A Closer Look at Pseudo-Polynomial Time and its Use in Real-Time Scheduling Theory

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Dedicated to Wang Yi to Celebrate his Scientific Career

Come again?

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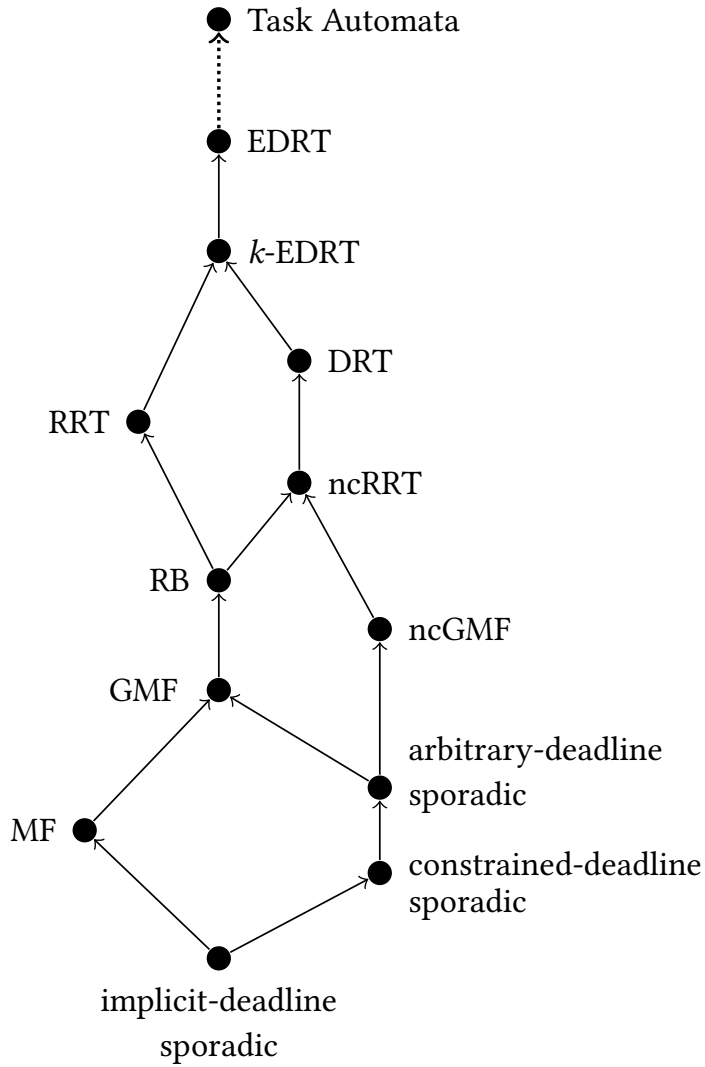
Pseudo-polynomial time has traditionally been considered tractable in real-time scheduling.

Pushing the boundary of pseudo-polynomial time



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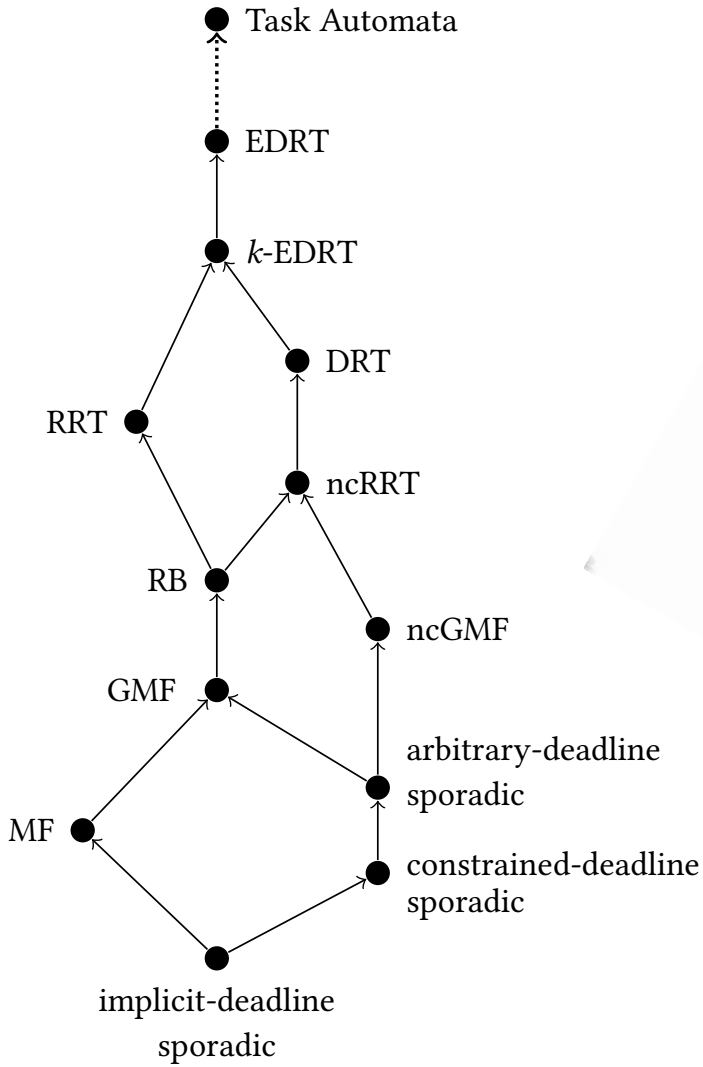

