Fixed-Priority Schedulability of Sporadic Tasks on Uniprocessors is NP-hard

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Uppsala University

RTSS 2017

Overview

	Implicit deadlines ($d = p$)	Constrained deadlines ($d \leqslant p$)	Arbitrary deadlines (d, p unrelated)
Arbitrary utilization			
Utilization bounded by a constant c			

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(*) Joseph and Pandya, 1986

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- Bounded utilization

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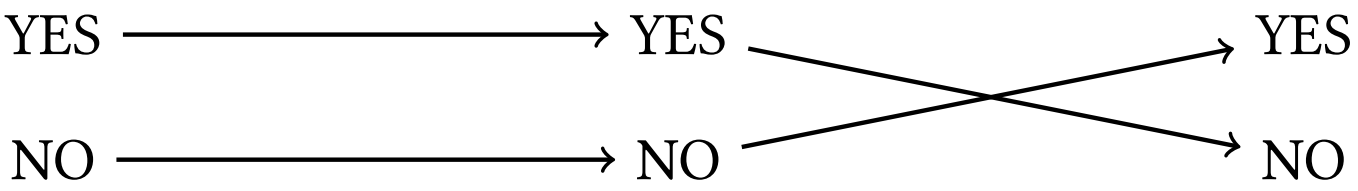

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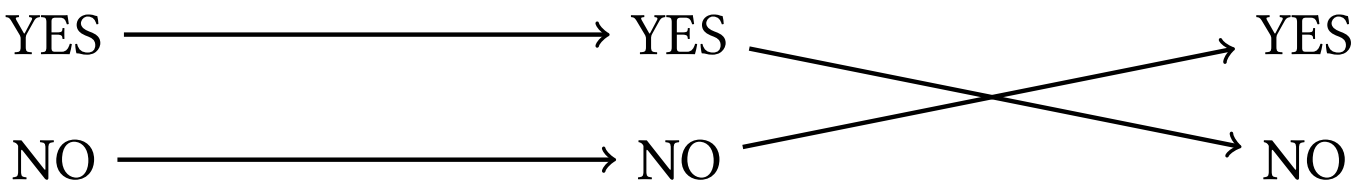

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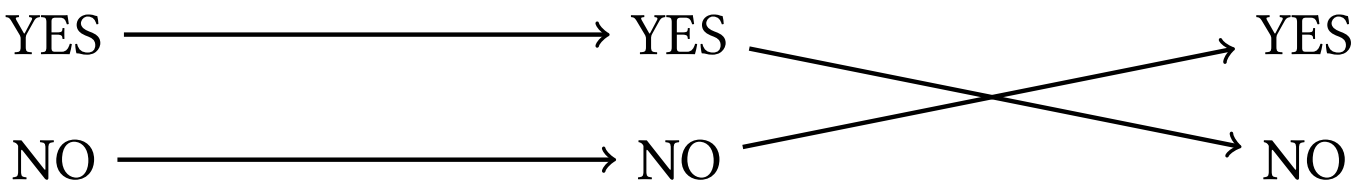

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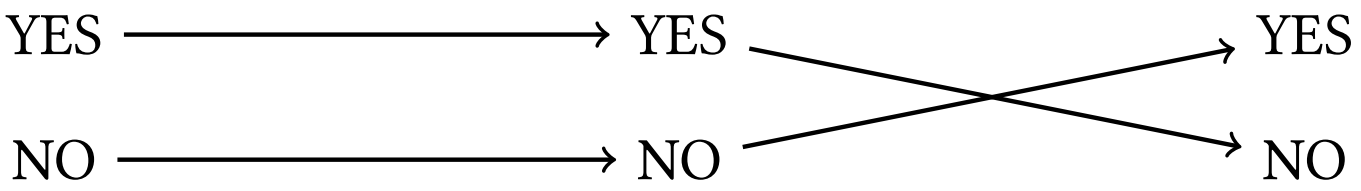

coNP-hard | coNP-hard | NP-hard

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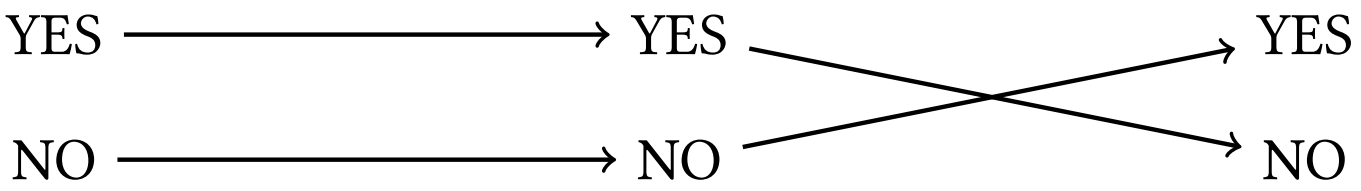

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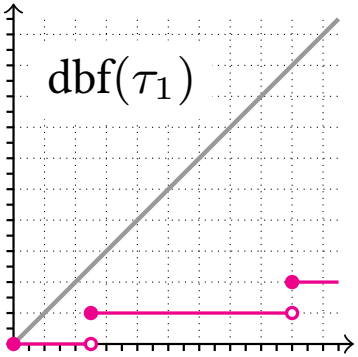

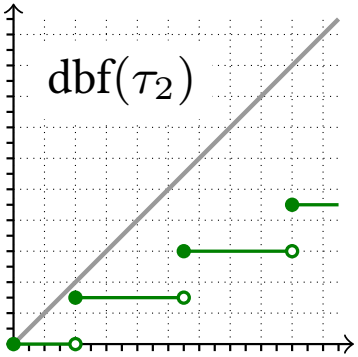

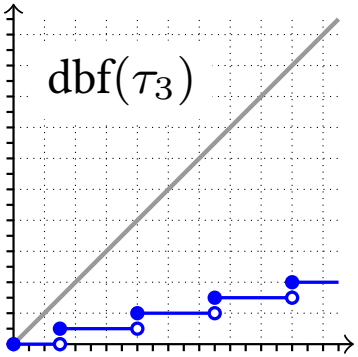

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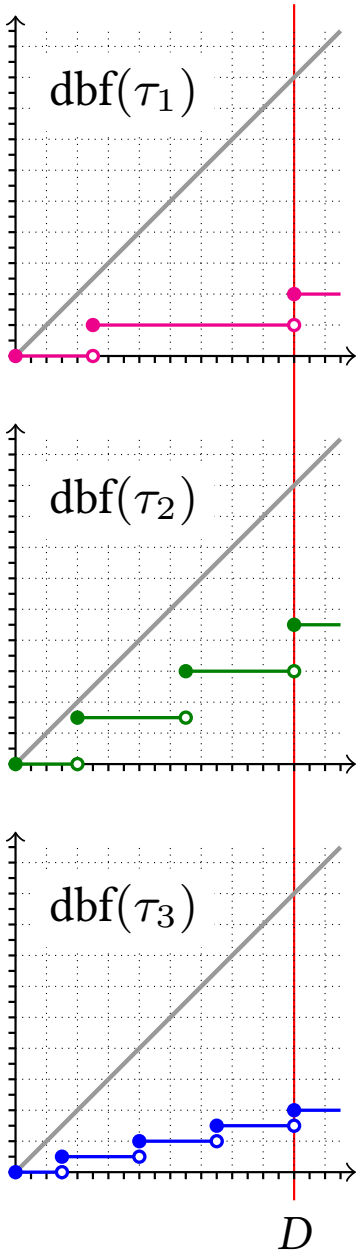

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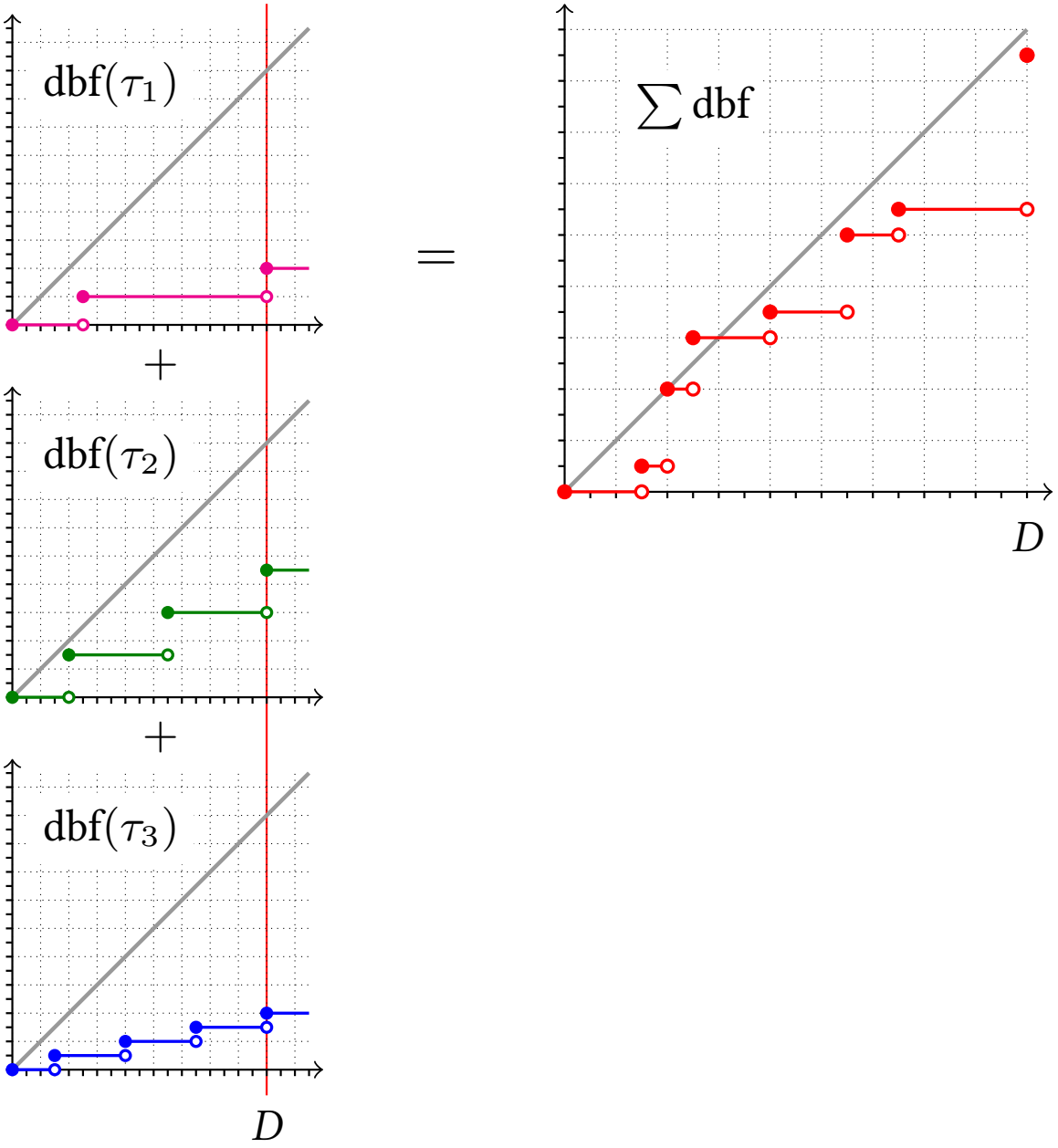

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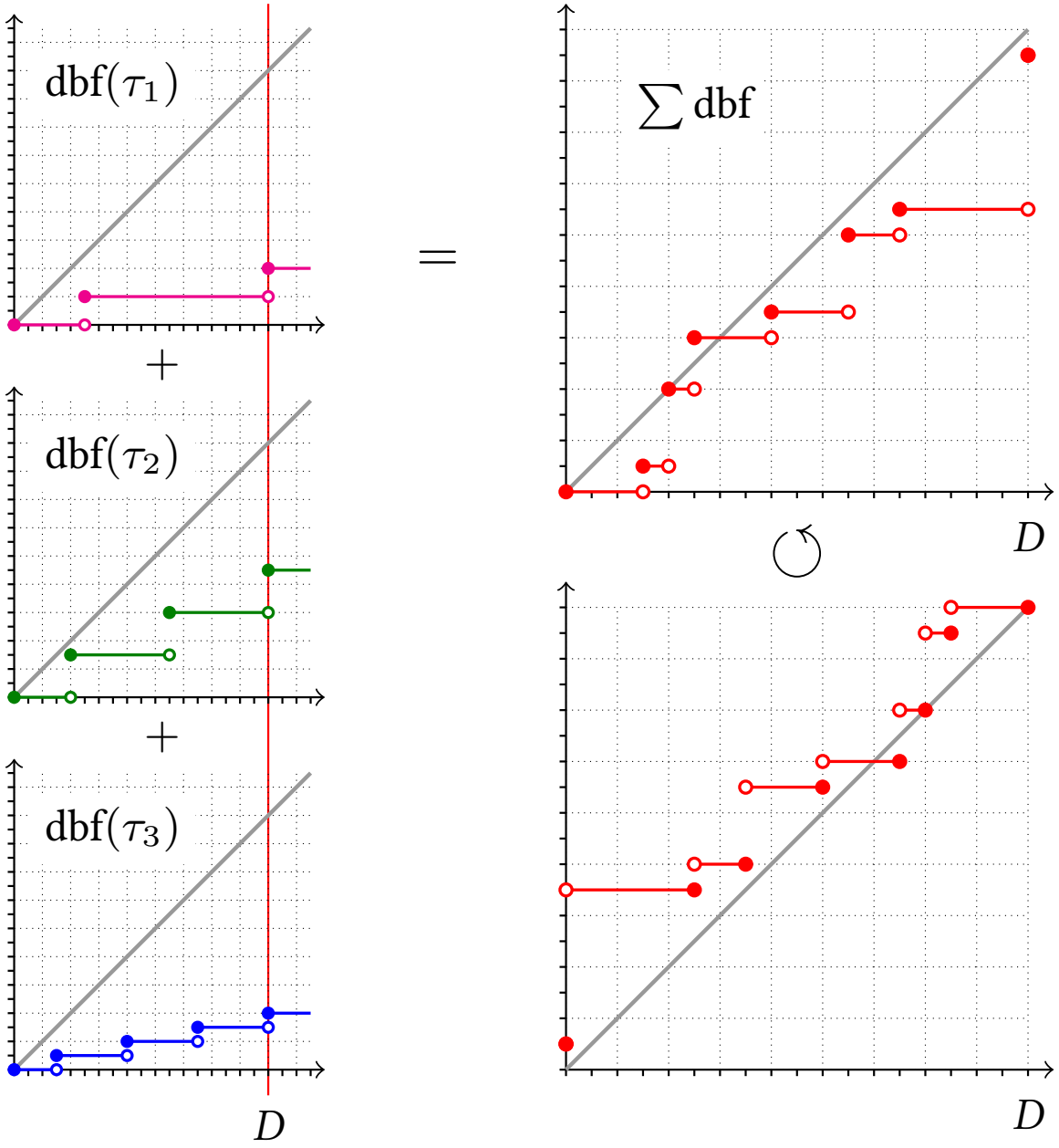

