Carrying Probabilities to the Infinite World

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(Joint work with Noomene Ben Henda, Richard Mayr, and Sven Sandberg)

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1 Background

2 Infinite-State Transition Systems

3 Decisive Markov Chains

4 Qualitative Reachability Analysis

5 Qualitative Repeated Reachability Analysis

6 Quantitative Reachability Analysis

7 Extensions

8 Conclusions and Current Challenges

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Background

Finite-State
Model Checking

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Background

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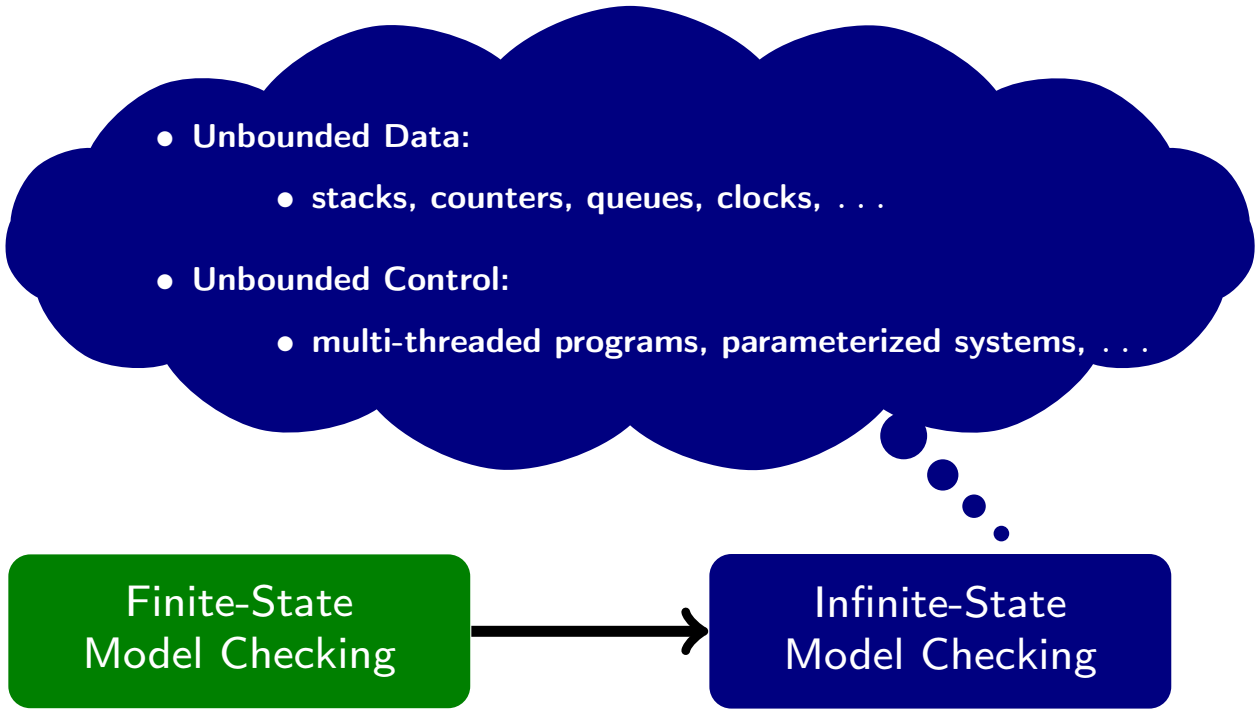
Background



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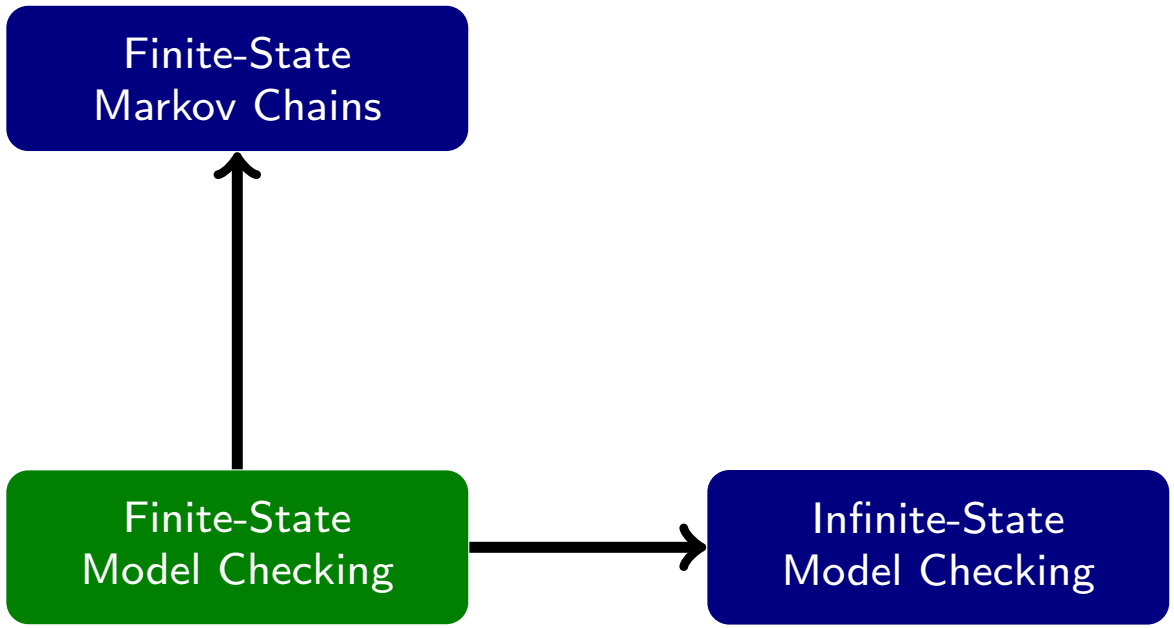