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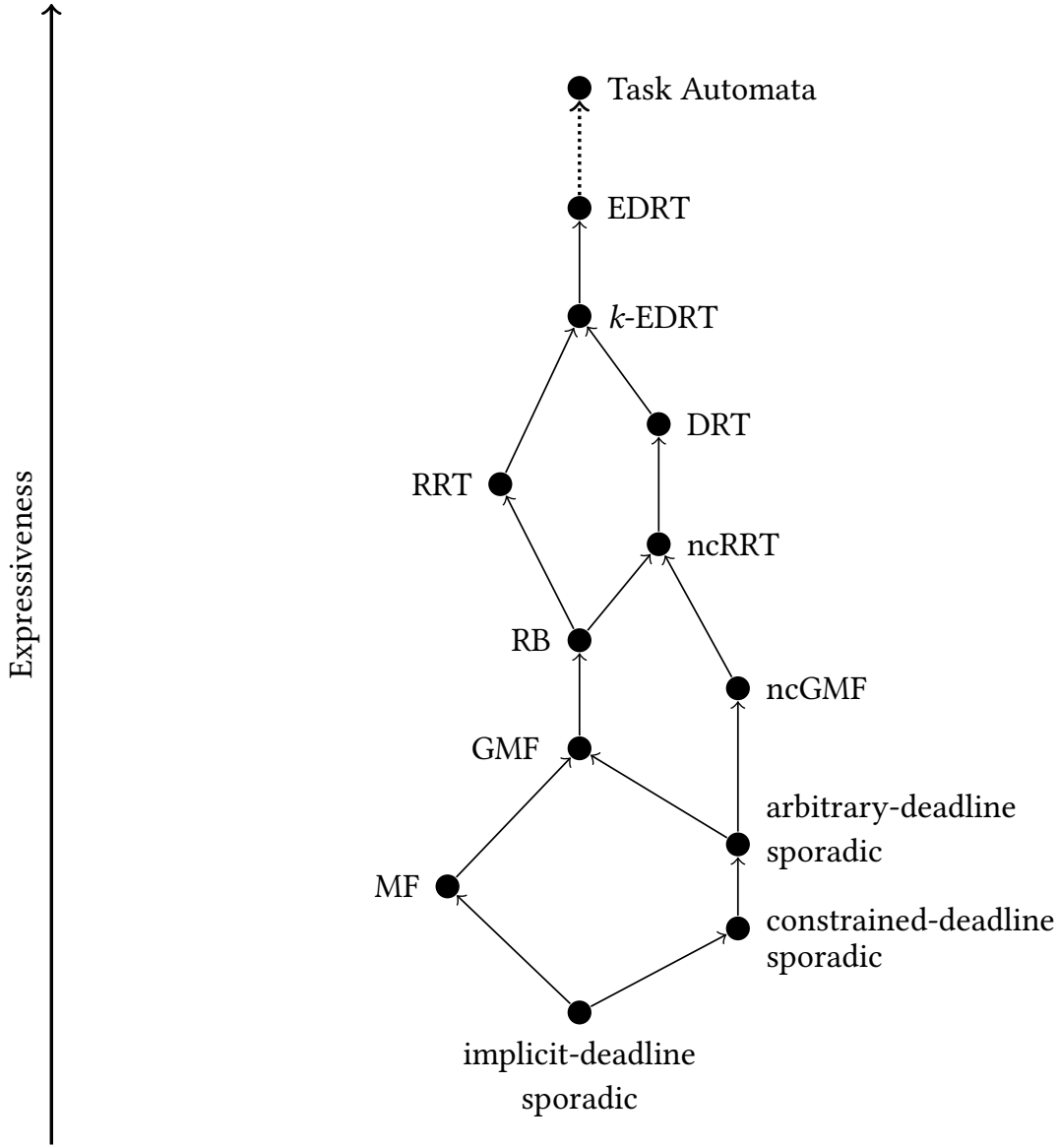