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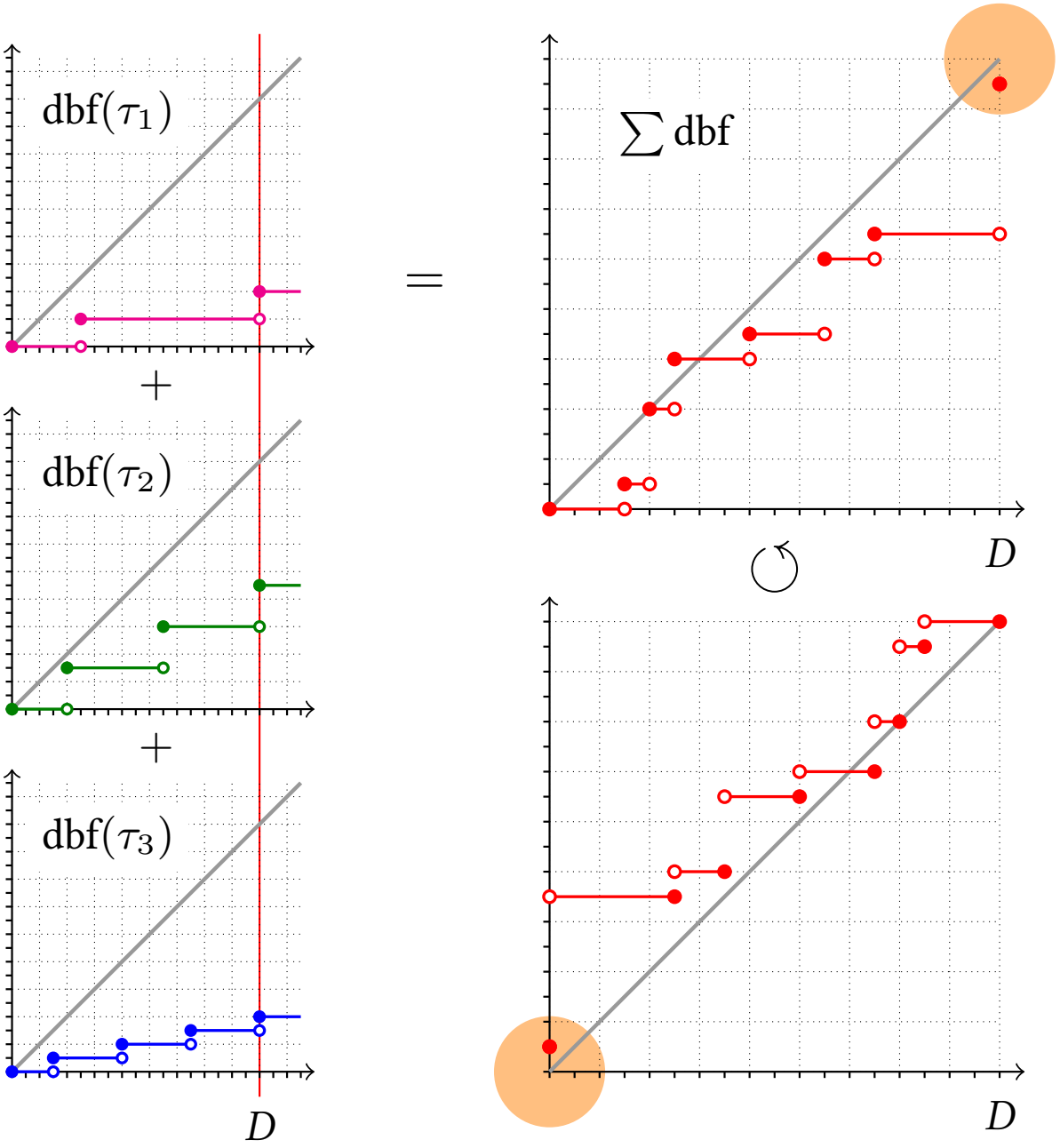

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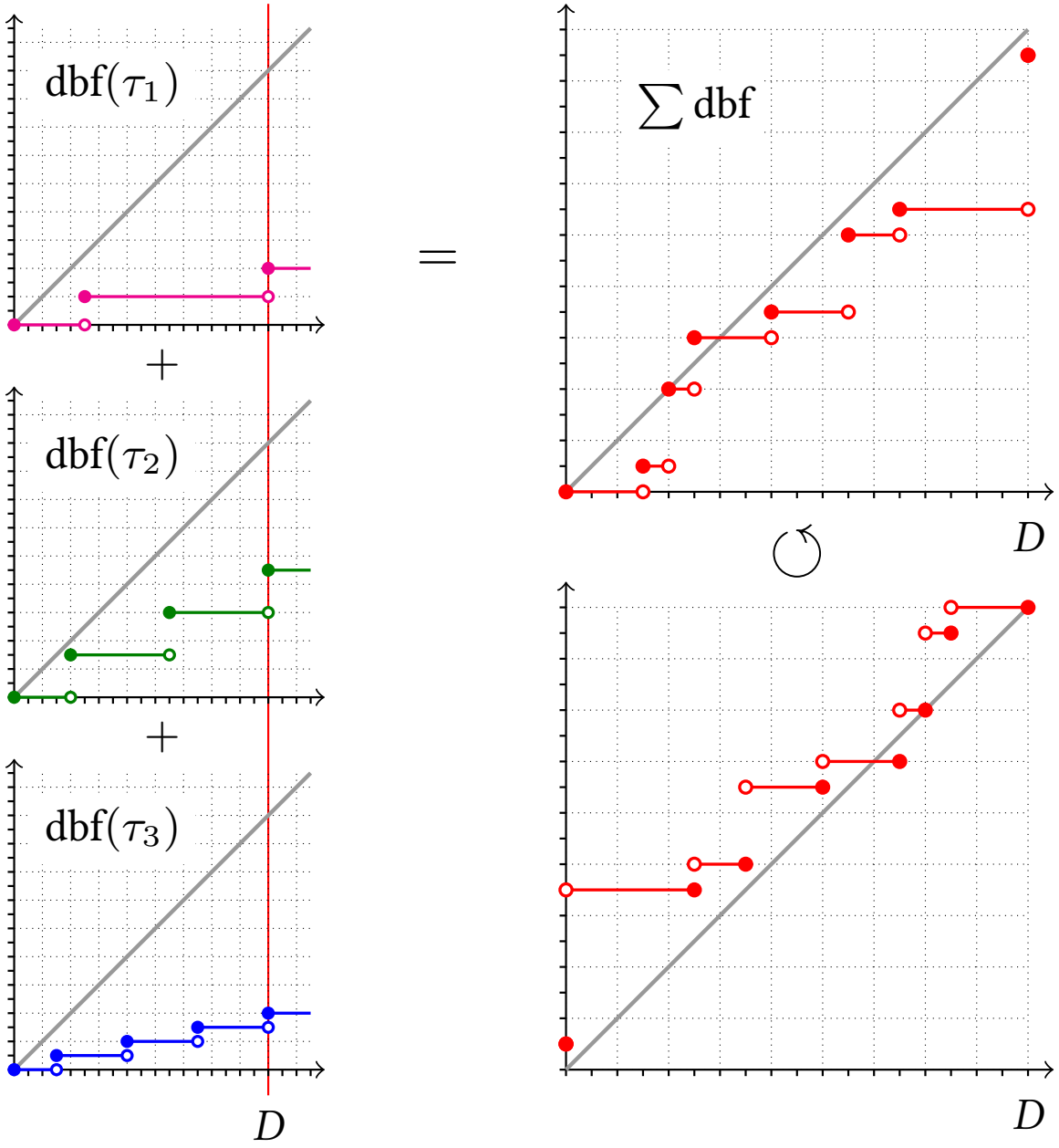

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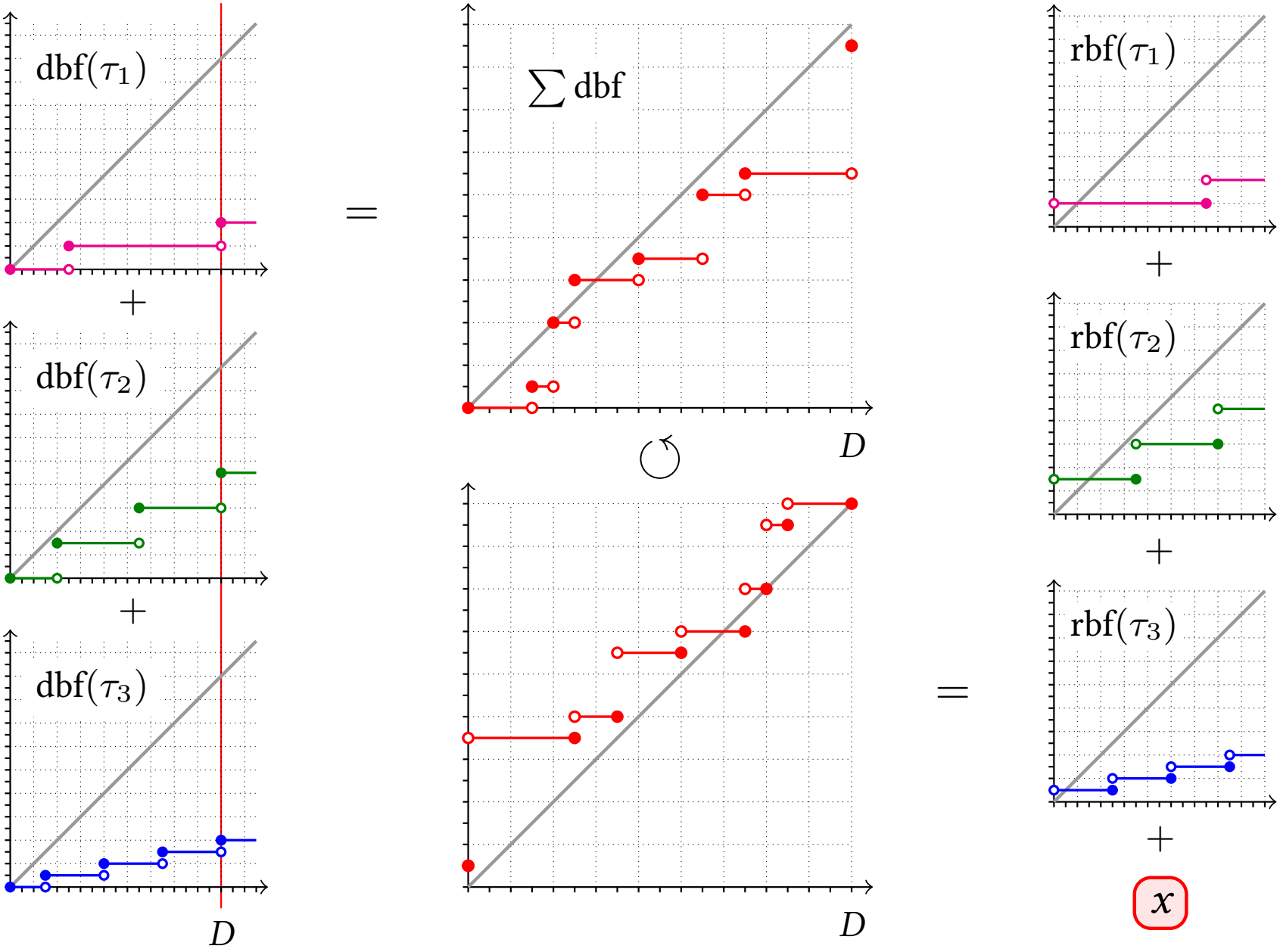

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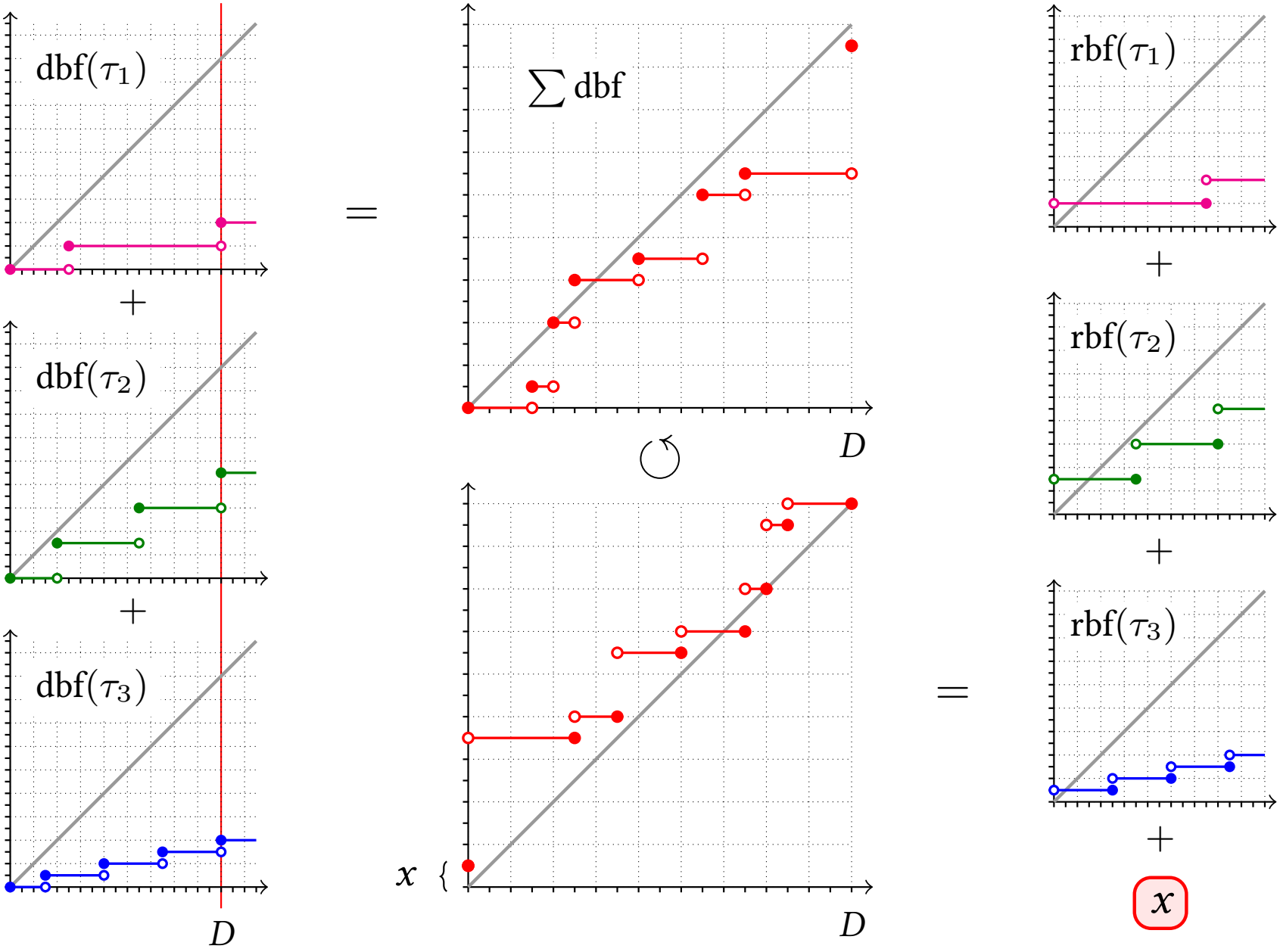

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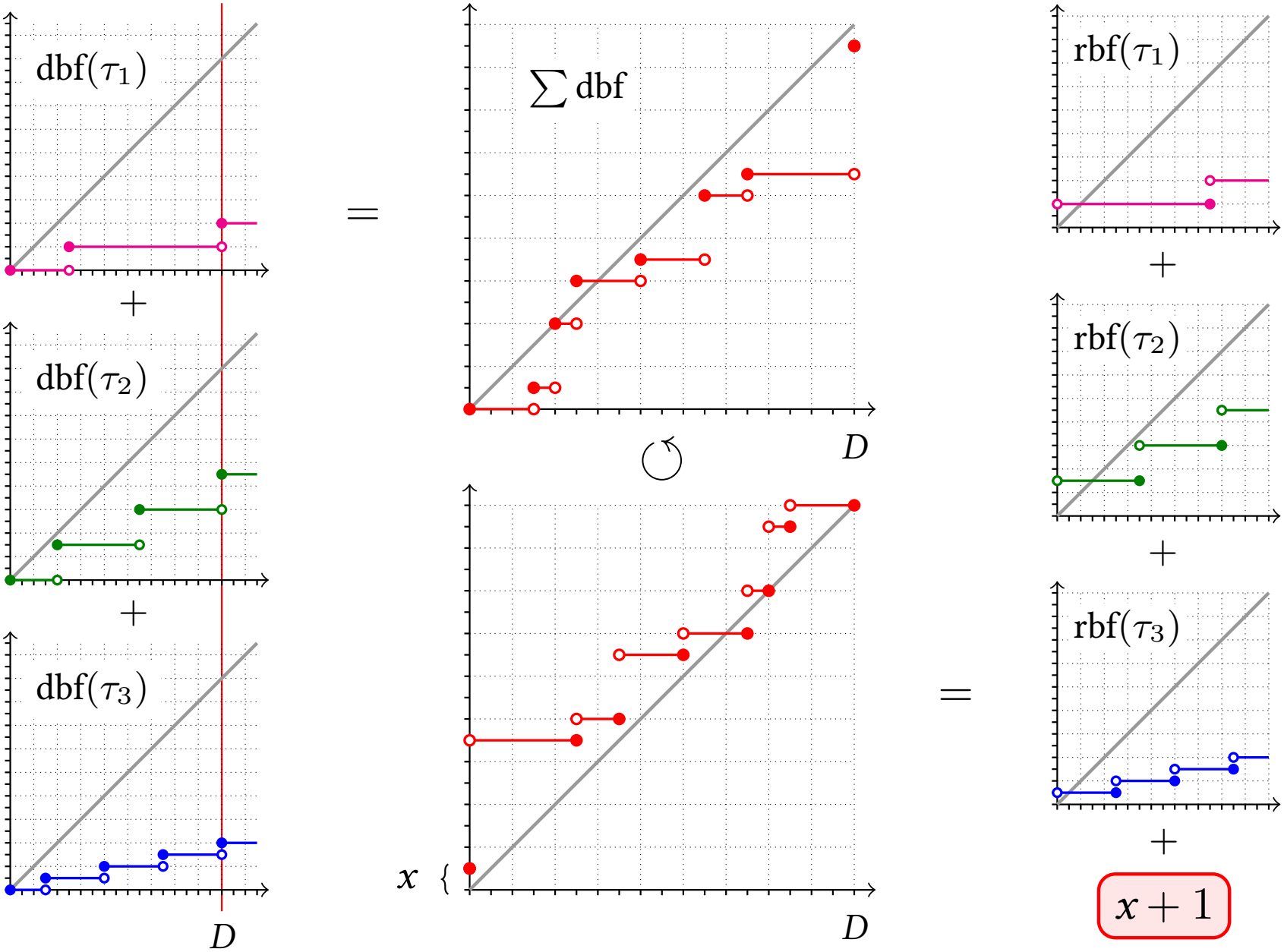