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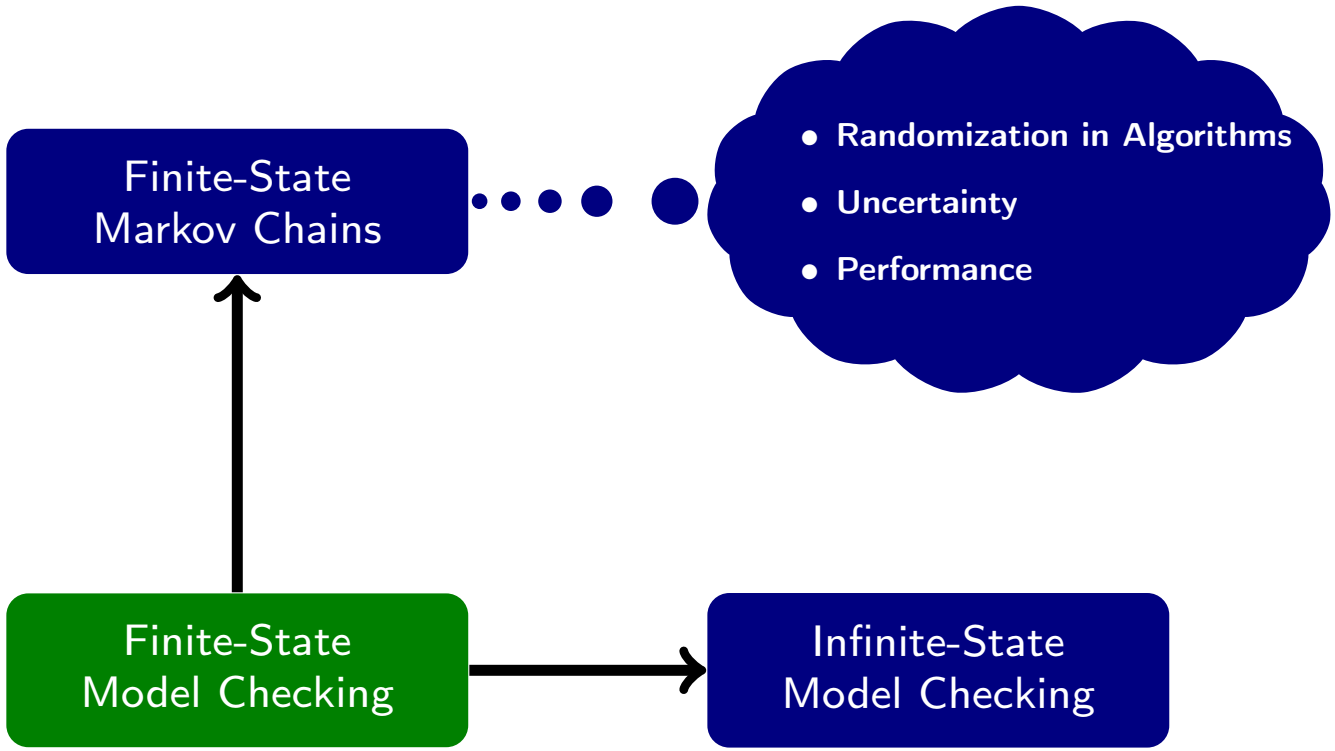

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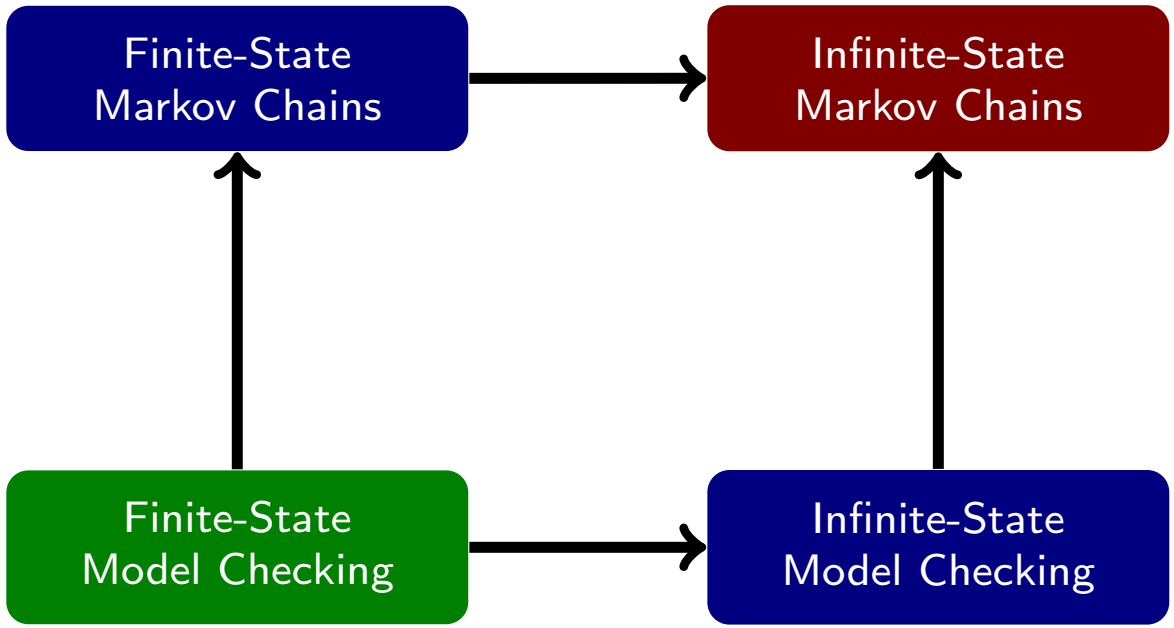