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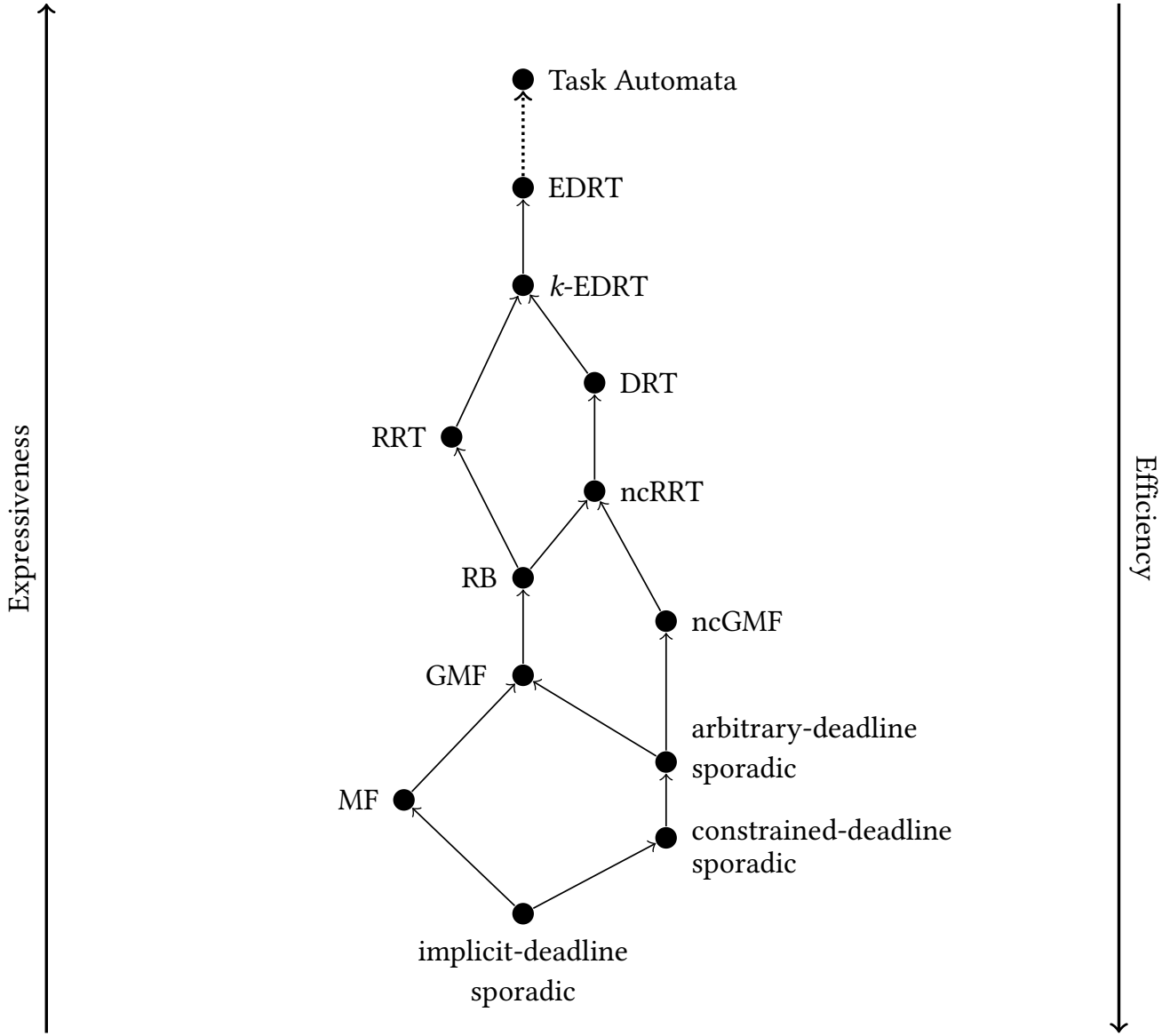

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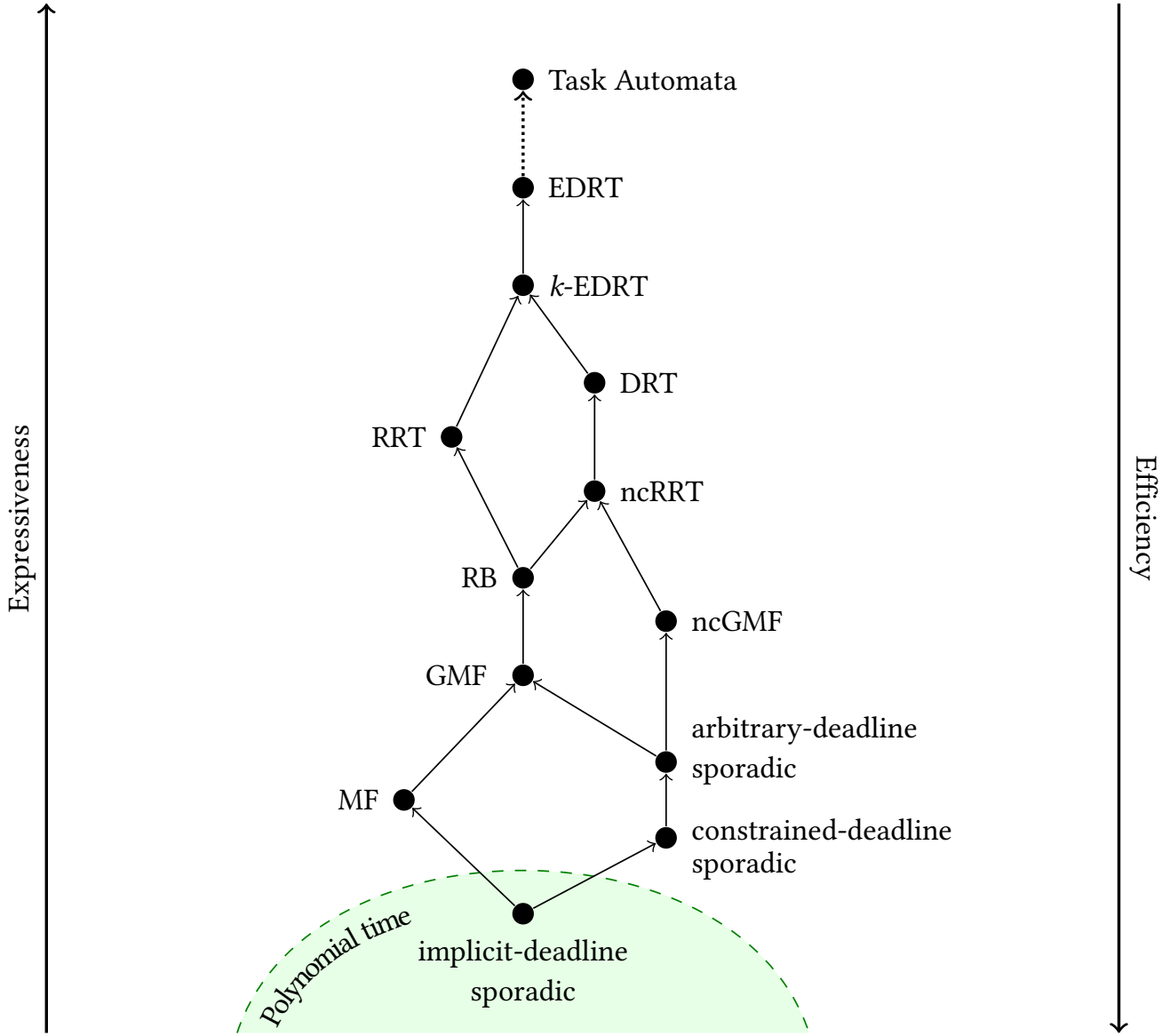