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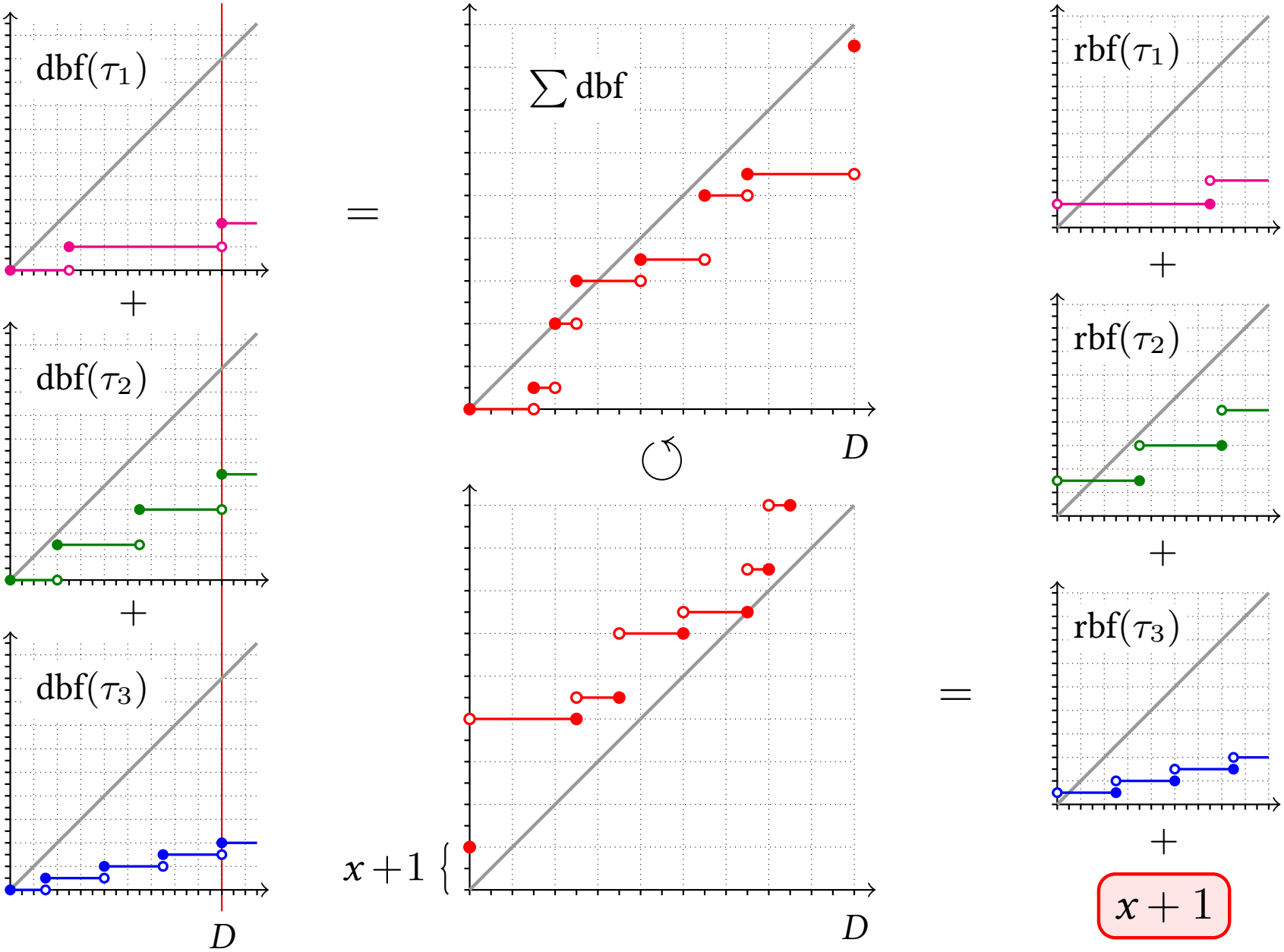

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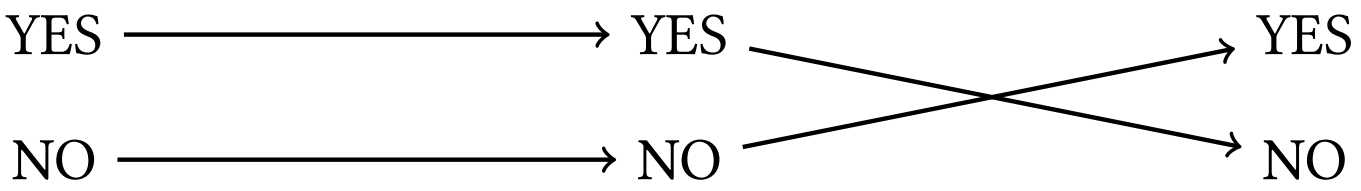

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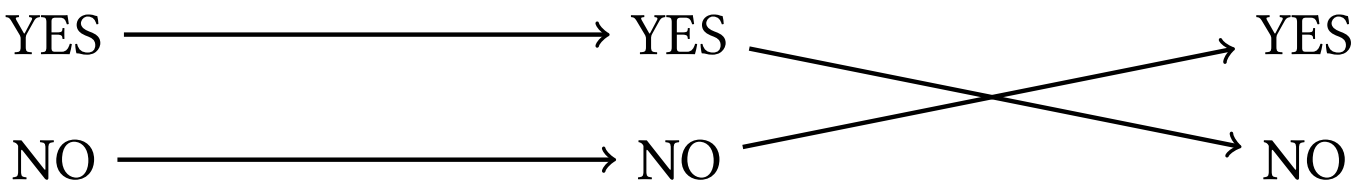

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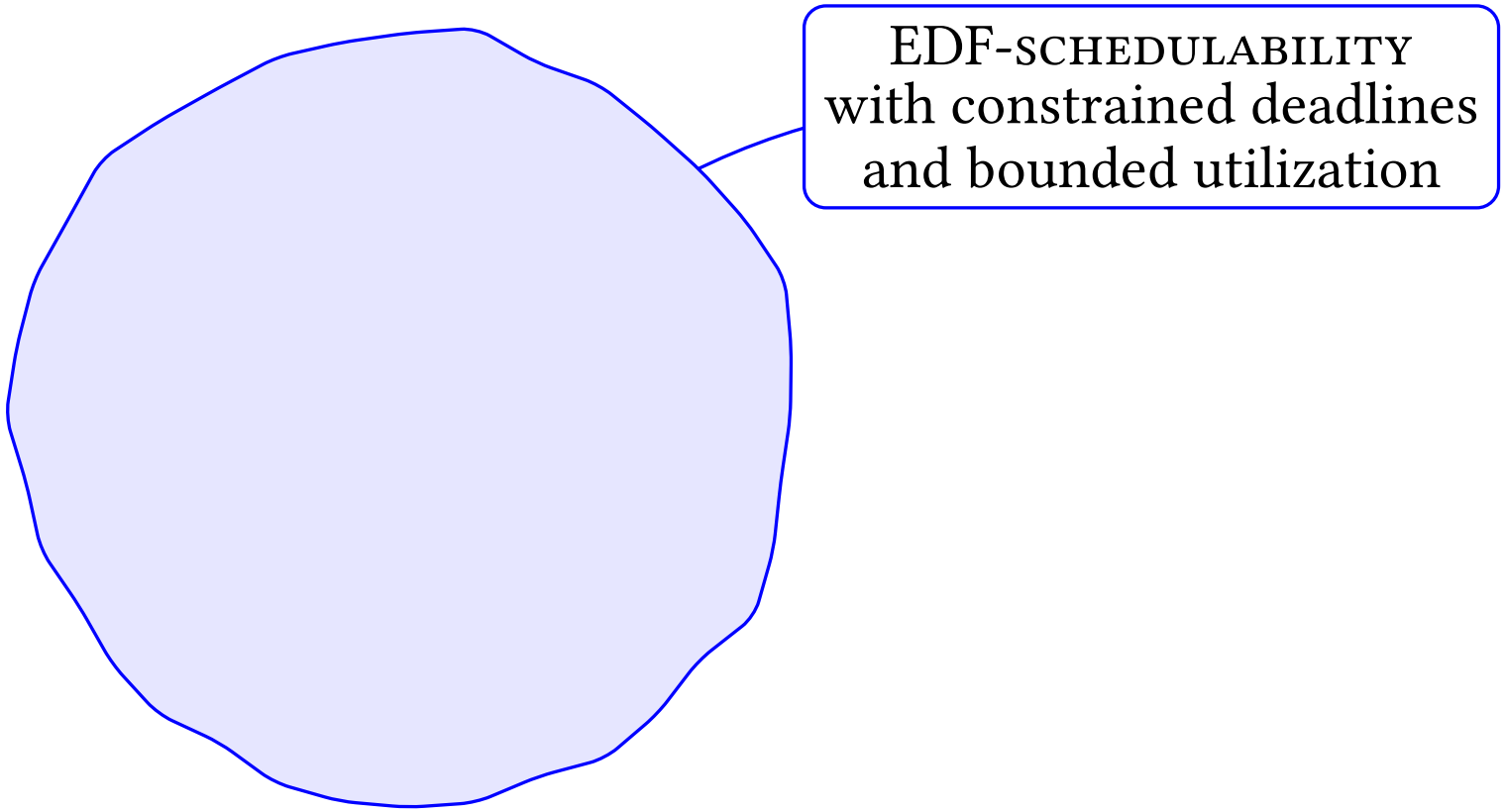

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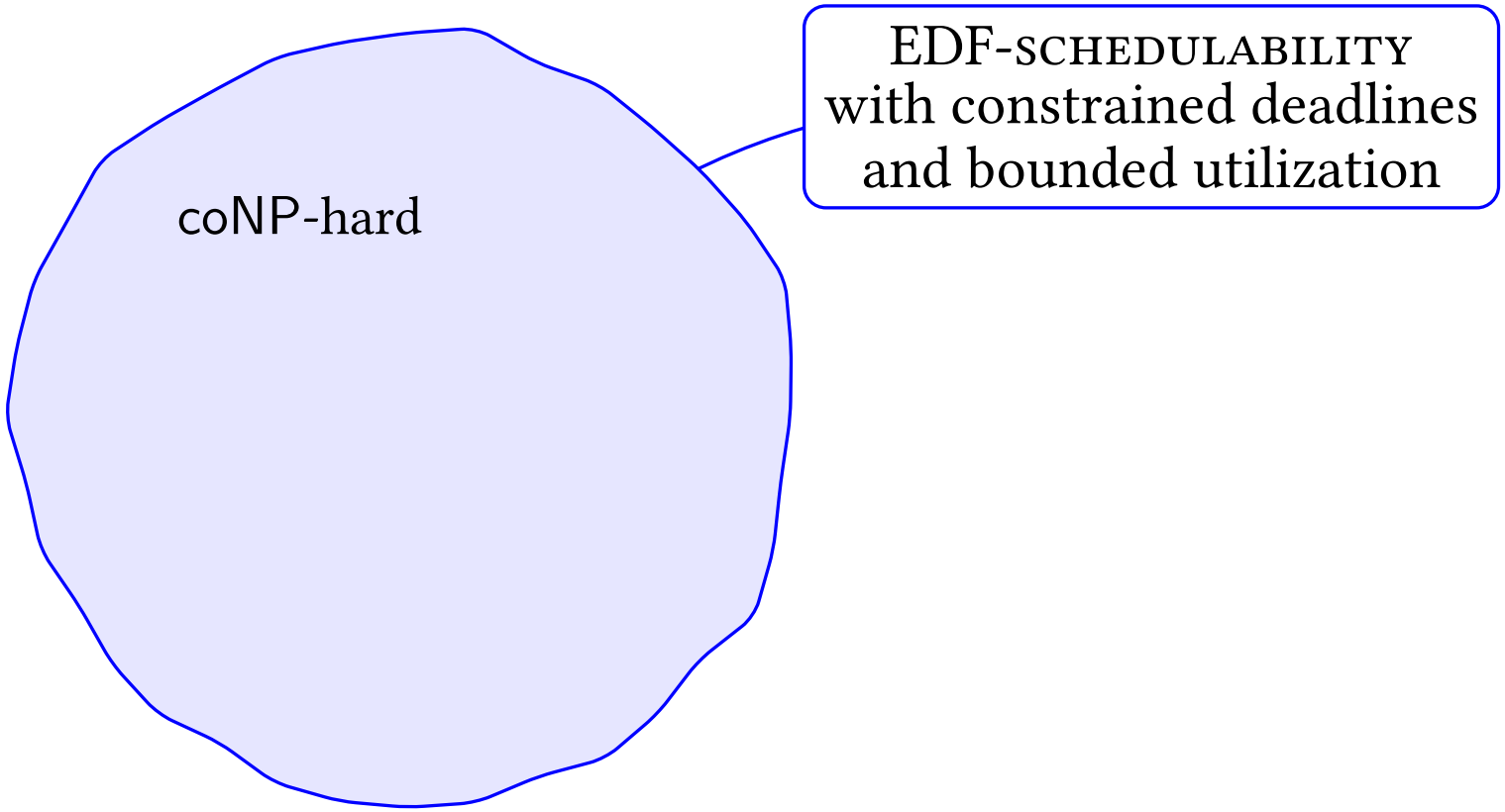

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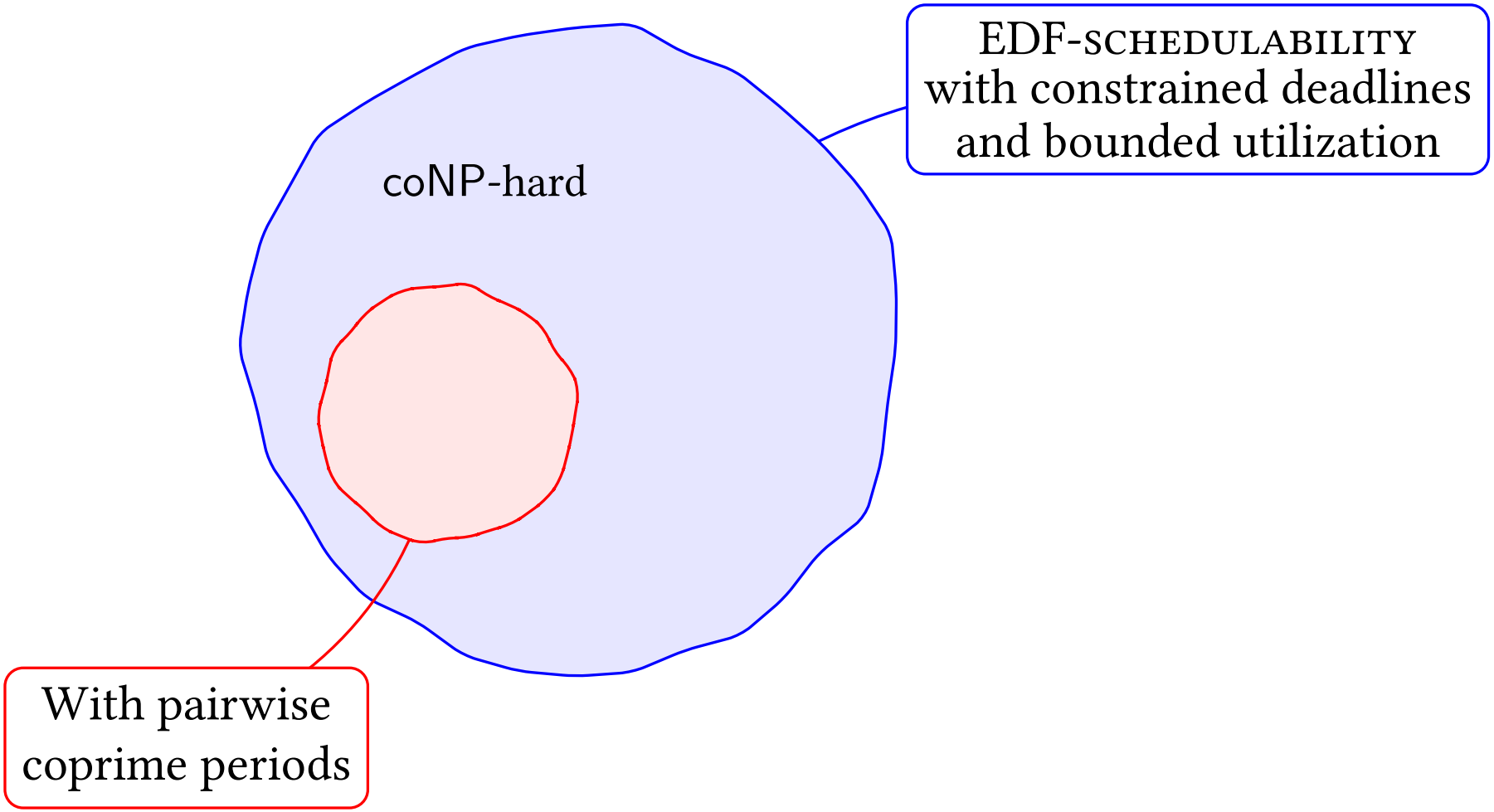