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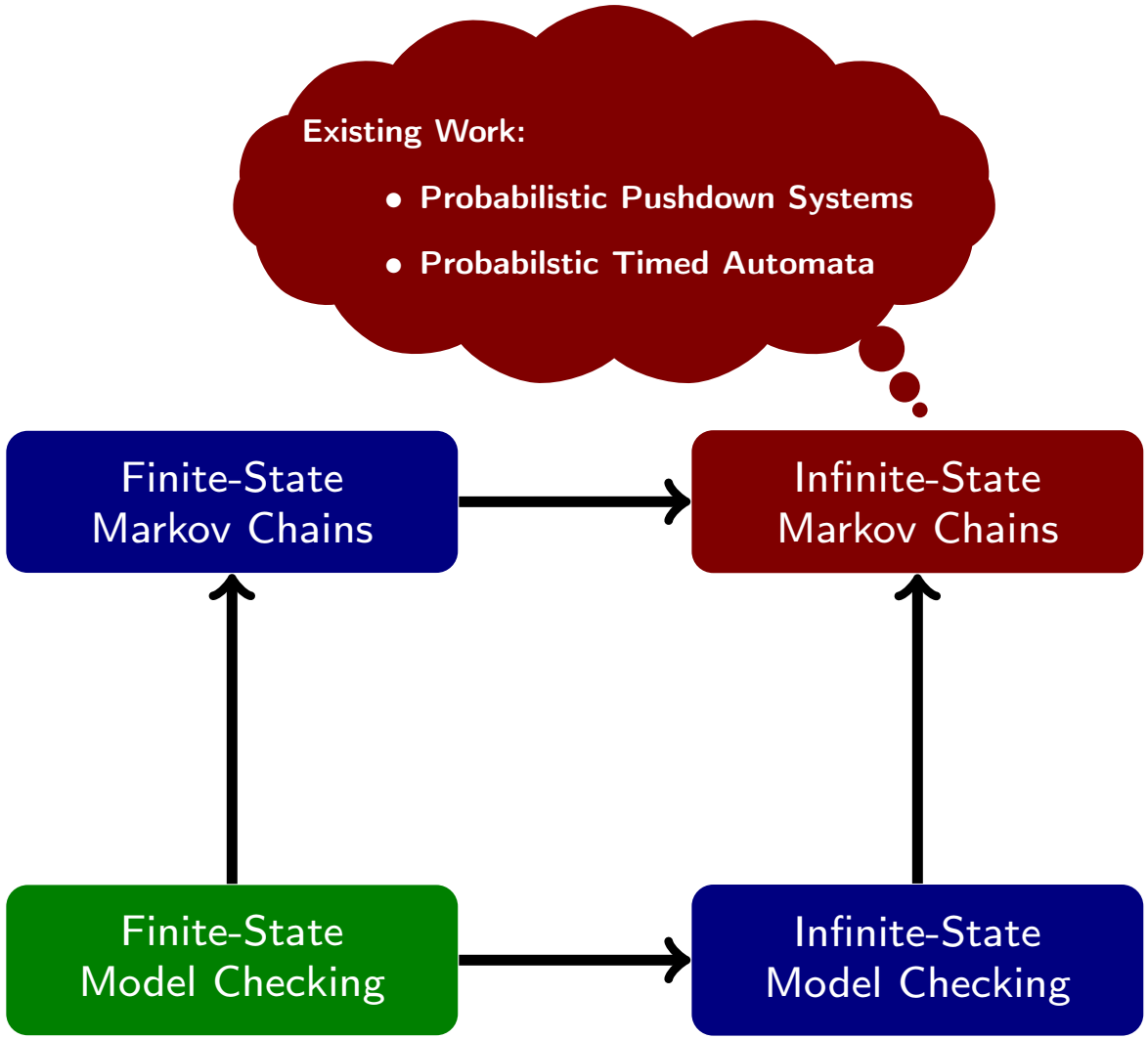

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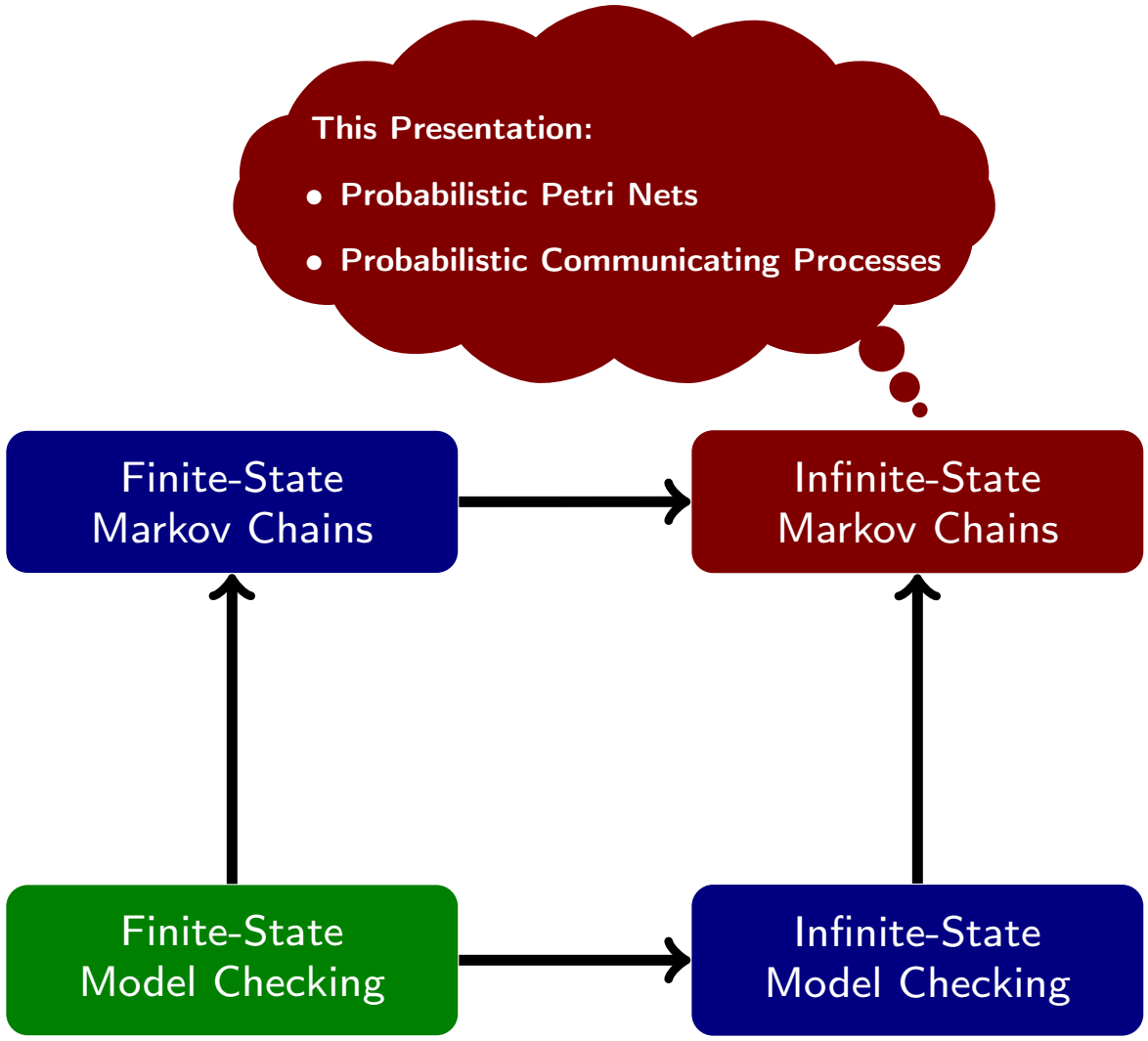

