Course 1DL441: Combinatorial Optimisation and Constraint Programming, whose part 1 is Course 1DL451: Modelling for Combinatorial Optimisation
Outline

1. Constraint Problems

2. Constraint Programming Technology

3. Constraint-Based Modelling

4. History & Success Stories & Opportunities
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4. History & Success Stories & Opportunities
Many important real-life problems are NP-hard or worse and can only be solved exactly & fast enough by intelligent search, unless $P = NP$:

- Personnel rostering, scheduling, time-tabling, ...
- Transportation logistics: vehicle routing, ...
- Packing: container or truck loading, carpet cutting, ...
- Configuration, design, experiment set-up, ...
- Alignment of bio-molecules, phylogeny, ...
- Financial investment instrument design, ...
- ...

**Definition**

In a constraint problem, values have to be found for all the variables within their given domains so that:

- All the given constraints on the variables are satisfied.
- Optionally: A cost is minimal, or a benefit is maximal.

Search spaces are often larger than the universe!

NP-hardness is not where the fun ends, but where it begins!
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Constraint programming (CP) offers methods & tools for:
what: Modelling constraint problems in a high-level language.
how: Solving constraint problems intelligently, by strategy-guided systematic search plus inference, or by strategy-guided local search plus inference.

Slogan of CP:

Constraint Program = Model [ + Search ]

CP solvers are complementary in strength to those of:
- Operations Research (OR): linear programming (LP), integer LP (ILP), mixed integer programming (MIP), . . .
- Boolean satisfiability (SAT), modulo theories (SMT)
- . . .

This leads to hybrid solving technologies!
☞ In Algorithms and Data Structures 3 (1DL481), taught in period 3 (January to March), there are assignments on local search as well as on MIP, SAT, and SMT modelling.
Scope of Constraint Programming

CP has a wide scope, as it addresses:

- satisfaction problems and optimisation problems
- discrete variables and continuous variables
- linear constraints and non-linear constraints

in principle in any combinations thereof, by:

- systematic search, if optimality more crucial than speed
- local search, if speed is more crucial than optimality
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The **constraint predicates** (Circuit, extensional, distinct, ...) and **structured variable types** (sets, ...) allow us *both* to model the structure of a problem *and* to exploit that structure when solving the problem.

Dozens of **constraint predicates** (see the **Catalogue**) **declaratively** encapsulate complex inference algorithms.

There is no standardised CP modelling language: distinct CP solvers may support distinct predicates, possibly under distinct names and signatures, as well as distinct types.
Pride:

*Constraint programming represents one of the closest approaches computer science has yet made to the Holy Grail of programming: the user states the problem, the computer solves it.*

— Eugene Freuder, a CP pioneer
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Prejudice:

The contribution of the article should be the reduction of an engineering problem to a known optimization format. […] showcases pseudo code […] submit this work to a journal interested in code semantics […].

— Reviewer of a paper of ours at a prestigious OR journal
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Stand-Alone Languages:

- **ALICE** by Jean-Louis Laurière, France, 1976
- **CHIP** at ECRC, Germany, 1987–1990; Cosytec, France
- **OPL**, by P. Van Hentenryck, USA, and ILOG, France: front-end to both CP Optimizer and CPLEX Optimizer
- **Comet**, by P. Van Hentenryck and L. Michel, USA
- **MiniZinc**, at Monash University, Australia
- ... 

**Libraries** (the ones listed before “;” are open-source):

- Prolog: ECLiPSe, ...; SICStus Prolog, ...
- C++: Gecode, CP-SAT; IBM CP Optimizer, CHIP, ...
- Java: Choco, Google CP-SAT, JaCoP, MiniCP, ...; ...
- Objective-C: Objective-CP; ...
- Scala: OscaR.cp, OscaR.cbls; ...
- ...
Success Stories by CP Users and Contributors:

Success stories: CP = technology of choice in routing, configuration, rostering, scheduling, timetabling, ...
You're watching

**The Americans**

*Season 6: Ep. 4*

**Mr. and Mrs. Teacup**

Oleg presses Philip to reveal what Elizabeth is working on. Paige joins her mom on an operation to steal a sensor. Philip breaks bad news to Henry.

And two: the concept of the propagation of constraints...
Opportunities for CP

Rapid prototyping (with high solving performance) when:
- Constraints are, still or again, subject to experiments
- Partition into hard & soft constraints yet undetermined

Combinatorial structure is impure, due to side constraints.

It is time to consider all or more problem constraints.

Domain knowledge exploitable for problem-specific search.

It is a configuration problem.

It is a personnel rostering problem.

It is a scheduling or timetabling problem.