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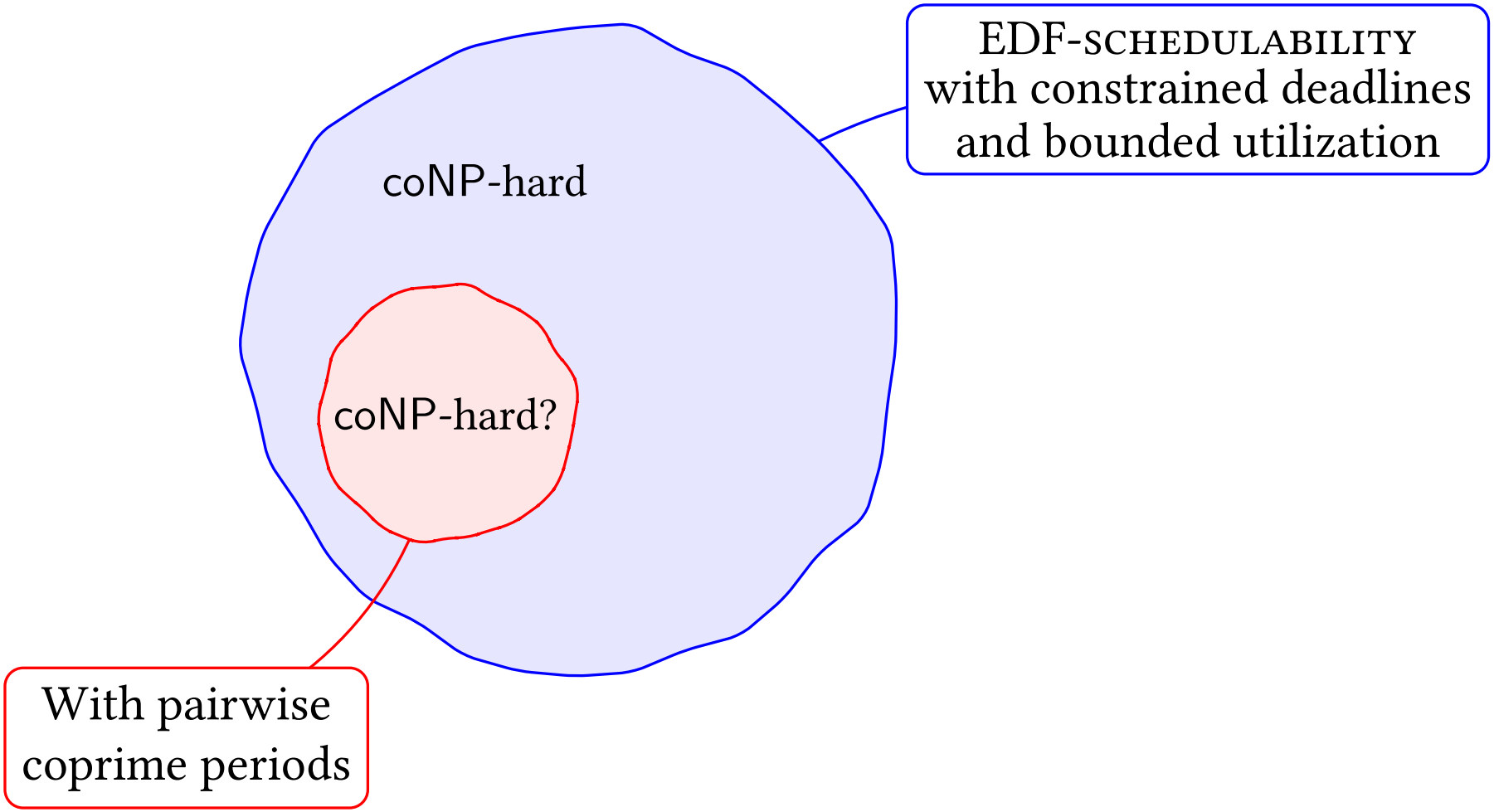

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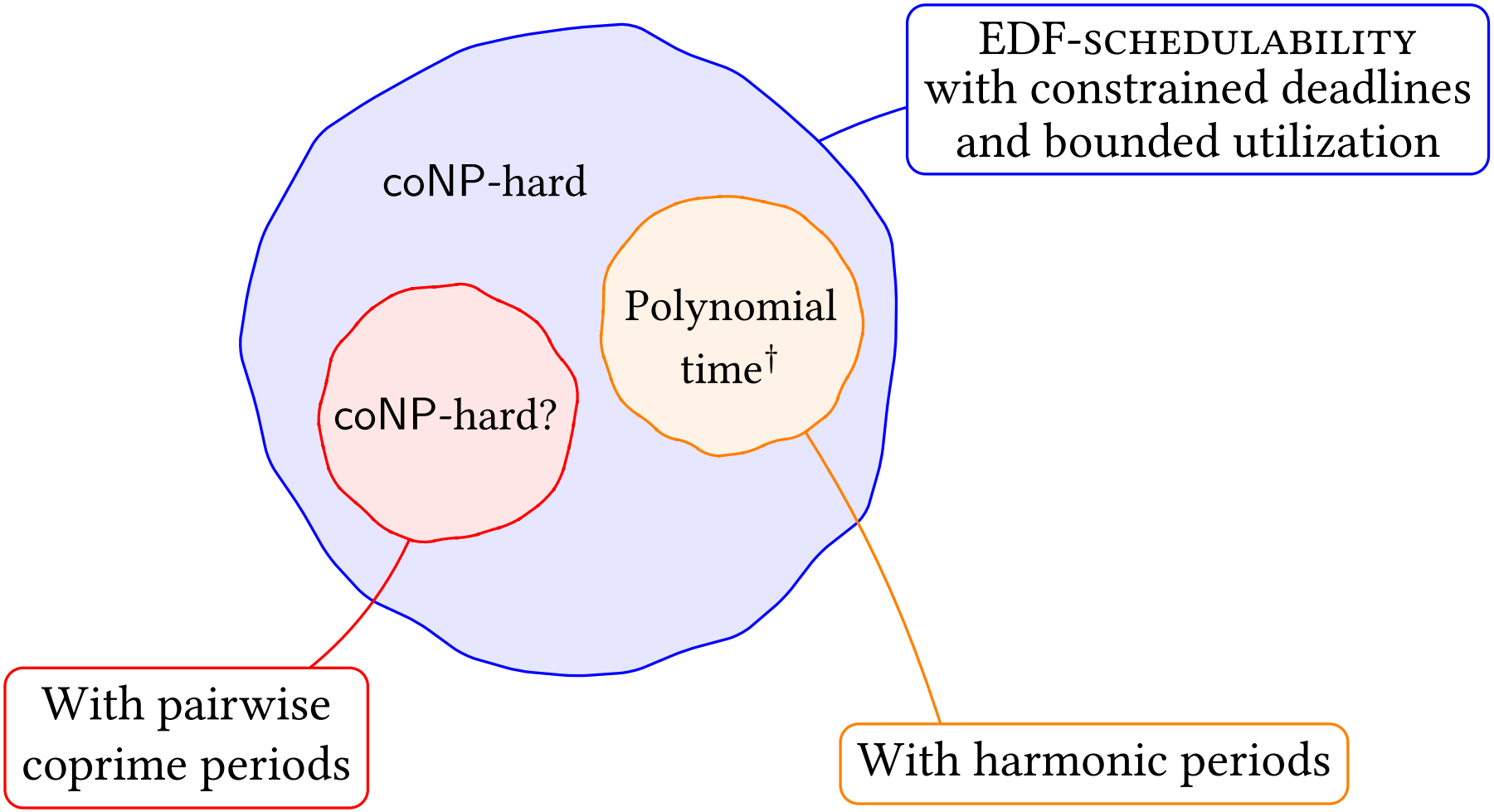

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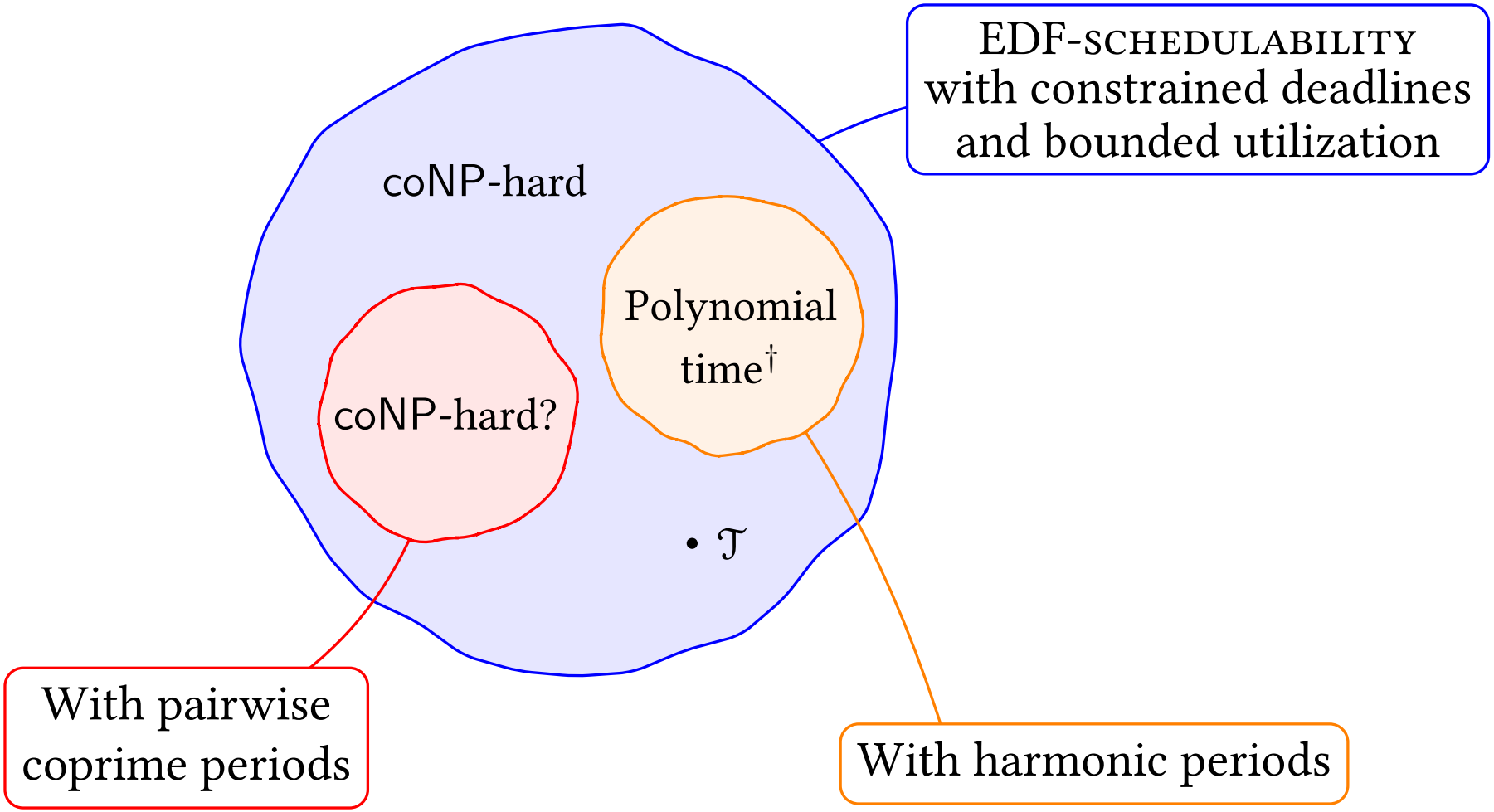

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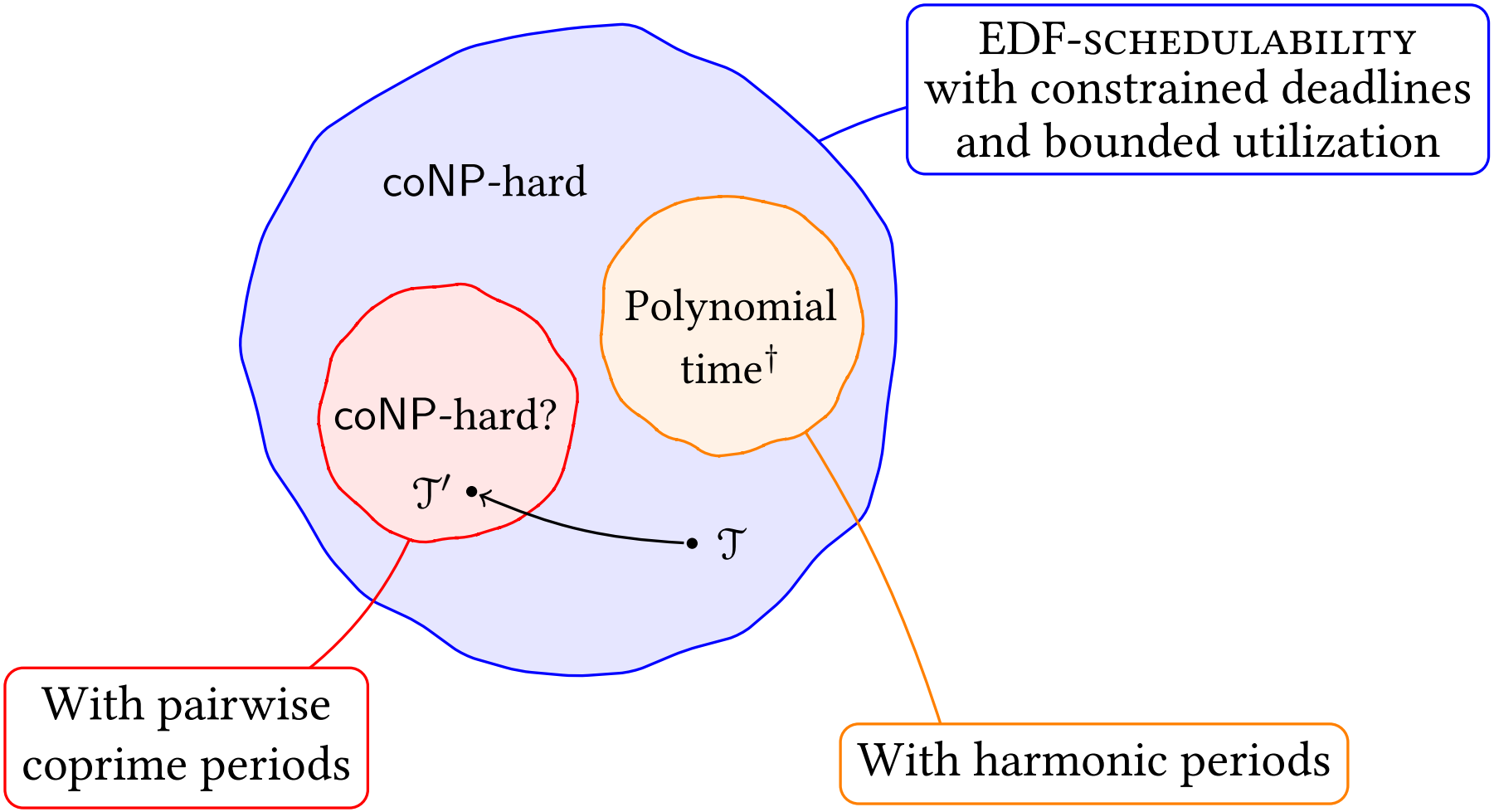