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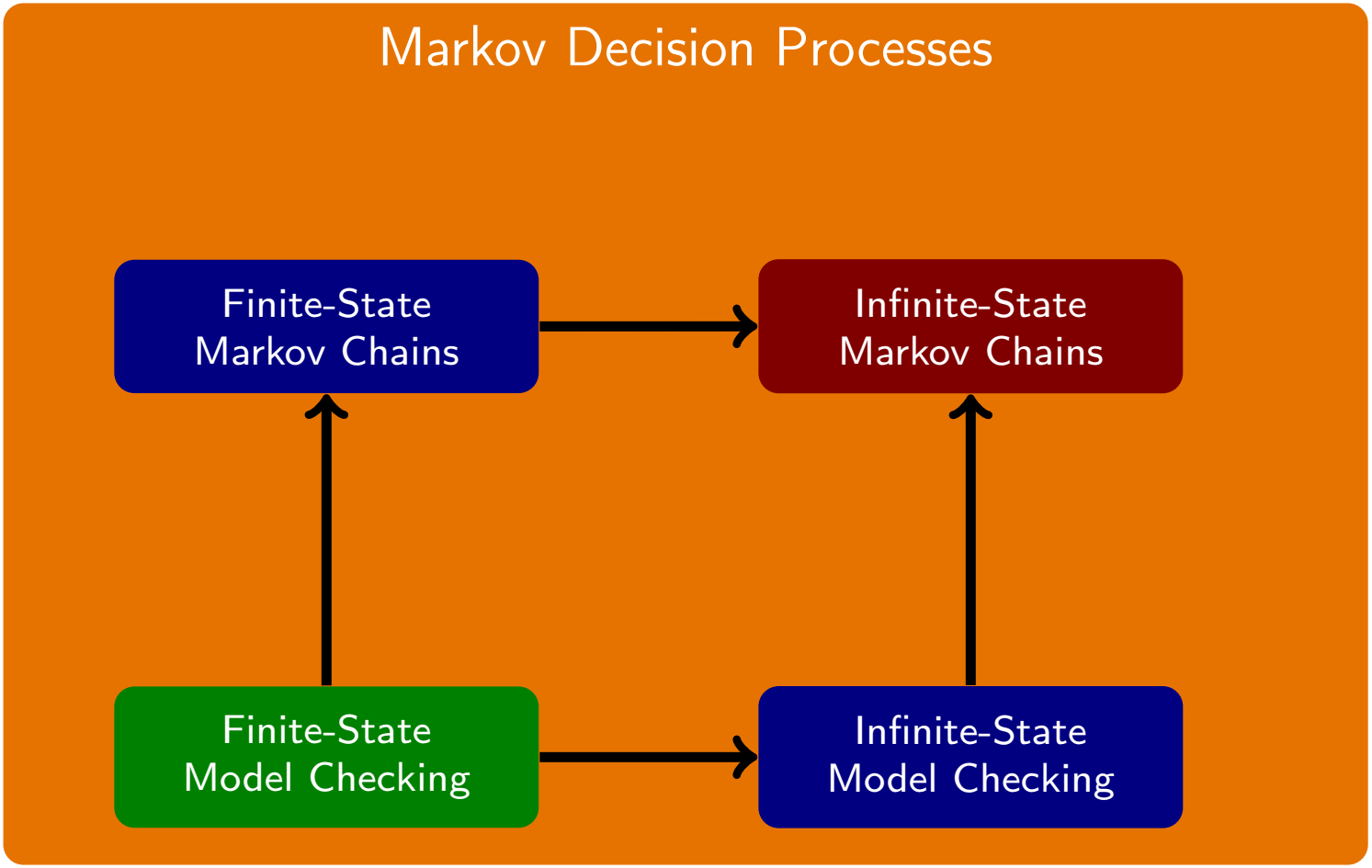

