On the asymptotic spectrum of stiffness matrices arising from Isogeometric Analysis

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The discretization of a given differential problem for some sequence of stepsizes h tending to zero leads to a sequence of systems of linear equations $A_m \mathbf{x}_m = \mathbf{b}_m$ with A_m some matrix of order d_m, where of course d_m depends on h, and tends to ∞ for $h \to 0$. To properly face their solution it is imperative to deeply understand the spectral properties of the matrices $\{A_m\}$:

- the eigenvalue of minimal modulus and the eigenvalue of maximal modulus,
- the conditioning,
- the localization of the spectrum,
- the asymptotic eigenvalue distribution.

The task of evaluating the asymptotic conditioning has a plain numerical motivation in understanding the numerical intrinsic difficulty of the problem, while the motivation of evaluating extremal eigenvalues and the localization of the spectrum is evident for obtaining reasonable bounds for the number of iterations when Krylov methods – such as the Conjugate Gradient or GMRES – are employed. On the other hand, the task of finding the asymptotic eigenvalue distribution is motivated by the analysis of multigrid methods [2, 5].

The spectral distribution of a sequence of matrices is a fundamental concept. Roughly speaking, saying that the sequence of matrices $\{A_m\}$ is distributed as the function f means that the eigenvalues of A_m behave as a sampling of f over an equispaced grid of the domain of f, at least if f is smooth enough. The function f is called the *symbol* of the sequence.

In this talk we discuss the case of stiffness matrices arising from the Isogeometric Analysis process [4]. As expected the sequence of matrices $\{A_m\}$ has an asymptotic spectrum, as in the case of Finite Difference and Finite Element approximations (see [5, 1] and references therein). We illustrate how this information can be applied to design/analyze optimal two-grid (multigrid) and optimal preconditioners [3].

References

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