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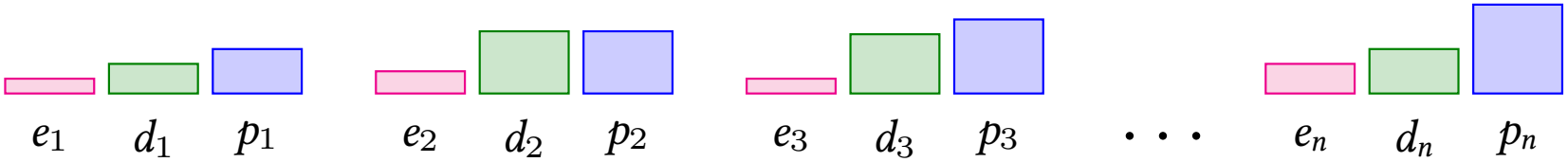
Outline of the Reduction



Outline of the Reduction

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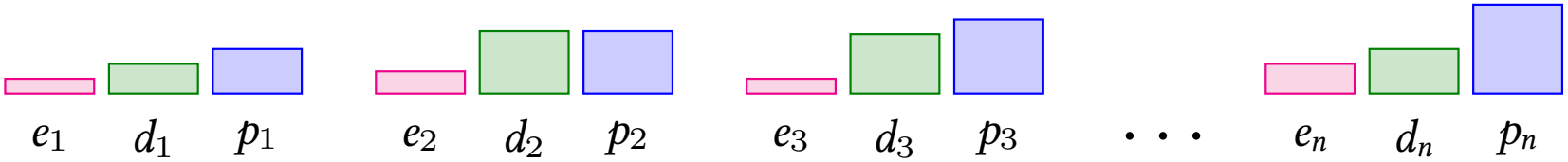

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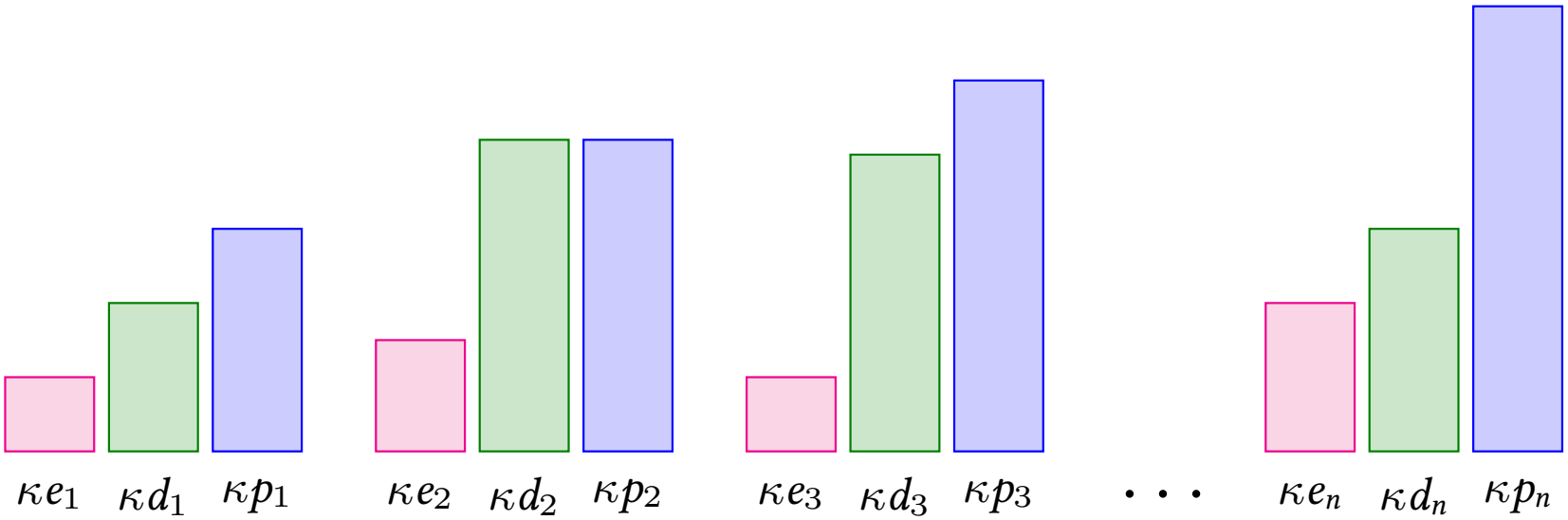

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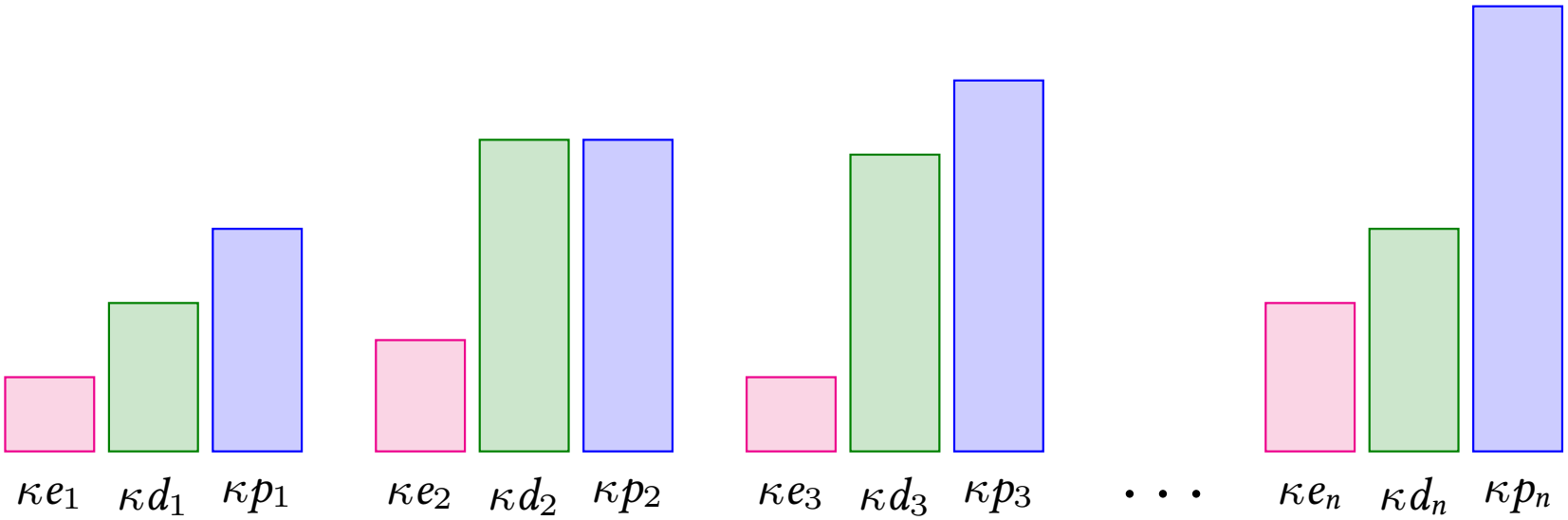

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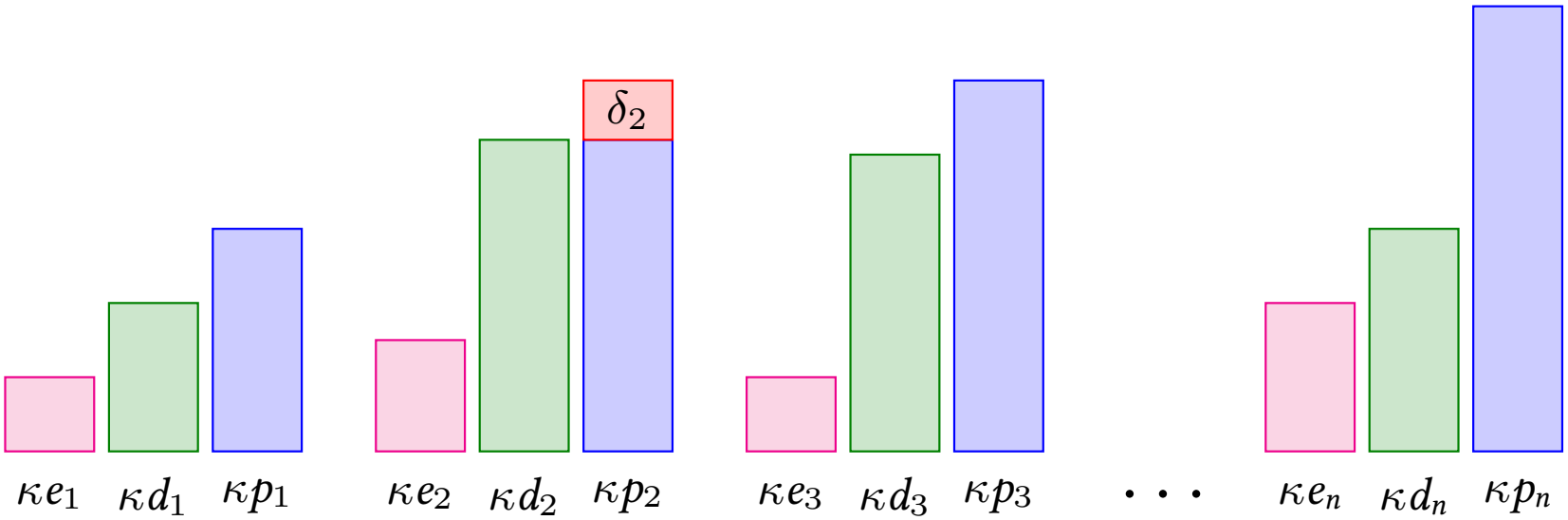

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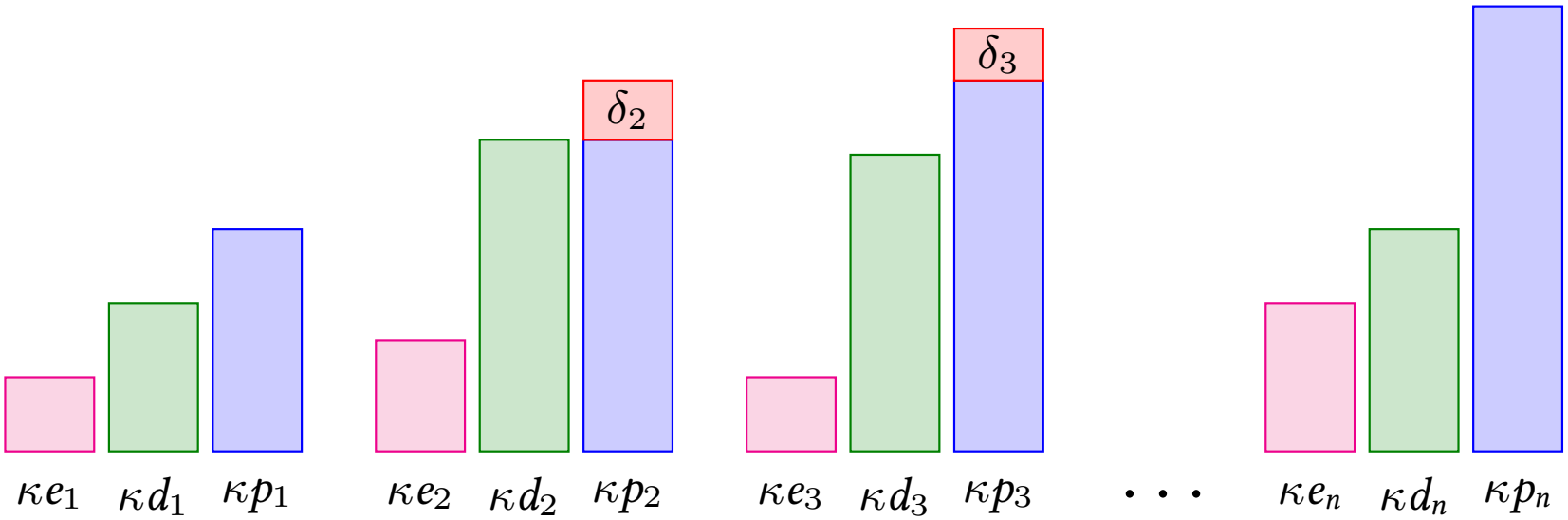

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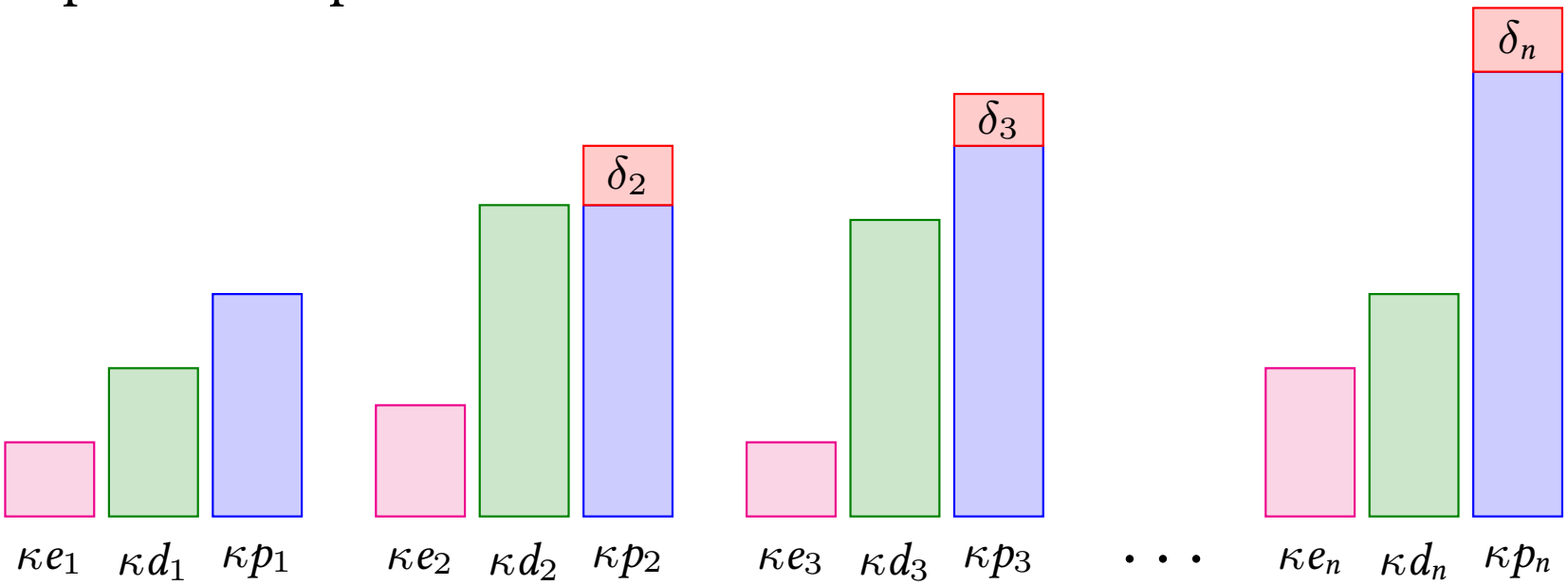

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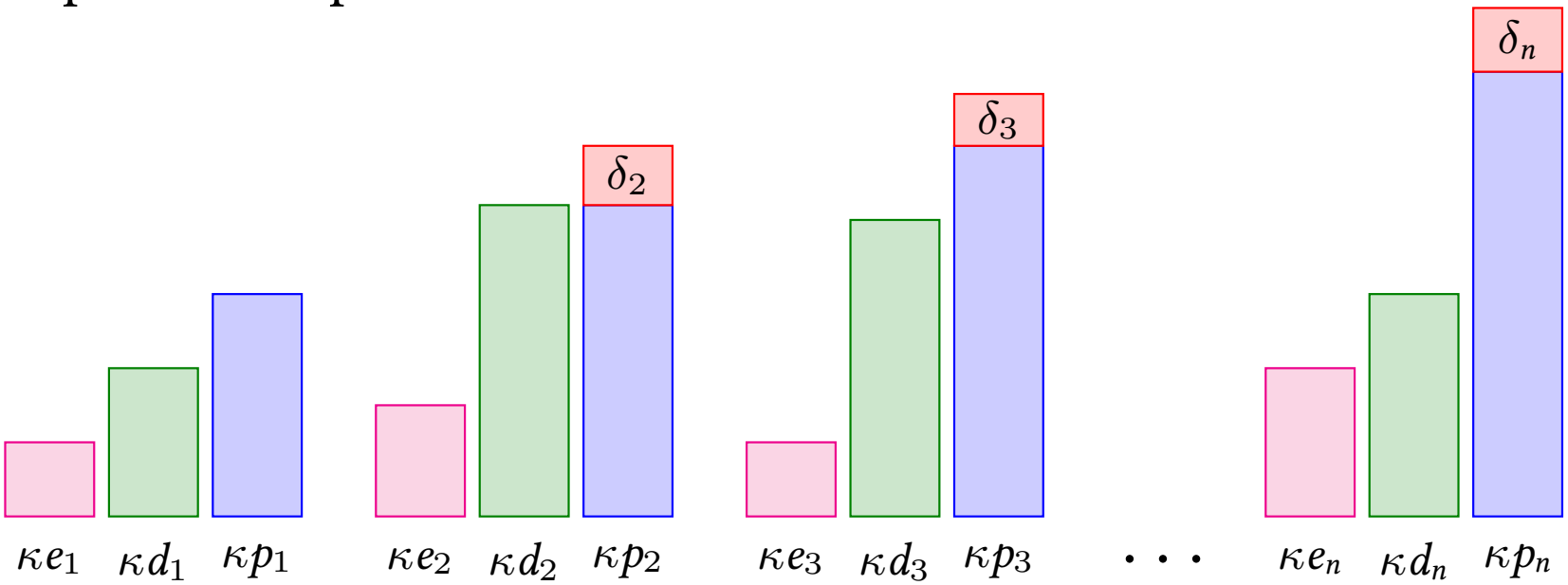

- The δ_i can be found in polynomial time.