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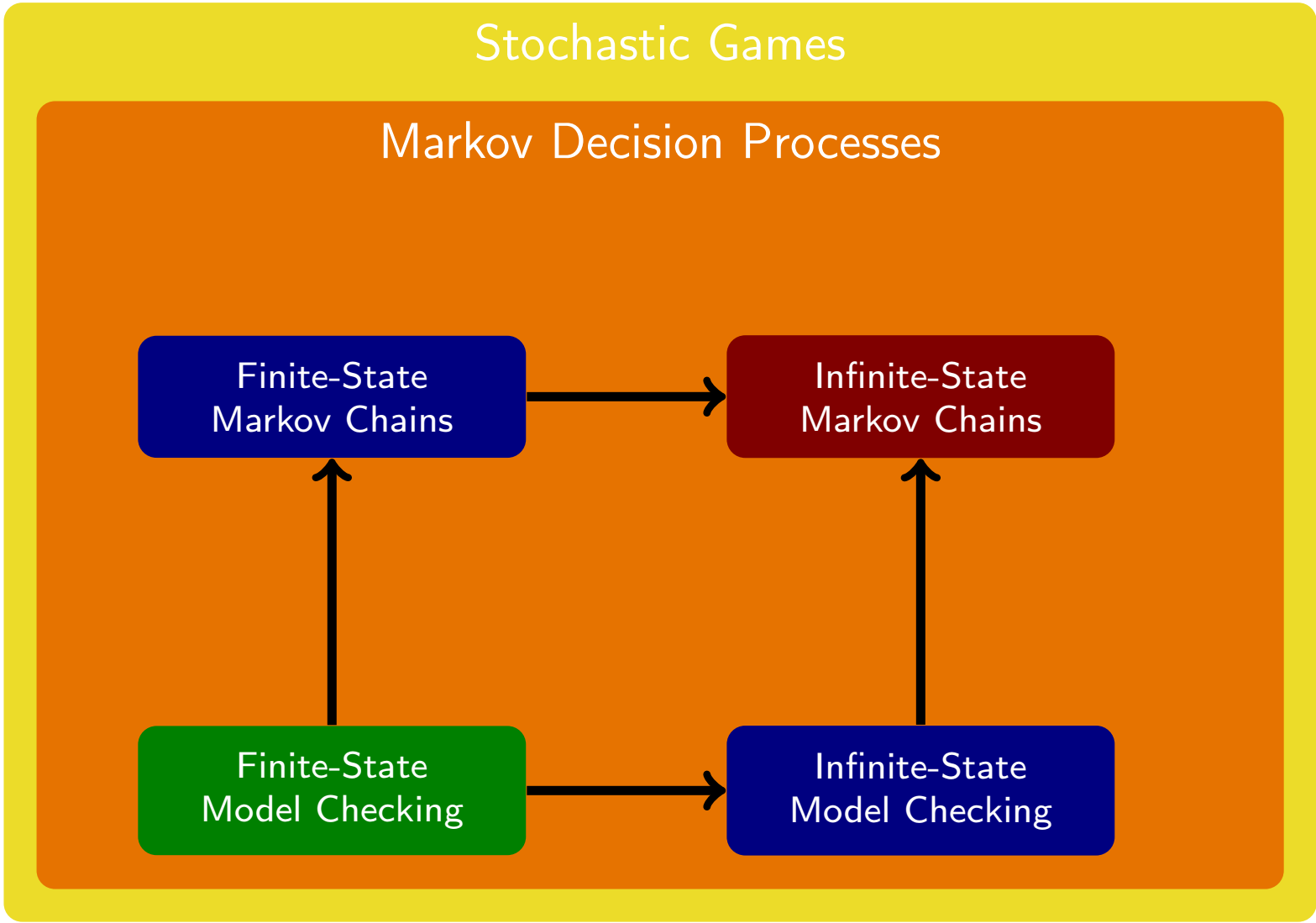
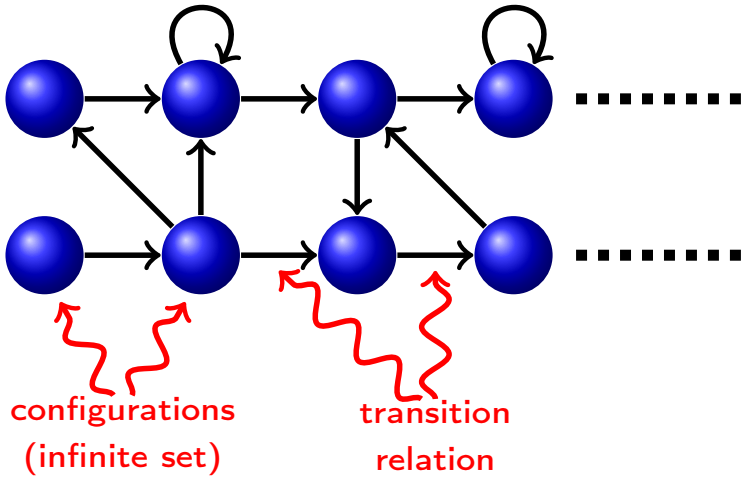


Infinite-State Transition Systems



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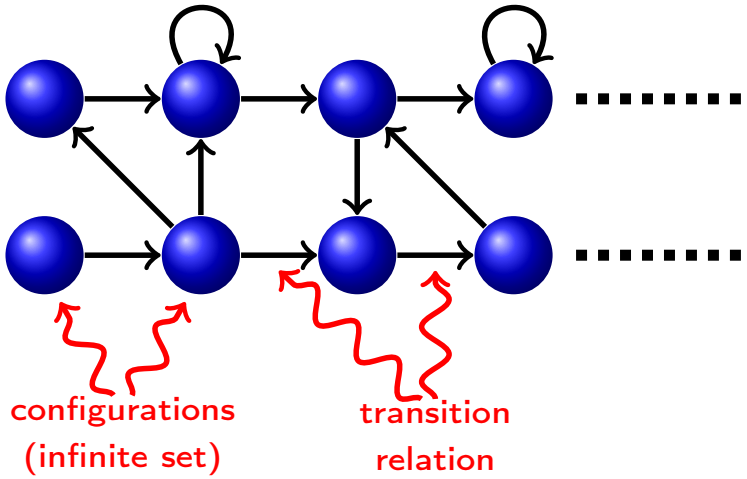
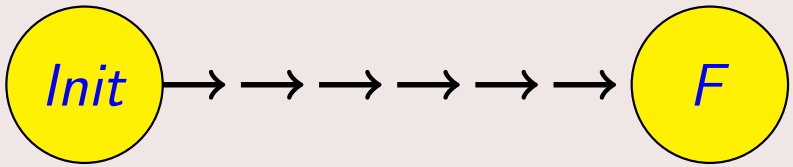
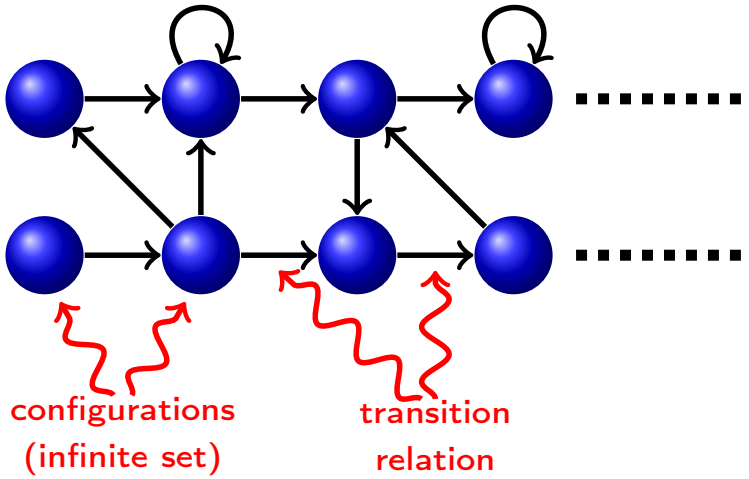