(Single processor Earliest Deadline First scheduling)

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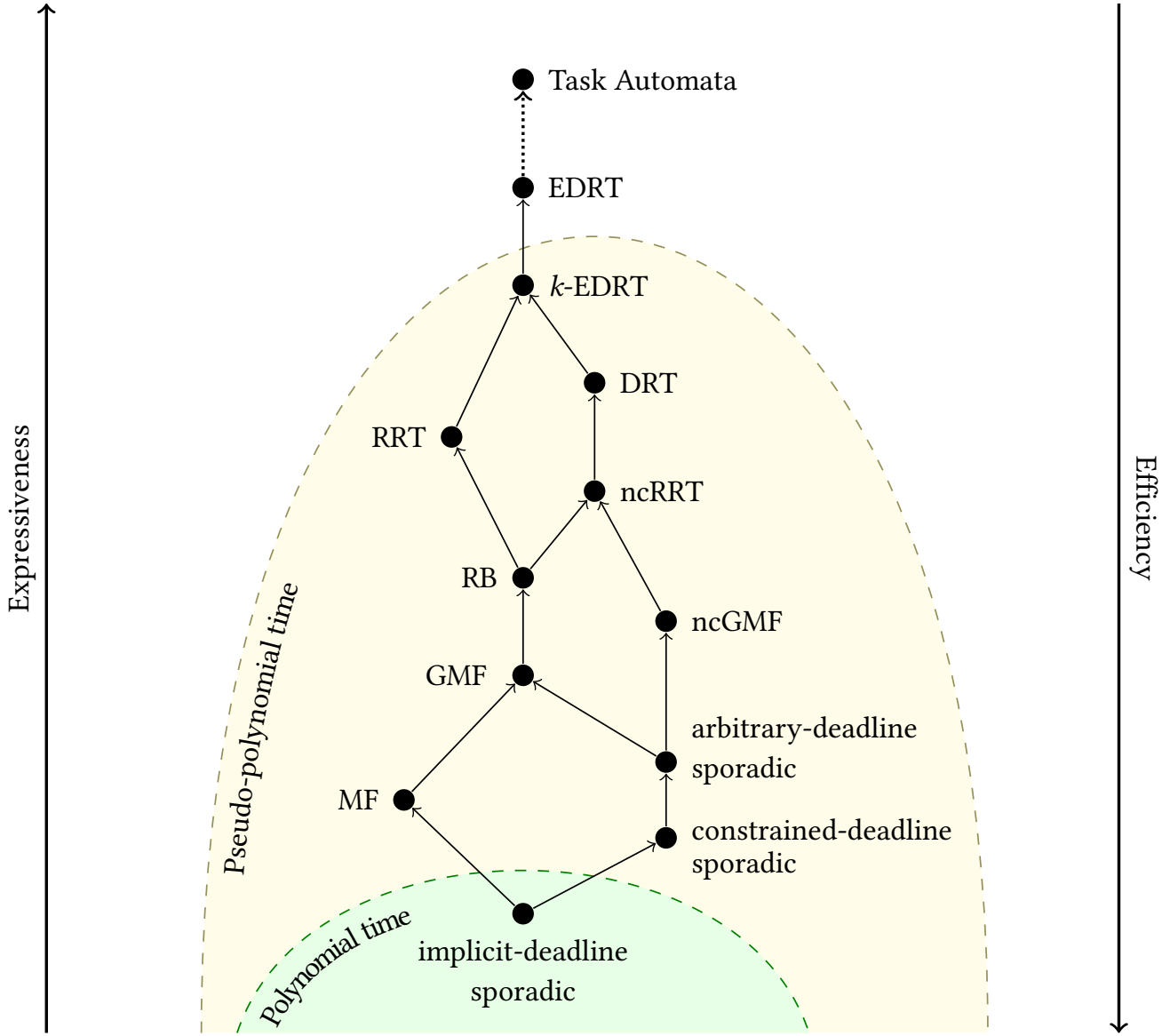