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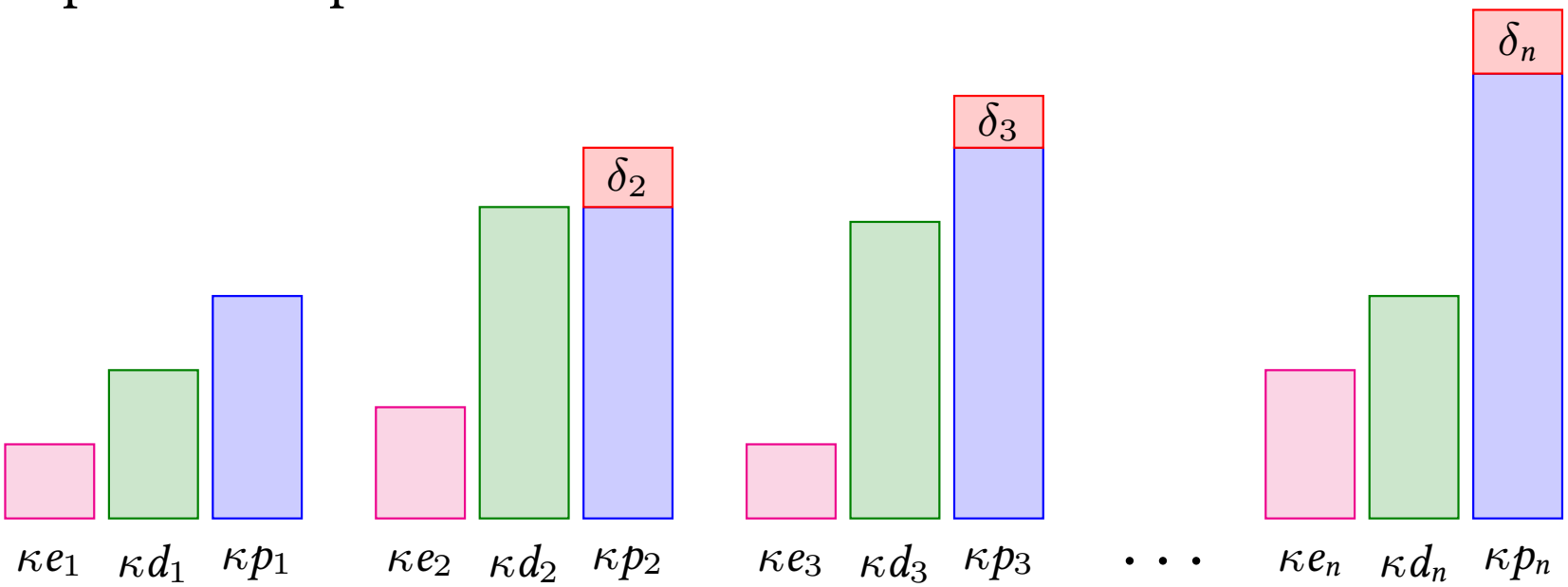

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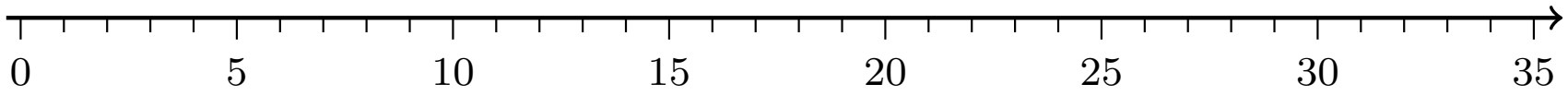

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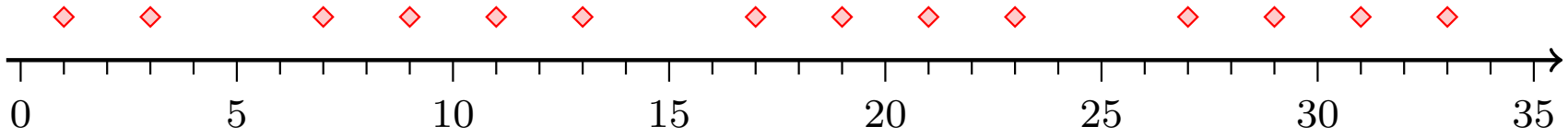

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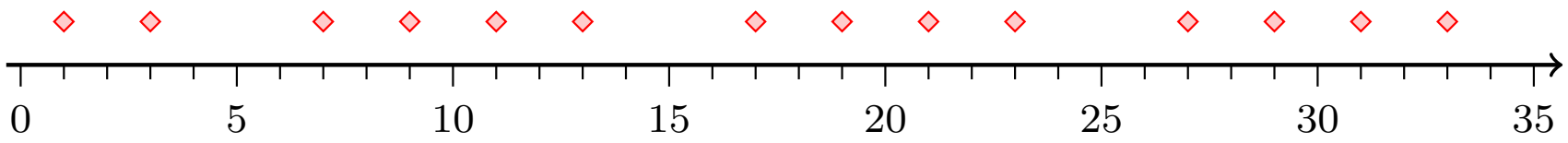

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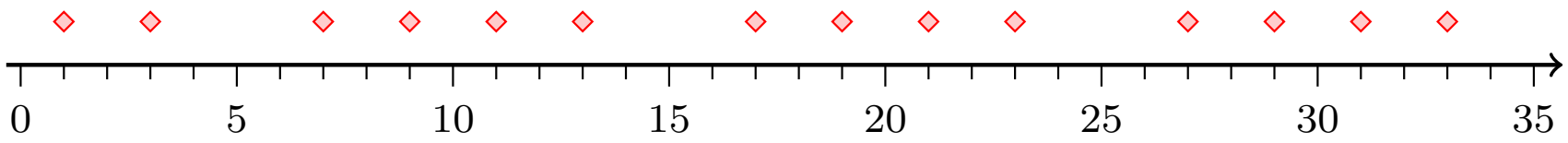

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Iwaniec, 1978

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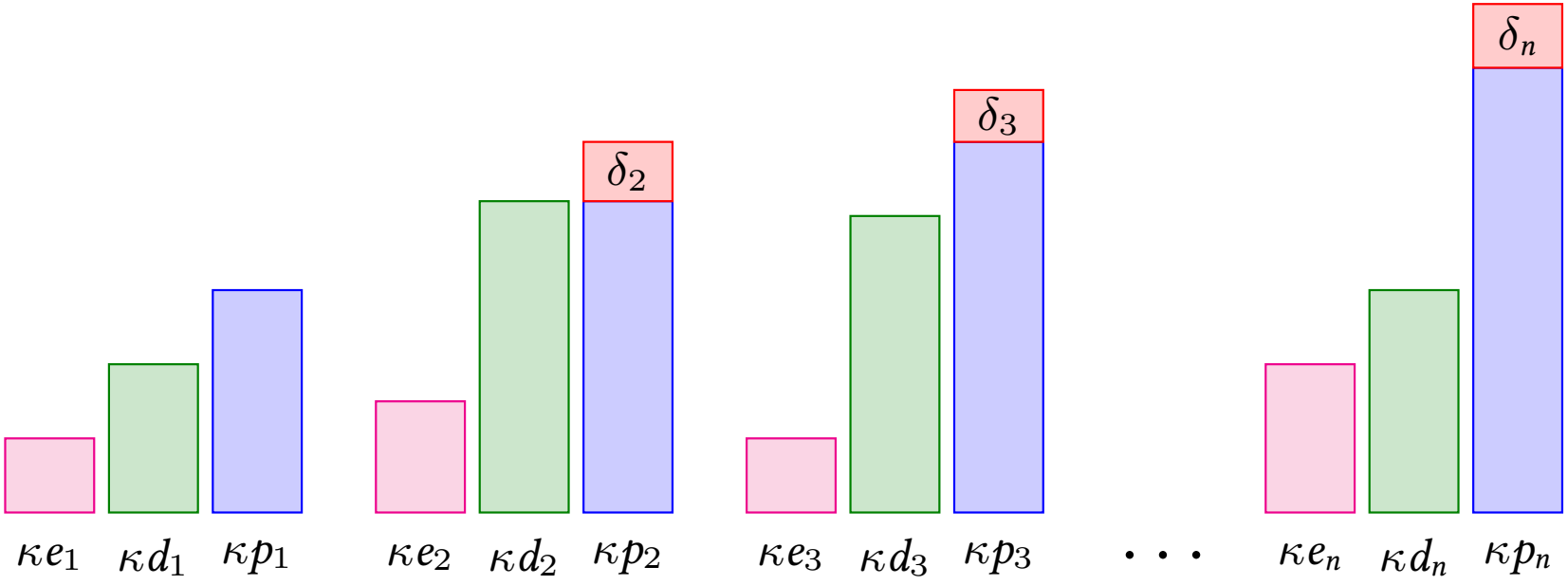

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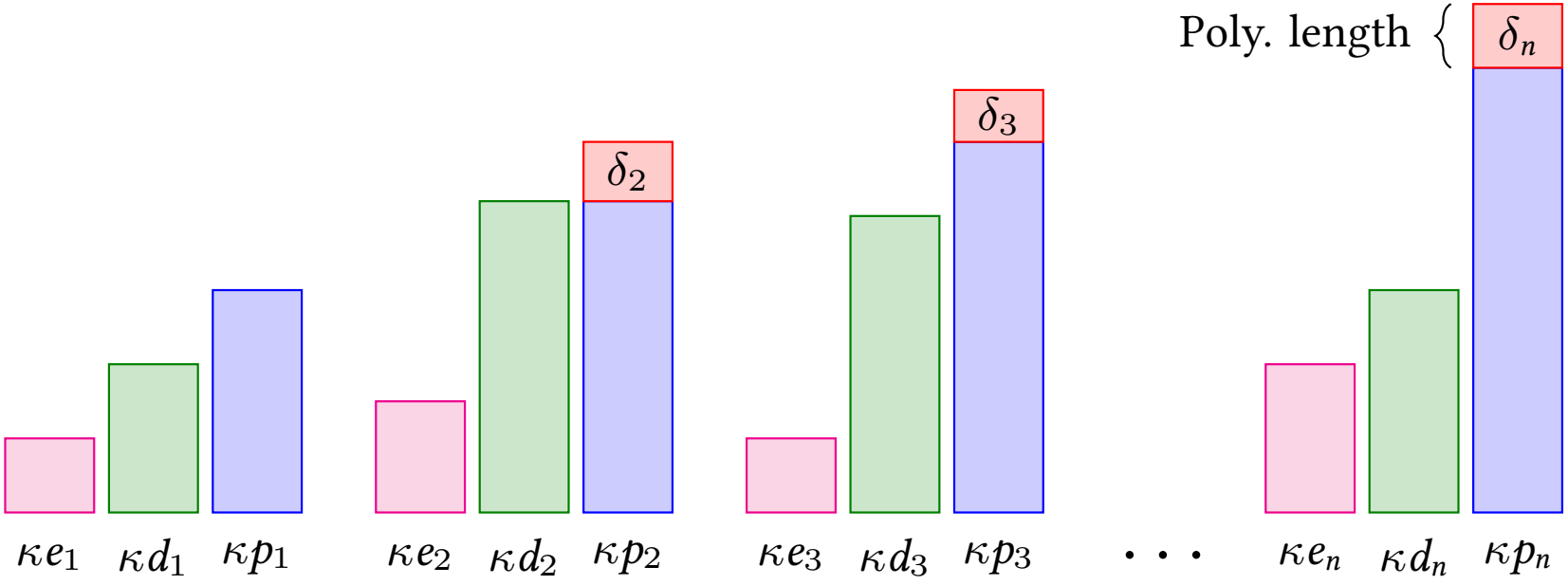

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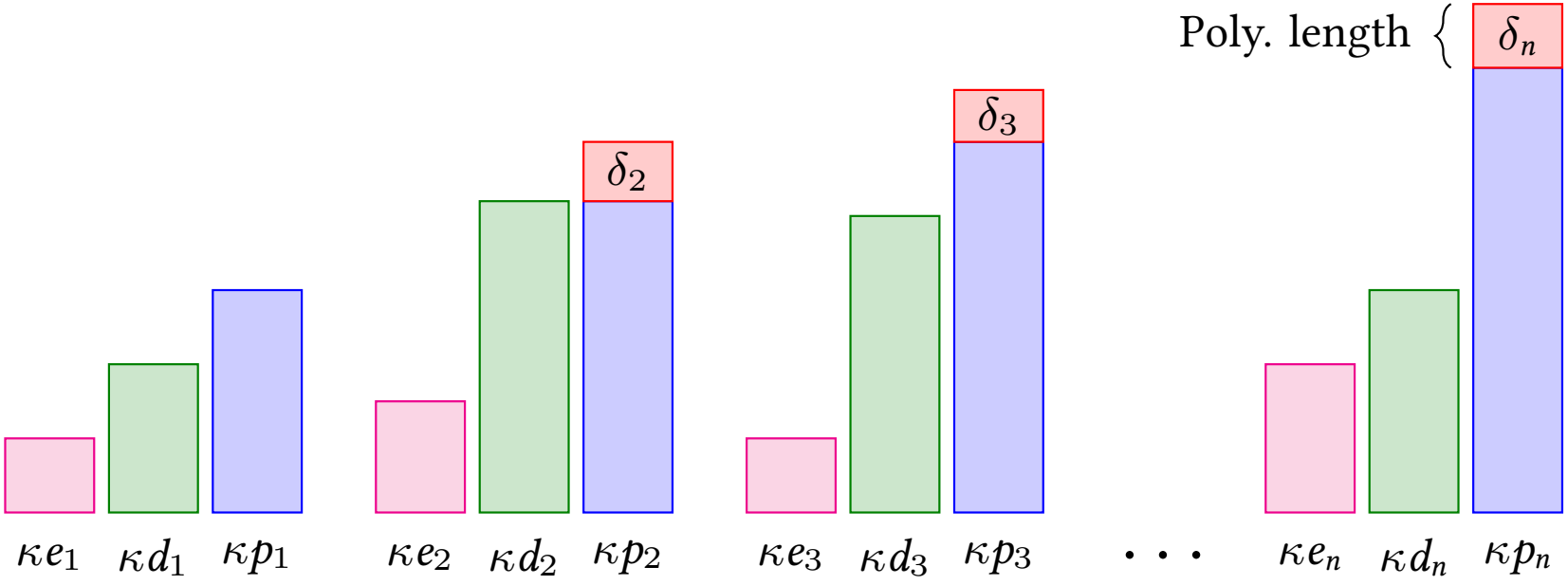