Reachability

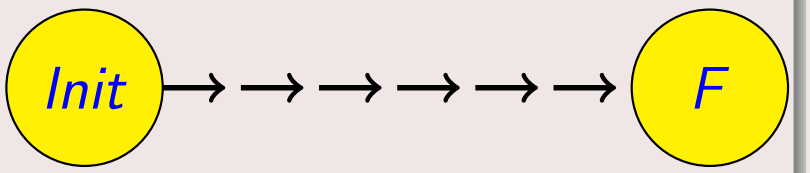


$Init \models \Diamond F$?

Infinite-State Transition Systems

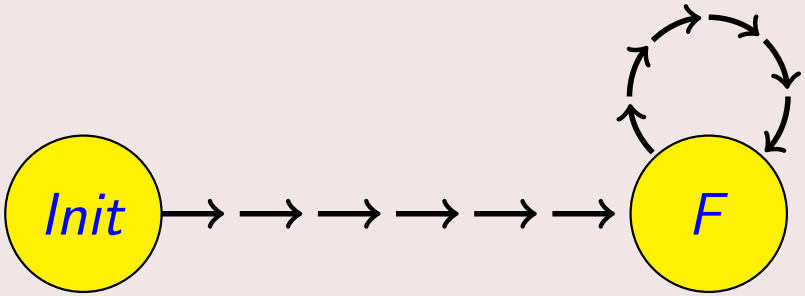


Reachability



$Init \models \Diamond F$?

Repeated Reachability



$Init \models \Box\Diamond F$?

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Petri Nets

Overview

- Syntax and Semantics
- Ordering
- Reachability Properties
- Coverability Algorithm
- VASS

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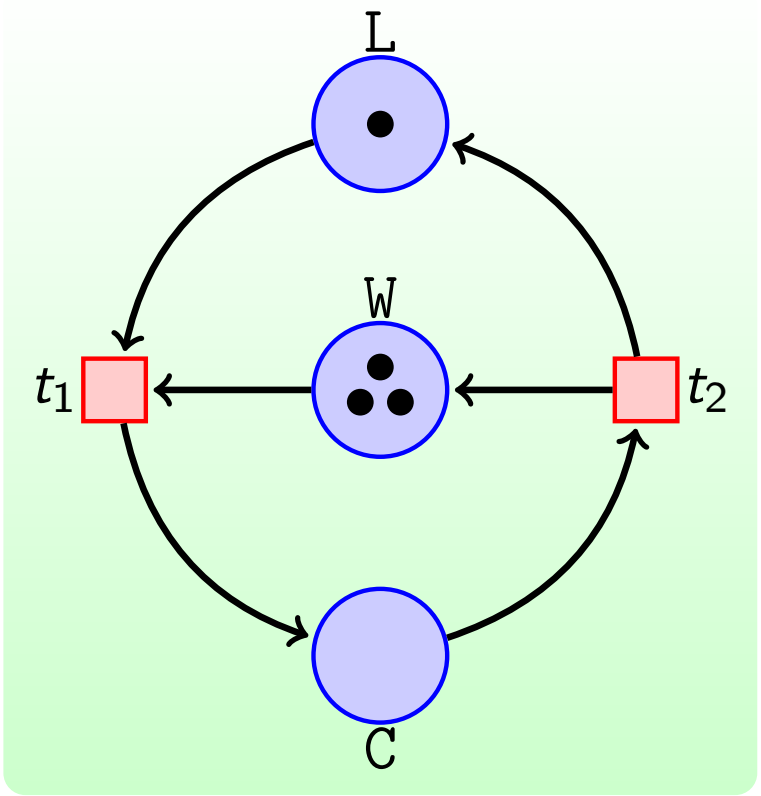
Syntax and Semantics



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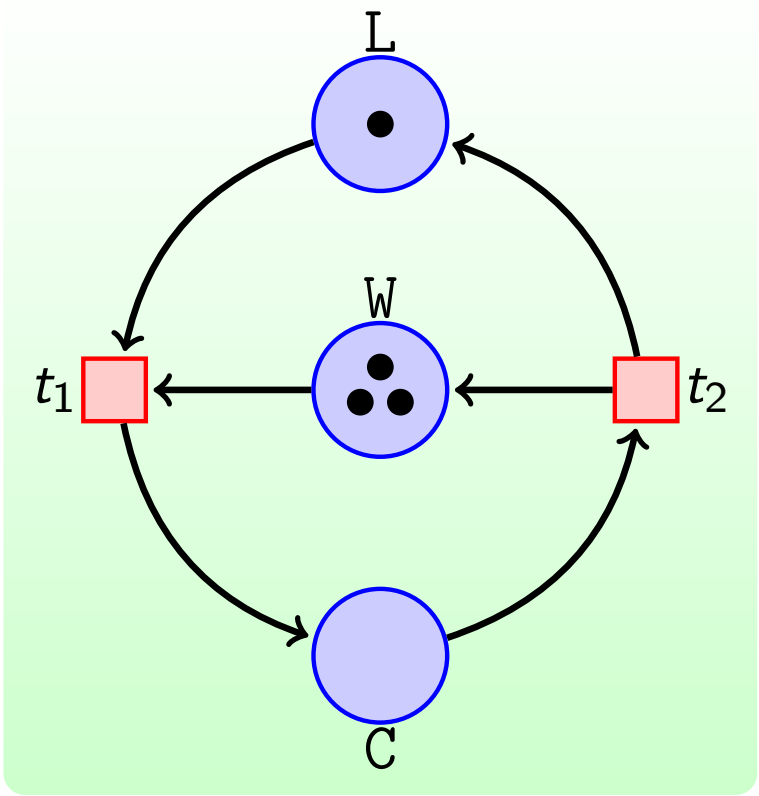
Syntax and Semantics