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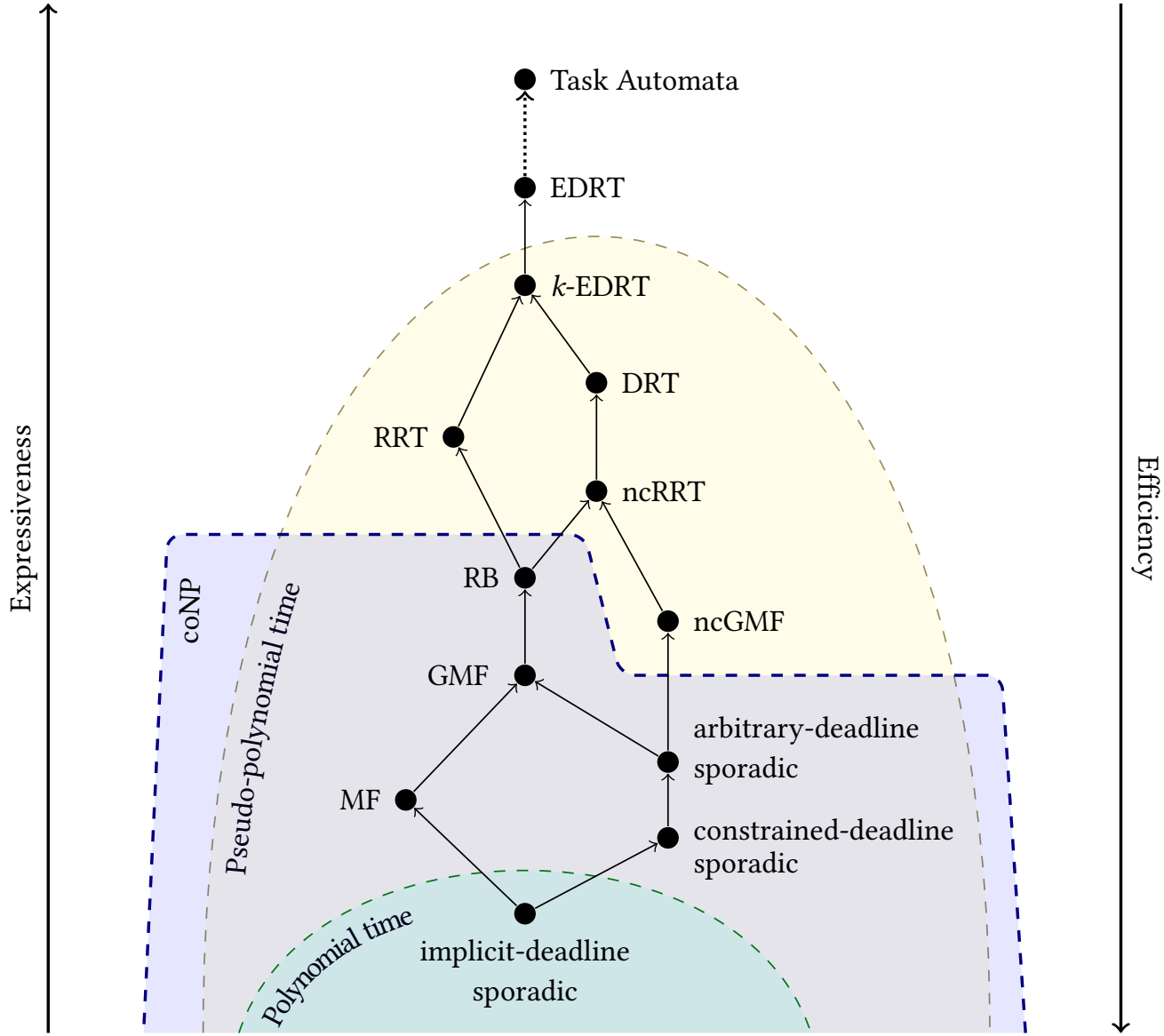

