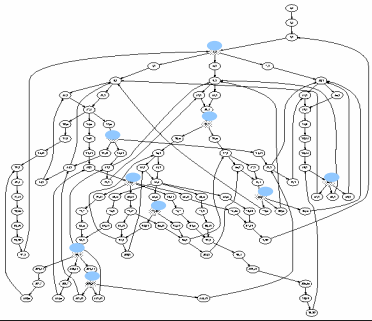


Cycles:
Only symbolic states involving loop-entry points need to be saved on **Passed** list

[RTSS97,CAV03]

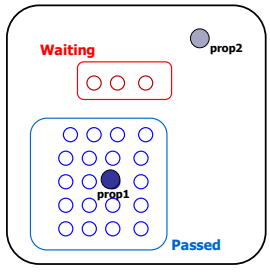
To Store Or Not To Store?

117 states$_{total}$
↓
81 states$_{entrypoint}$
↓
9 states



Time OH less than 10%
(need to re-explore some states)

Reuse of State Space

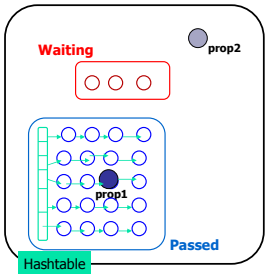



```
A[]  prop1

A[]  prop2
A[]  prop3
A[]  prop4
A[]  prop5
.
.
.
A[]  propn
```

Search in existing **Passed** list before continuing search

Which order to search?

Reuse of State Space

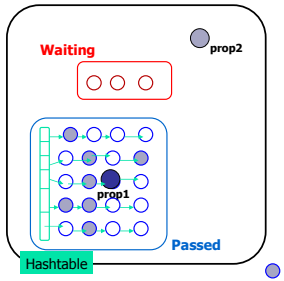



```
A[]  prop1

A[]  prop2
A[]  prop3
A[]  prop4
A[]  prop5
.
.
.
A[]  propn
```

Search in existing **Passed** list before continuing search

Which order to search?

Reuse of State Space



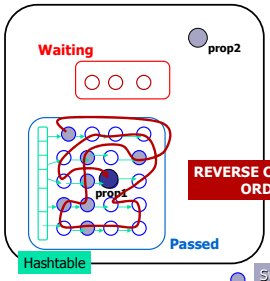

```
A[]  prop1

A[]  prop2
A[]  prop3
A[]  prop4
A[]  prop5
.
.
.
A[]  propn
```

Search in existing **Passed** list before continuing search

Which order to search?

Swapped to secondary memory

Reuse of State Space




```
A[]  prop1

A[]  prop2
A[]  prop3
A[]  prop4
A[]  prop5
.
.
.
A[]  propn
```

Search in existing **Passed** list before continuing search

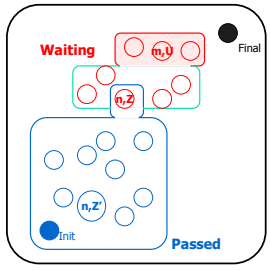
Which order to search?

generation order

Swapped to secondary memory

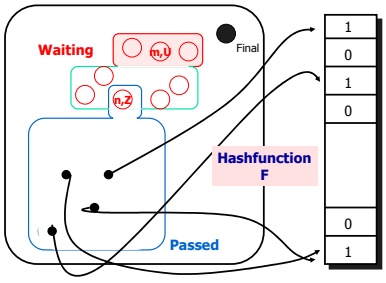
Under-approximation

Bitstate Hashing (Holzman,SPIN)



Under-approximation

Bitstate Hashing



Passed= Bitarray

UPPAAL 8 Mbits

Bit-state Hashing

INITIAL **Passed** := Ø;
Waiting := {(n0,Z0)}

REPEAT
- pick (n,Z) in **Waiting**
- **if** for some Z' ⊇ Z (n,Z') in **Passed** **then** **STOP**
- **else** /explore/ add { (m,U) : (n,Z) => (m,U) } to **Waiting**;
  Add (n,Z) to **Passed**

UNTIL **Waiting** = Ø or Final is in **Waiting**

Passed(F(n,Z)) = **1**

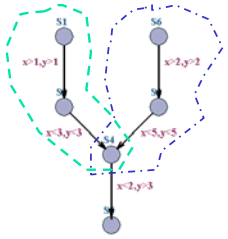
Passed(F(n,Z)) := **1**

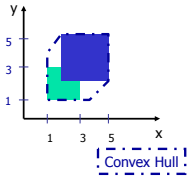
Under Approximation
(good for finding Bugs quickly, debugging)

- Possitive answer is safe (you can trust)
  - You can trust your tool if it tells:
    a state is reachable (it means Reachable!)
- Negative answer is Inconclusive
  - You should not trust your tool if it tells:
    a state is non-reachable
  - Some of the branch may be terminated by conflict (the same hashing value of two states)

Over-approximation

Convex Hull





Over-Approximation
(good for safety property-checking)

- Possitive answer is Inconclusive
  - a state is reachable means Nothing
    (you should not trust your tool when it says so)
  - Some of the transitions may be enabled by Enlarged zones
- Negative answer is safe
  - a state is not reachable means Non-reachable
    (you can trust your tool when it says so)