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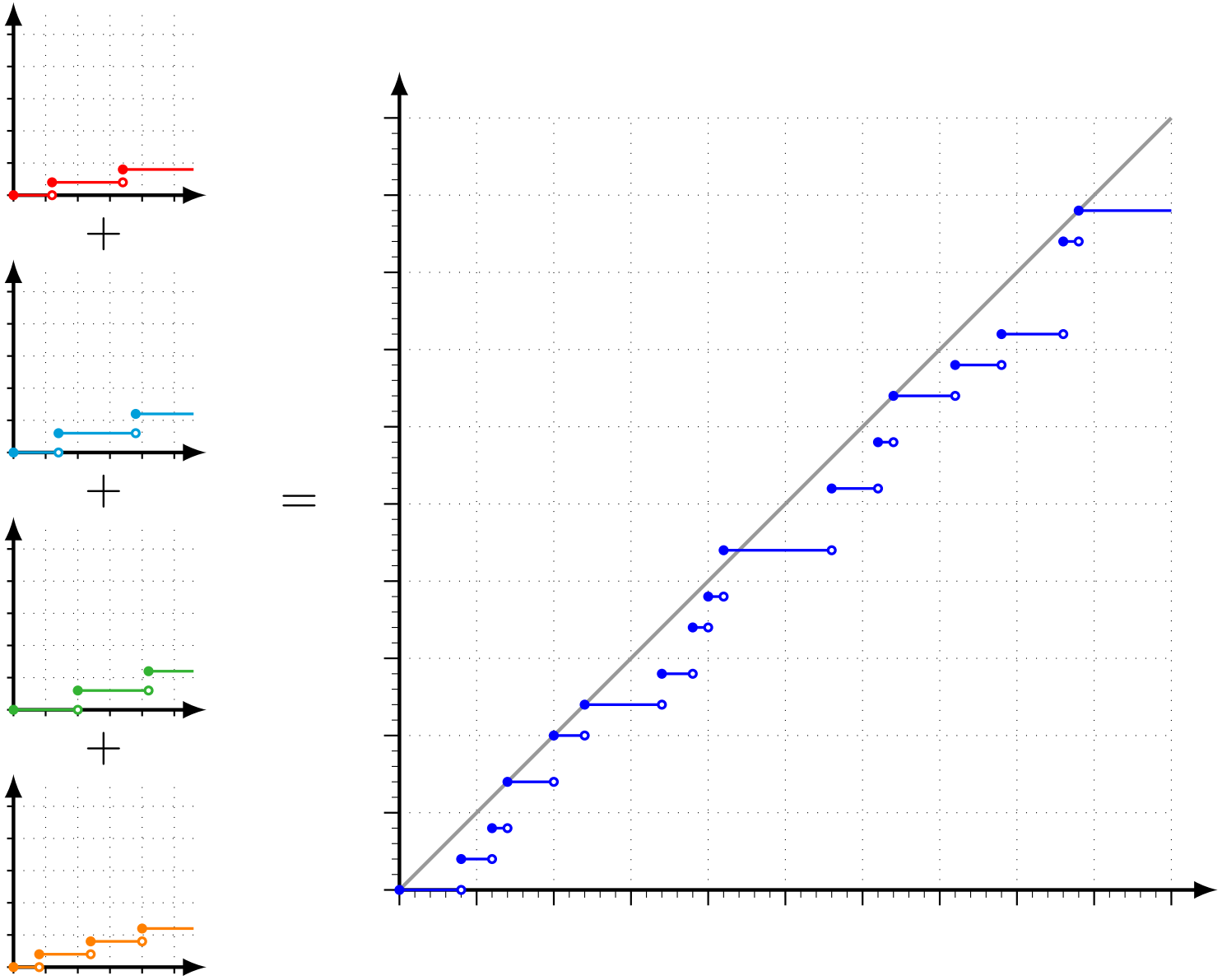

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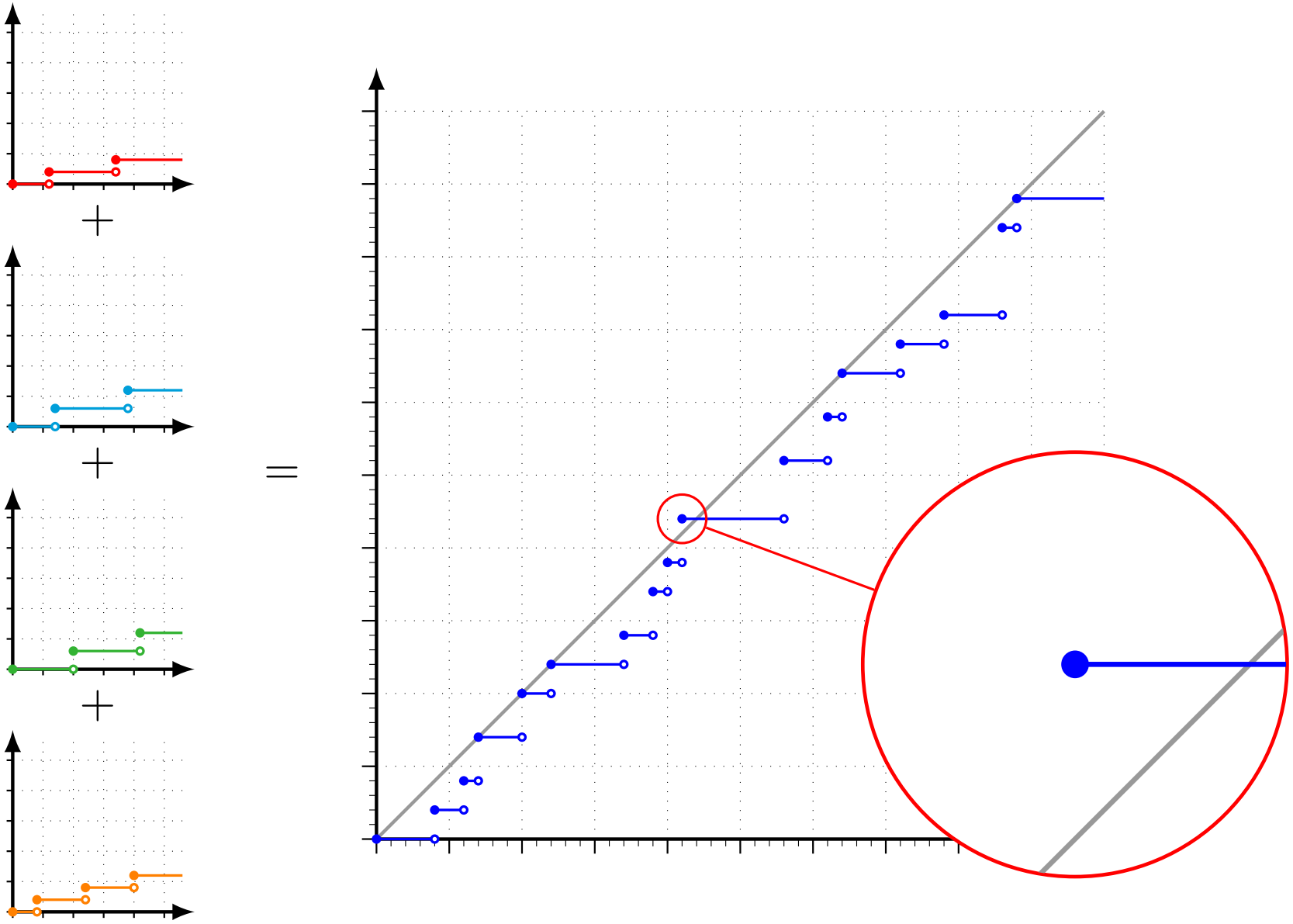

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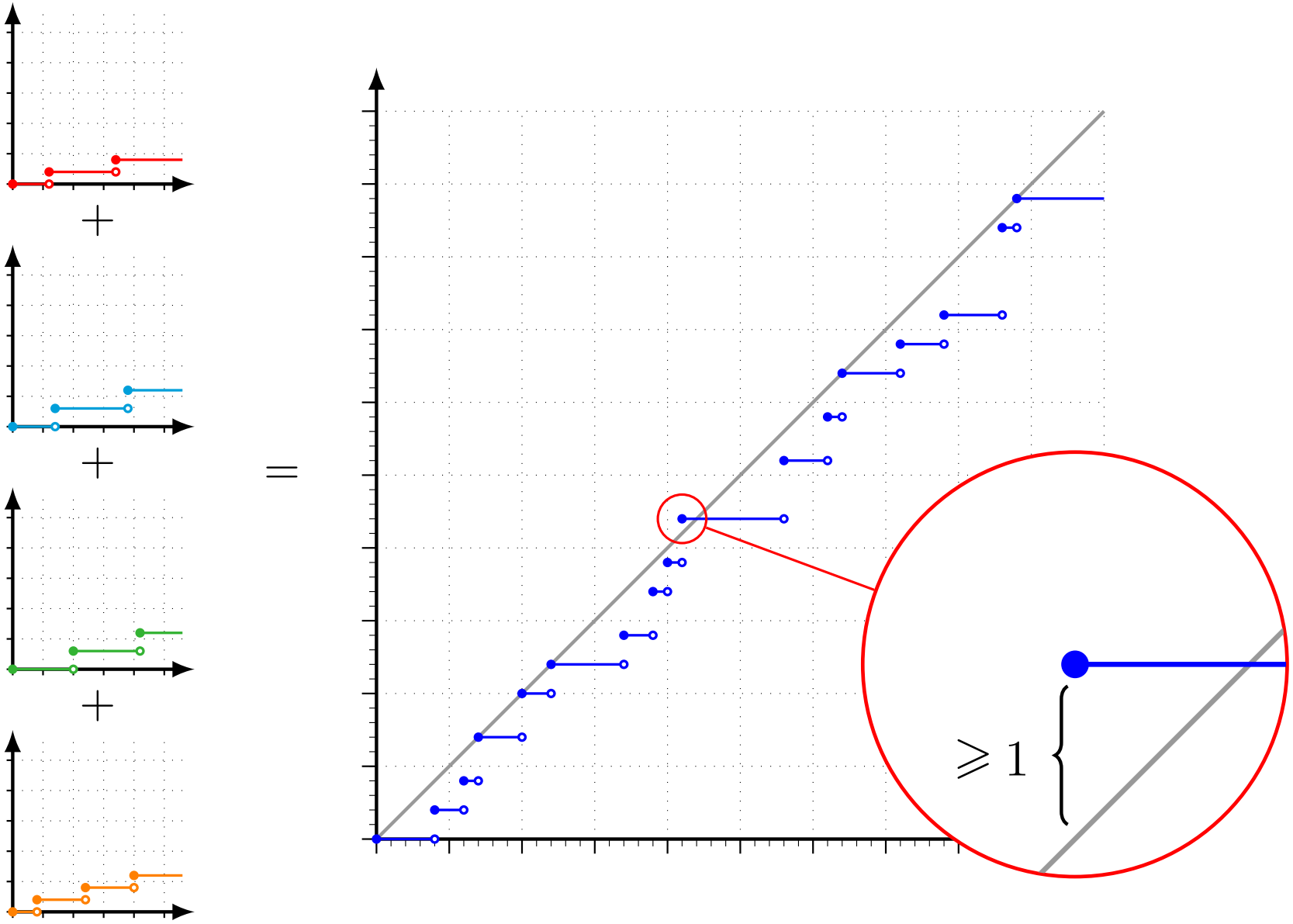

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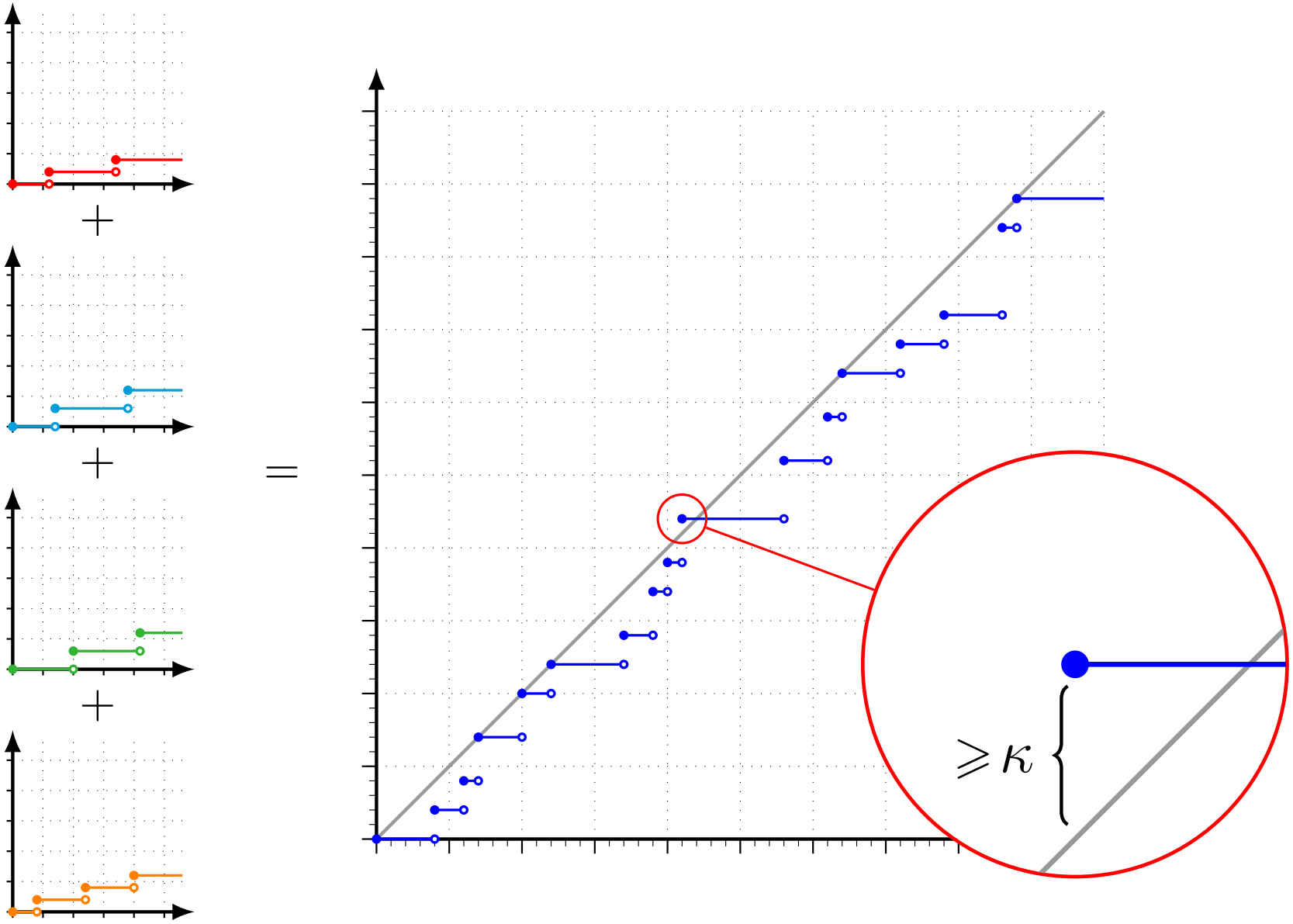