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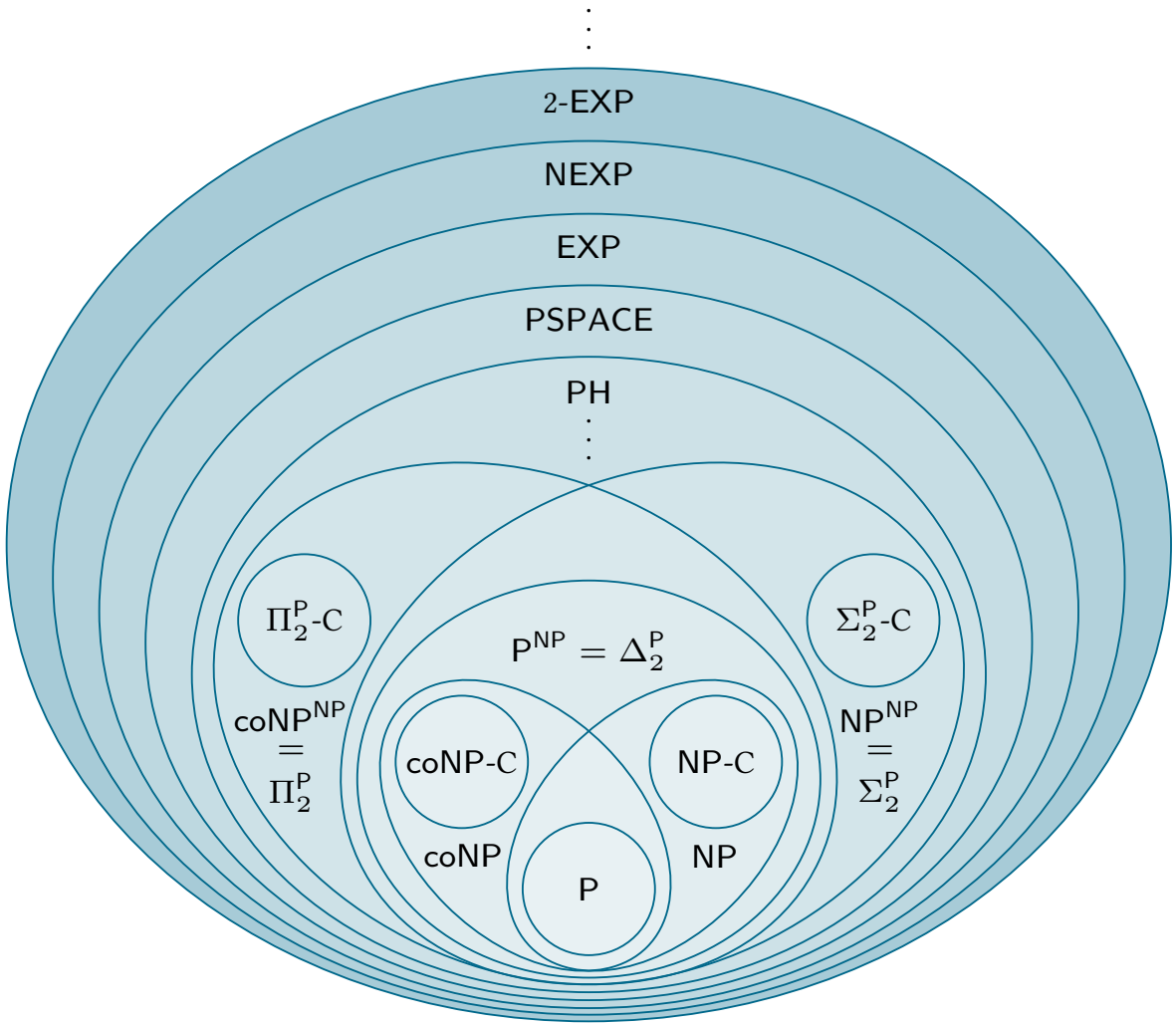
Where is pseudo-polynomial time?



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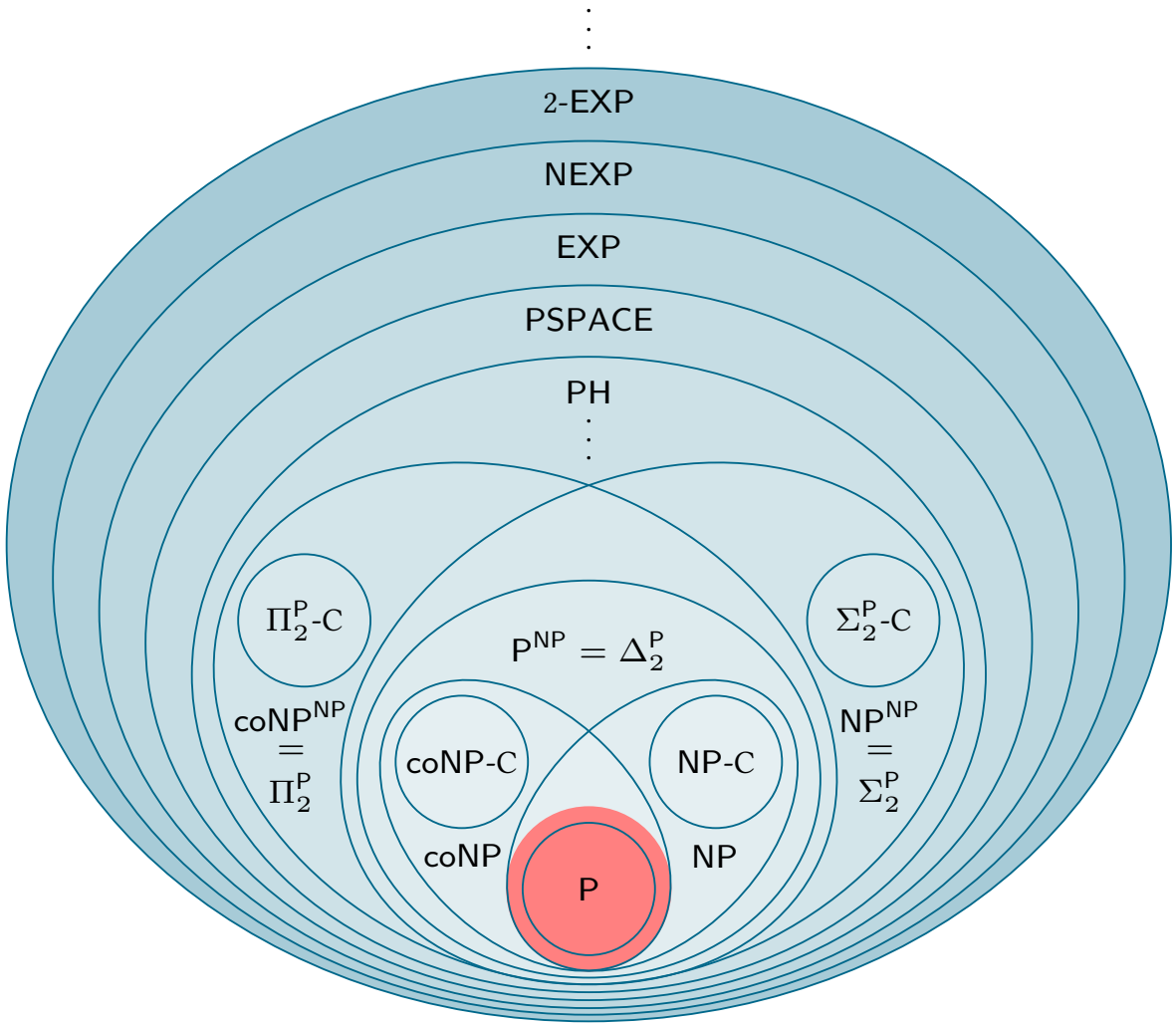

