

TWO STUDIES OF THE ACCURACY AND COMPLEXITY OF MONTE CARLO METHODS:

HÅKON HOEL*

ABSTRACT. In this talk we present two studies of the accuracy and complexity of Monte Carlo methods:

(1) Derivation of a robust Monte Carlo sequential stopping rule

C. Bayer, H. Hoel, E. von Schwerin, and R. Tempone

We consider the setting of estimating the mean of a random variable by a sequential stopping rule Monte Carlo (MC) method. The performance of a typical second moment based sequential stopping rule MC method is shown to be unreliable in such settings both by numerical examples and through analysis. By combining explicit and asymptotic error bounds for the central limit theorem, we construct a higher moment based stopping rule which in numerical examples is shown to perform more reliably and only slightly less efficiently than the second moment based stopping rule.

(2) Weak approximation of SDE by multilevel Monte Carlo and strong error adaptive numerical integration:

H. Hoel, J. Häppölä, and R. Tempone

We present a multilevel Monte Carlo (MLMC) method for weak approximation of stochastic differential equations (SDE) that uses an *a posteriori* strong error adaptive Euler–Maruyama step-size control in the numerical integration of SDE realizations. *Strong* error adaptivity provides a reliable and efficient way to control the statistical error of the weak approximation MLMC estimator. For a large set of low-regularity weak approximation problems the adaptive MLMC method produces output whose weak error is bounded by $\mathcal{O}(\epsilon)$ at the cost $\mathcal{O}(\epsilon^{-2} \log(\epsilon)^4)$, which is a lower asymptotic cost than what typically can be obtained by the uniform time-step Euler–Maruyama MLMC method on the given set of problems. The adaptive MLMC method’s potential cost reduction is illustrated in a numerical example of a low-regularity problem.

*Applied Mathematics and Computational Sciences, KAUST, Thuwal, Saudi Arabia (hakon.hoel@kaust.edu.sa).