Example Used in Exemplifying the Marginalized (Rao-Blackwellized) Particle Filter

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In order to illustrate the marginalized (Rao-Blackwellized) particle filter, we provide the MATLAB code for solving the following examples,

$$x_{t+1}^{\mathrm{n}} = \arctan x_t^{\mathrm{n}} + \begin{pmatrix} 1 & 0 & 0 \end{pmatrix} x_t^{\mathrm{l}} + w_t^{\mathrm{n}}, \tag{1a}$$

$$x_{t+1}^{\mathrm{l}} = \begin{pmatrix} 1 & 0.3 & 0 \\ 0 & 0.92 & -0.3 \\ 0 & 0.3 & 0.92 \end{pmatrix} x_t^{\mathrm{l}} + w_t^{\mathrm{l}}, \tag{1b}$$

$$y_t = \begin{pmatrix} 0.1(x_t^{\mathrm{n}})^2 \operatorname{sgn}(x_t^{\mathrm{n}}) \\ 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix} x_t^{\mathrm{l}} + e_t, \tag{1c}$$

where

$$w_t = \begin{pmatrix} w_t^{\mathrm{n}} \\ w_t^{\mathrm{l}} \end{pmatrix} \sim \mathcal{N}\left(0,\, 0.01 I_{4\times 4}\right), \tag{1d}$$

$$e_t \sim \mathcal{N}(0,\, 0.1 I_{2\times 2}), \tag{1e}$$

$$x_0^{\mathrm{n}} \sim \mathcal{N}(0,\, 1), \tag{1f}$$

$$x_0^{\mathrm{l}} \sim \mathcal{N}(0_{3\times 1},\, 0_{3\times 3}), \tag{1g}$$

$$\tag{1h}$$

Looking at the notation used in Model 3 in [2], that is the model specified in

(18) and (19) of [2], we have,

$$
\begin{aligned}
f_t^{\mathrm{n}}(x_t^{\mathrm{n}}) &= \arctan x_t^{\mathrm{n}}, && \text{(2a)}\\
A_t^{\mathrm{n}}(x_t^{\mathrm{n}}) &= \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}, && \text{(2b)}\\
G_t^{\mathrm{n}}(x_t^{\mathrm{n}}) &= 1, && \text{(2c)}\\
f_t^{\mathrm{l}}(x_t^{\mathrm{n}}) &= \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T, && \text{(2d)}\\
A_t^{\mathrm{l}}(x_t^{\mathrm{n}}) &= \begin{pmatrix} 1 & 0.3 & 0 \\ 0 & 0.92 & -0.3 \\ 0 & 0.3 & 0.92 \end{pmatrix}, && \text{(2e)}\\
G_t^{\mathrm{l}}(x_t^{\mathrm{n}}) &= I_{3\times 3}, && \text{(2f)}\\
h_t(x_t^{\mathrm{n}}) &= \begin{pmatrix} 0.1(x_t^{\mathrm{n}})^2 \operatorname{sgn}(x_t^{\mathrm{n}}) \\ 0 \end{pmatrix}, && \text{(2g)}\\
C_t(x_t^{\mathrm{n}}) &= \begin{pmatrix} 0 & 0 & 0 \\ 1 & -1 & 1 \end{pmatrix}, && \text{(2h)}\\
Q_t &= 0.01 I_{4\times 4}, && \text{(2i)}\\
R_t &= 0.1 I_{2\times 2}, && \text{(2j)}\\
\bar{x}_0 &= \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}^T, && \text{(2k)}\\
\bar{P}_0 &= 0_{3\times 3}. && \text{(2l)}
\end{aligned}
$$

It is worth noting that we also used this example in [1] in order to illustrate how to perform maximum likelihood identification in mixed linear/nonlinear models.

References

[1] F. Lindsten and T. B. Schön. Maximum likelihood identification in mixed linear/nonlinear state-space models. In *Proceedings of the 49th IEEE Conference on Decision and Control (CDC)*, Atlanta, USA, December 2010.

[2] T. Schön, F. Gustafsson, and P.-J. Nordlund. Marginalized particle filters for mixed linear/nonlinear state-space models. *IEEE Transactions on Signal Processing*, 53(7):2279–2289, July 2005.