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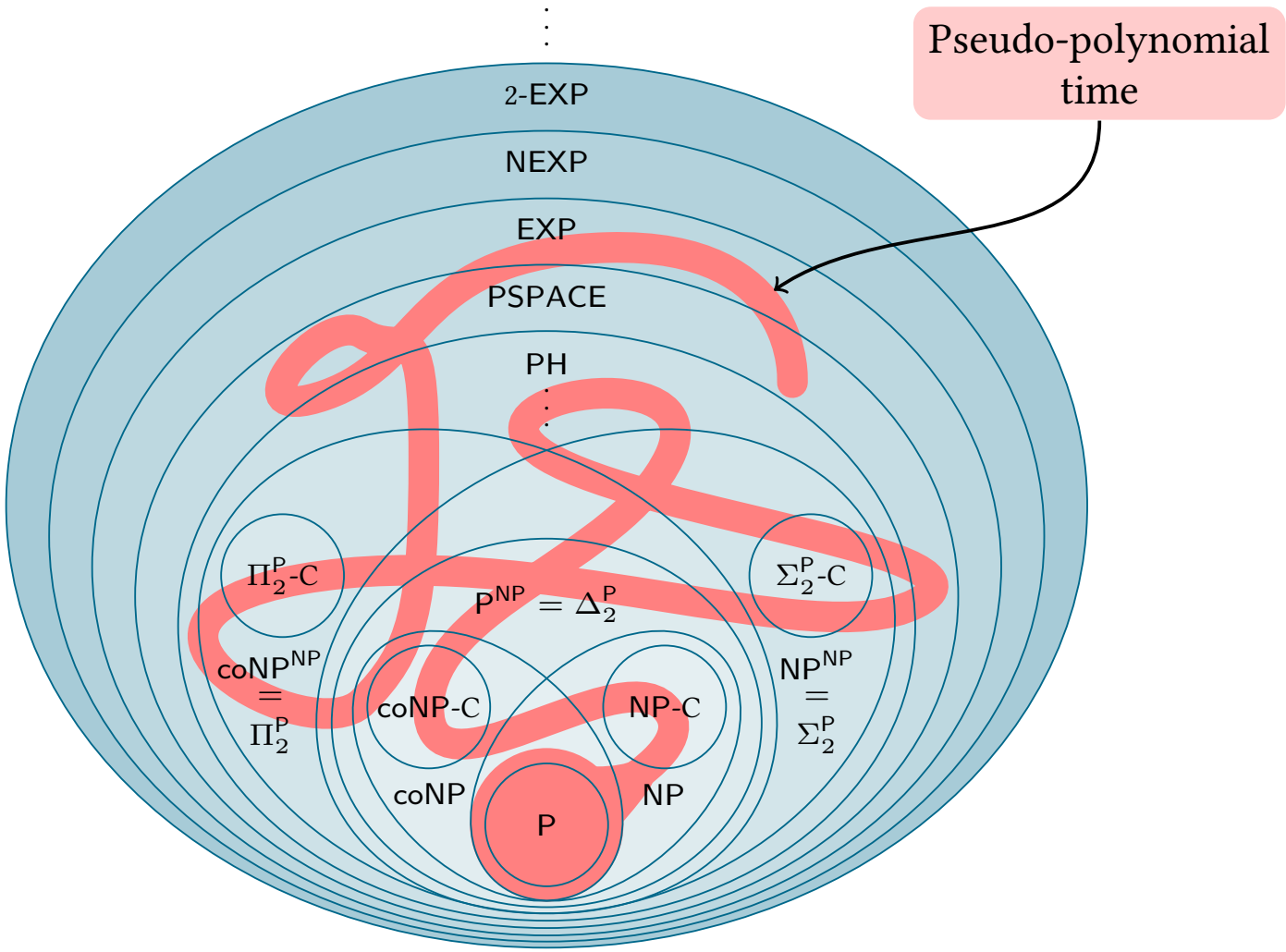