Marking: $[\mathtt{L}, \mathtt{W}^3]$

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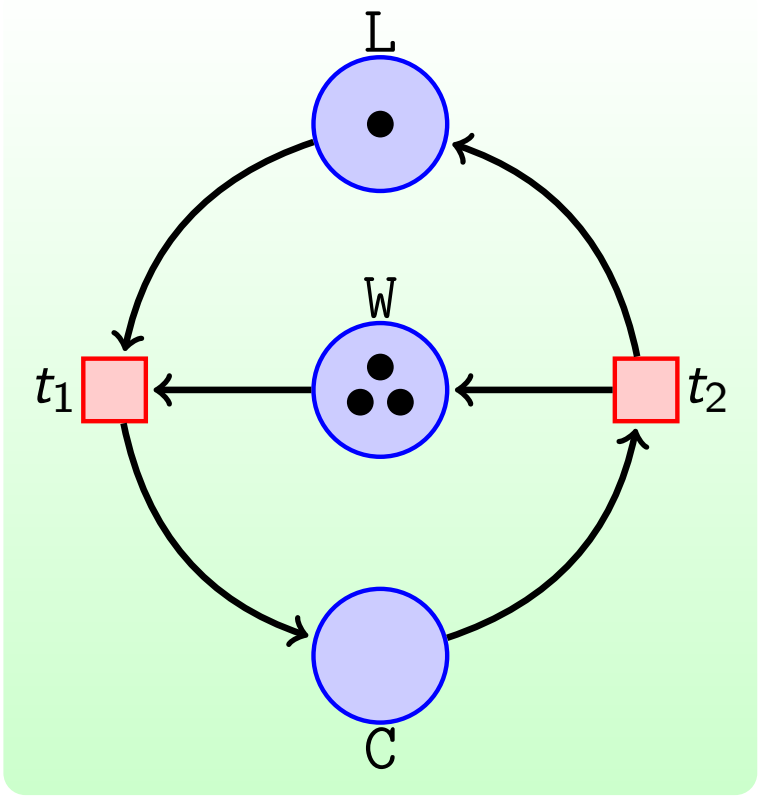
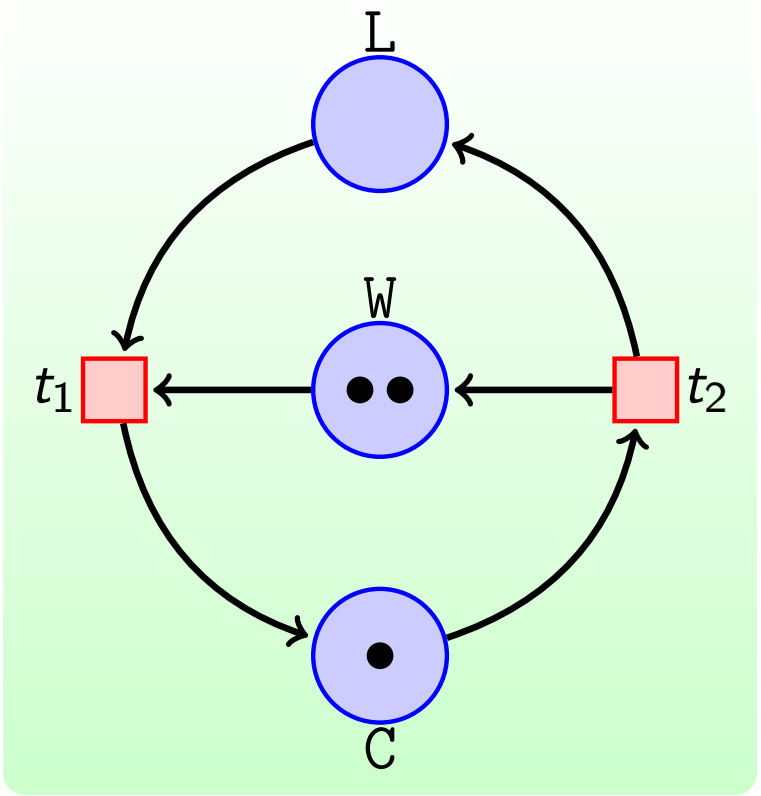
Syntax and Semantics





Firing t_1: $\left[\mathtt{L}, \mathtt{W}^3\right] \longrightarrow \left[\mathtt{W}^2, \mathtt{C}\right]$