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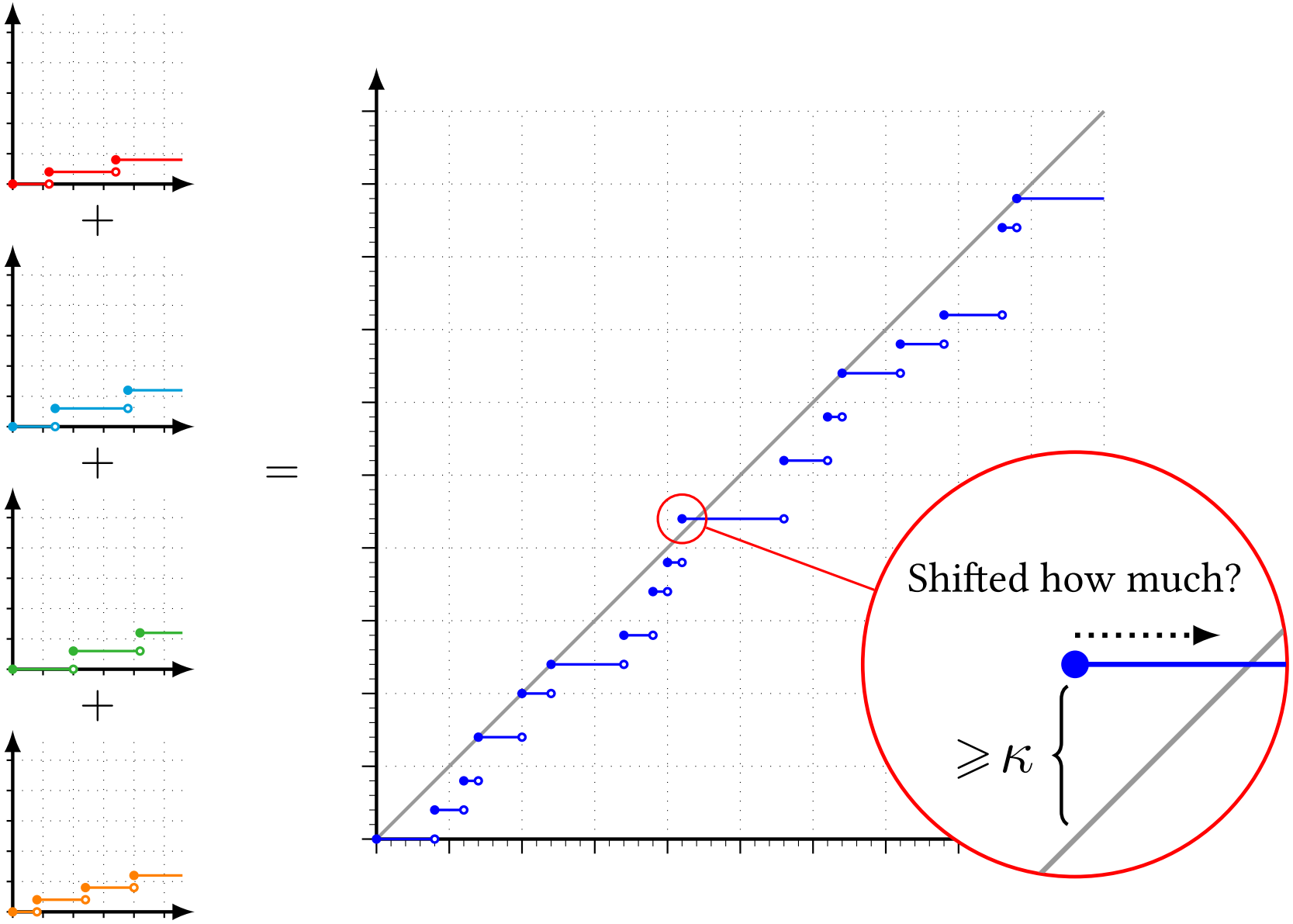

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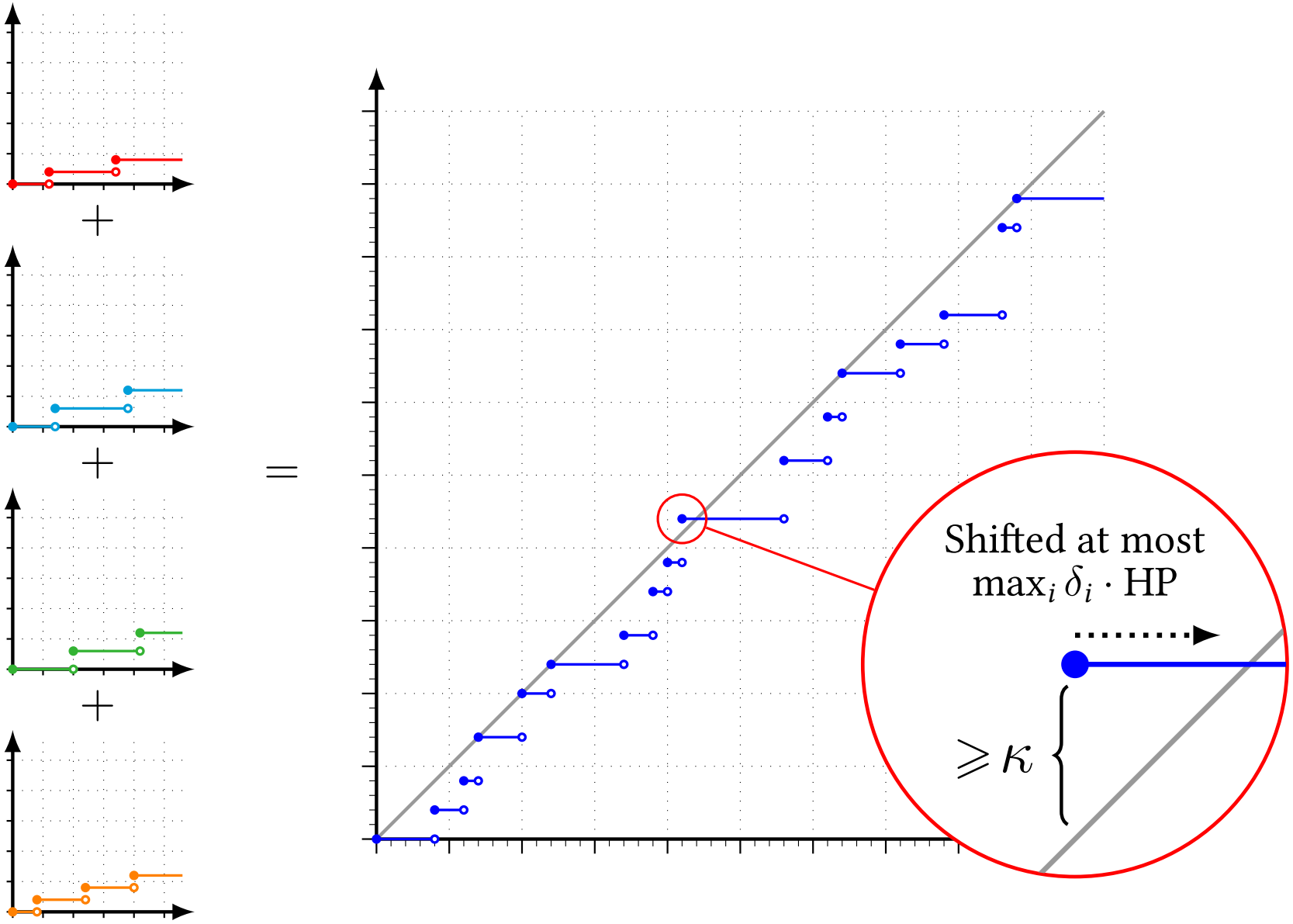

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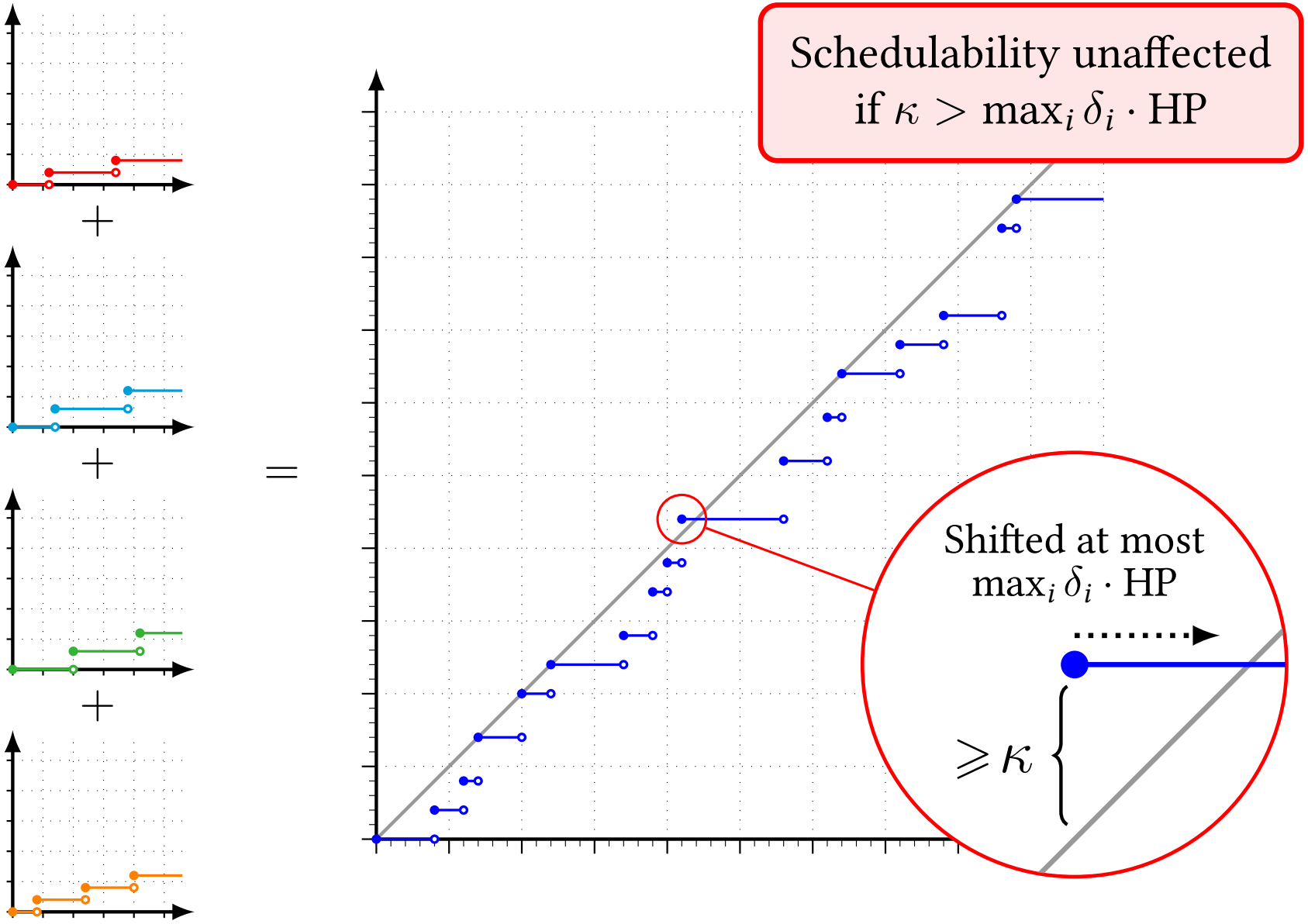

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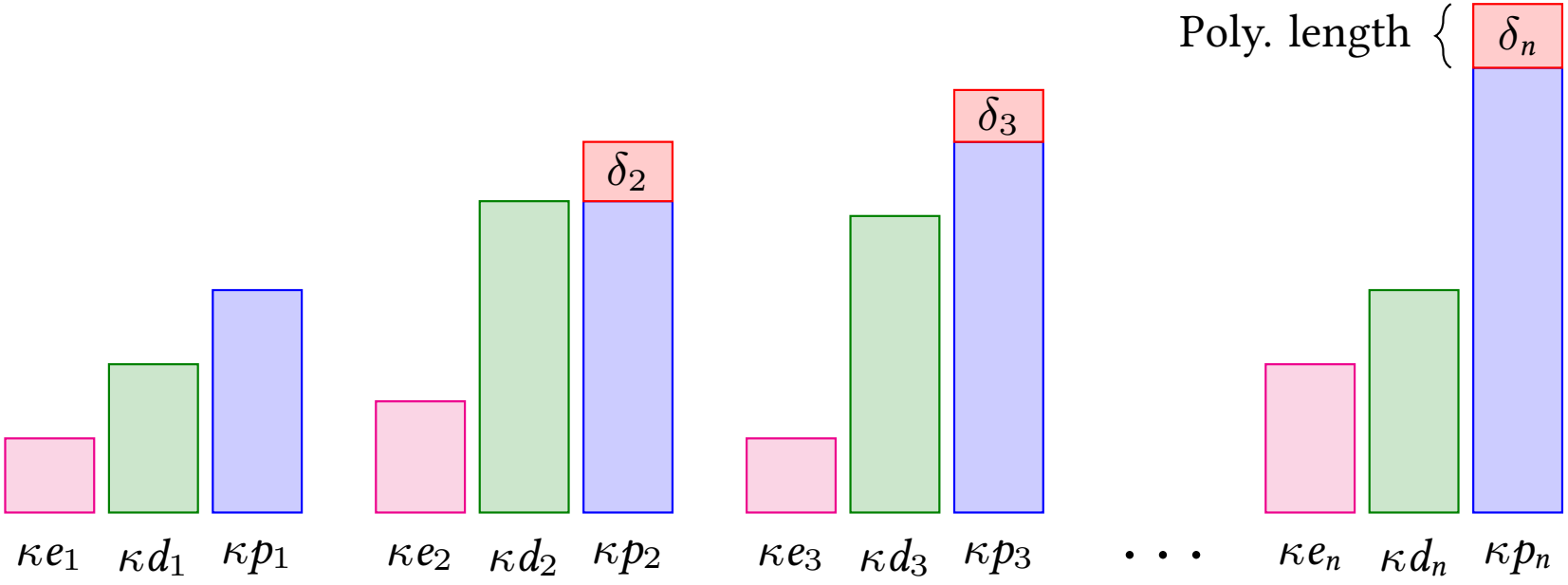

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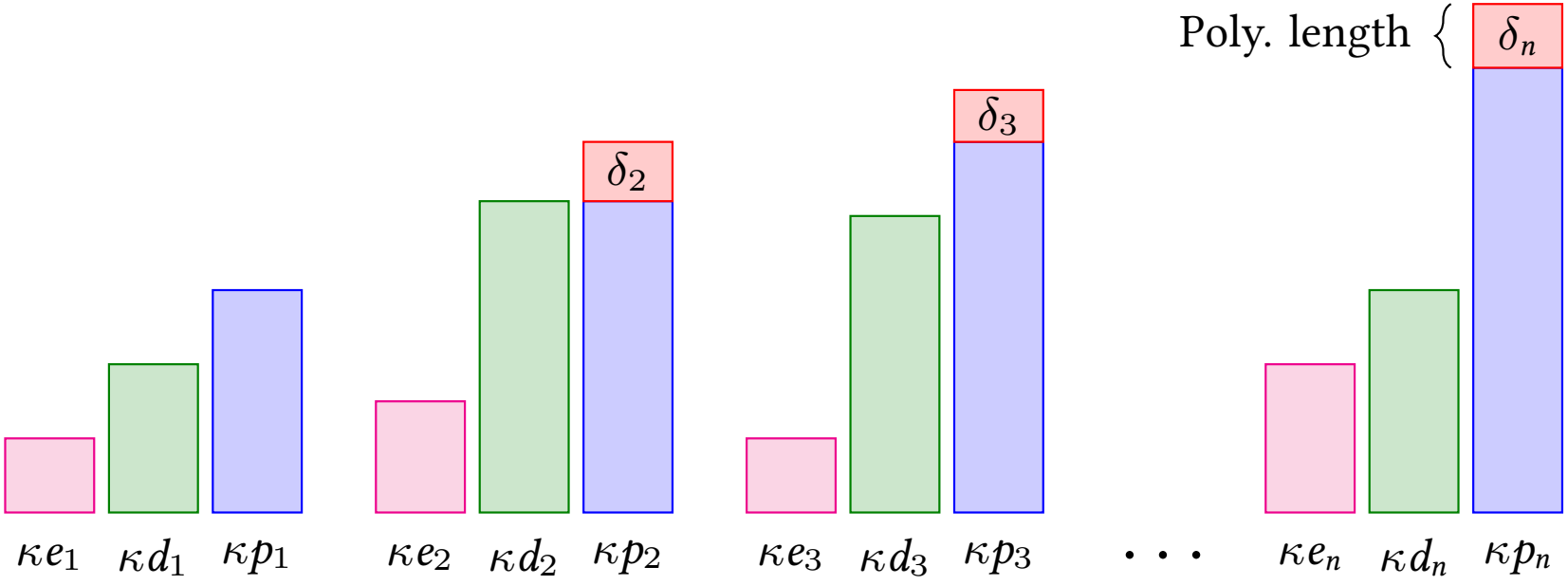

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A Tale of Two Reductions

EDF-schedulability

- Constrained deadlines
- Bounded utilization

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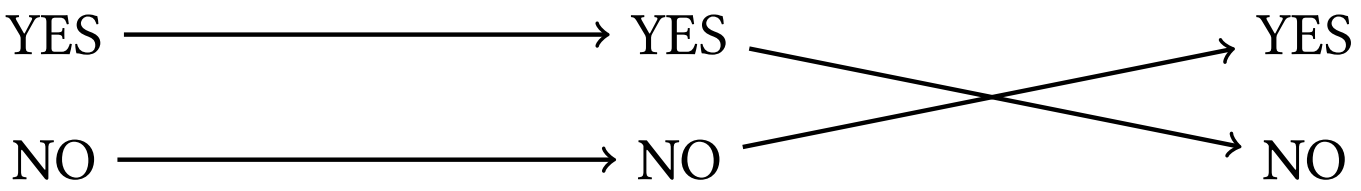
- **Pairwise coprime periods**

FP-schedulability

- Implicit deadlines

—or—

- Constrained deadlines
- Bounded utilization



coNP-hard

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NP-hard

Conclusions

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FP	Arbitrary utilization	Weakly NP-complete	Weakly NP-complete	Weakly NP-hard
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$\diamond$

$\exists$Questions?