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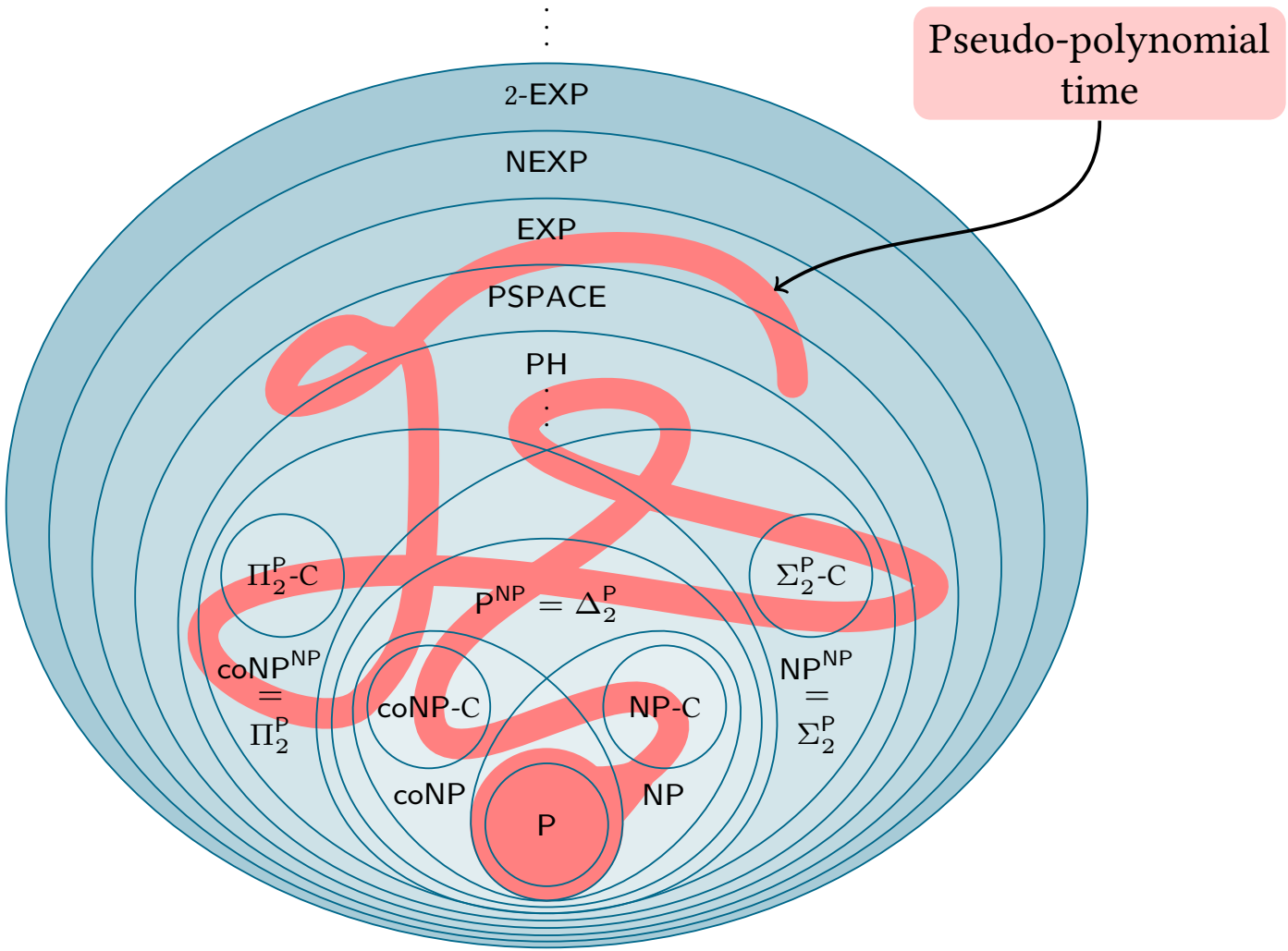

Every C-complete class has pseudo-polynomial problems if $C \subseteq EXP$ and C is closed under polynomial-time reductions.

A more fine-grained take on pseudo-polynomial time

Pseudo-polynomial time: $\mathsf{poly}(n, N)$

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- An algorithm is *pseudo-linear* if it is $\mathcal{O}(n^k \times N)$.
- An algorithm is *pseudo-quadratic* if it is $\mathcal{O}(n^k \times N^2)$.

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$$\mathsf{TIME}(\mathsf{poly}(n) \times N^a) \subset \mathsf{TIME}(\mathsf{poly}(n) \times N^b), \text{ if } 0 < a < b$$

$$\text{If } f(N) = g(N) \times \log^k(N), \text{ then pseudo-}f = \text{pseudo-}g$$

A few examples

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Response time analysis (RTA)

Processor demand analysis (PDA)

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(with optimization trick)

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A running time of $\mathsf{poly}(n, N/G)$ is *scale invariant*.

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Running times should be $\mathsf{poly}(n, N/G)$, where
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Definition

A running time of $\mathsf{poly}(n, N/G)$ is *scale invariant*.

Not all pseudo-polynomial time problems are scale-invariant.

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But several others seem to be

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1. Pseudo-polynomial problems can be found in unexpected places.
2. There is plenty of structure inside pseudo-polynomial time.
   - Significant practical consequences that should not be ignored.
   - New and interesting classification tasks!

What was this really about?

Pseudo-Intellectual Ravings on Pseudo-Polynomial Time

Sanjoy Baruah[1] and Pontus Ekberg[2]

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Abstract. Mention Wang's role in schedulability analysis – pushing the boundaries of pseudo-polynomial time

Keywords: First keyword · Second keyword · Another keyword.

1 Introduction

The **schedulability analysis** problem in one of the foundational problems studied in real-time scheduling theory; it may be described as follows:

> GIVEN (i) the specifications of the computational demands of, and the timing constraints upon, a workload; (ii) the platform upon which this workload is to be implemented; and (iii) the run-time resource allocation and scheduling

$\forall$Thank you!

$\exists$Questions?