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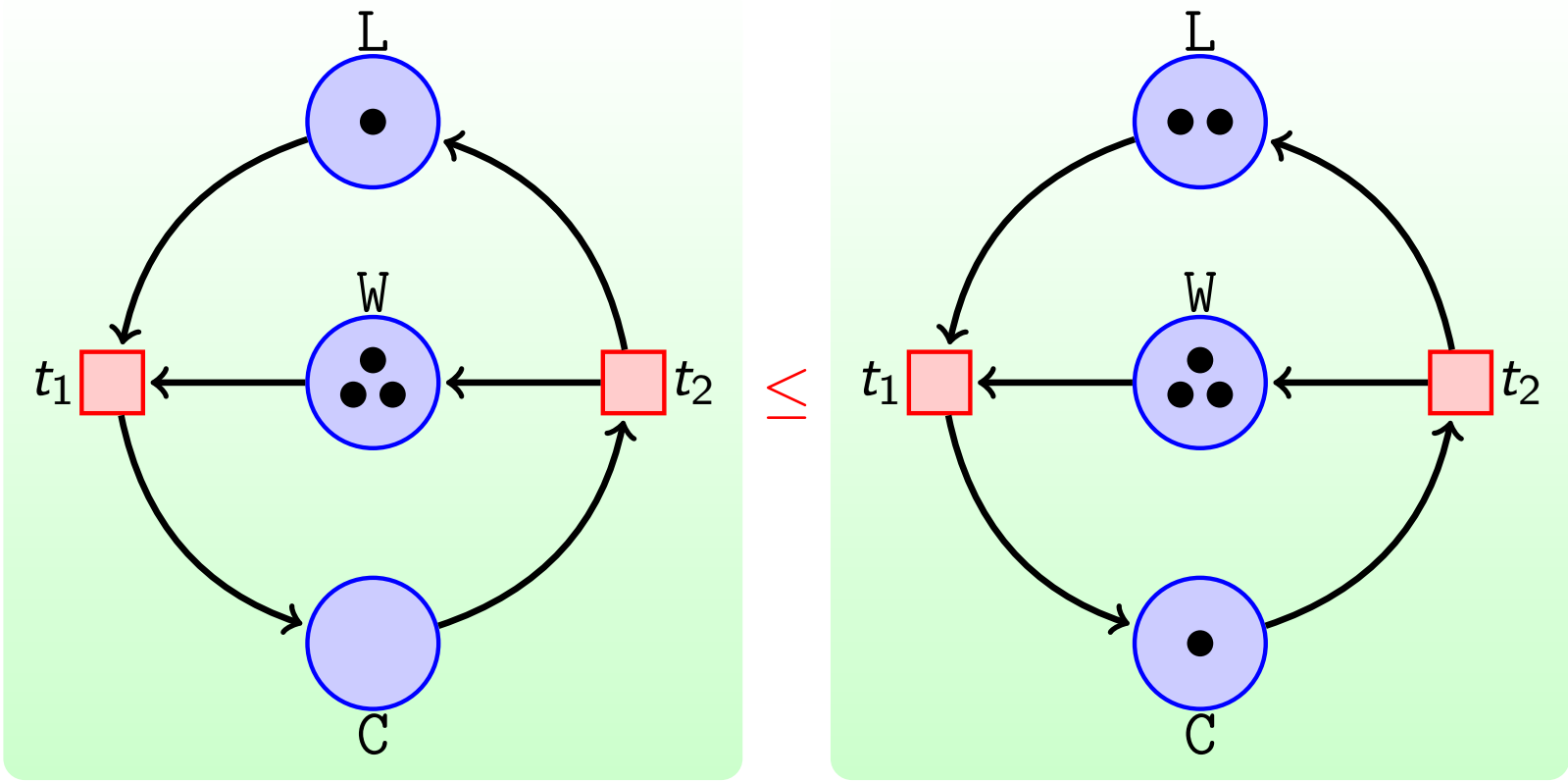
Ordering



$$\left[\mathtt{L}, \mathtt{W}^3\right] \leq \left[\mathtt{L}^2, \mathtt{W}^3, \mathtt{C}\right]$$

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Ordering

Upward Closed Sets

$$(m \in U) \wedge (m \leq m') \implies (m' \in U)$$

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Upward Closure

- $m{\uparrow} := \{m' | m \leq m'\}$
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Minimal Elements

- $\min(U)$:= minimal elements of U wrt. $\leq$.

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Minimal Elements

- $\min(U)$:= minimal elements of U wrt. $\leq$.
- Properties:
  - $\min(U)$ is finite
  - $\min(U){\uparrow} = U$.

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Reachability

K-Reachability

- $m_1 \xrightarrow{K} m_2$: m_1 can reach m_2 within K steps

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Reachability

- $m_1 \xrightarrow{*} m_2$: m_1 can reach m_2

Finite Span (wrt. F)

- $\exists K.\, \forall m.\, m \xrightarrow{*} F$ implies $m \xrightarrow{K} F$

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Finite Span (wrt. F)

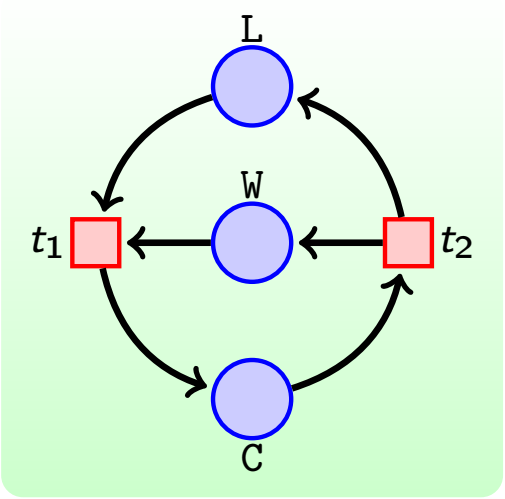
- $\exists K.\, \forall m.\, m \xrightarrow{*} F$ implies $m \xrightarrow{K} F$

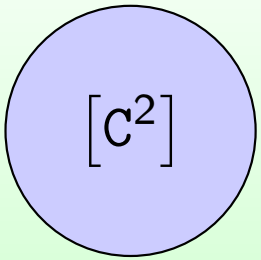
The Coverability Problem

- Instance: m_1, m_2: markings
- Question: $m_1 \xrightarrow{*} m_2{\uparrow}$?

Petri Nets

Compute $Pre^*\left(\left[C^2\right]\uparrow\right)$ - Backward Reachability Analysis





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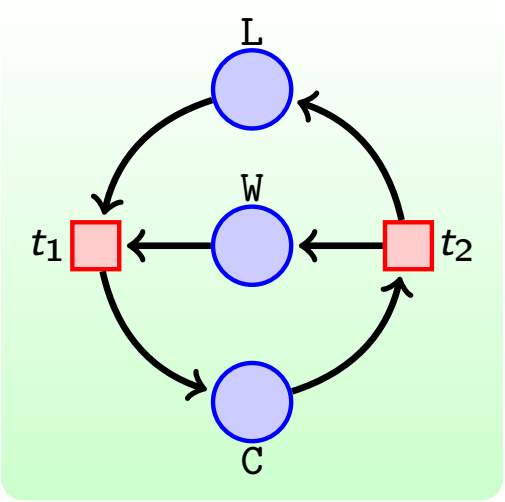

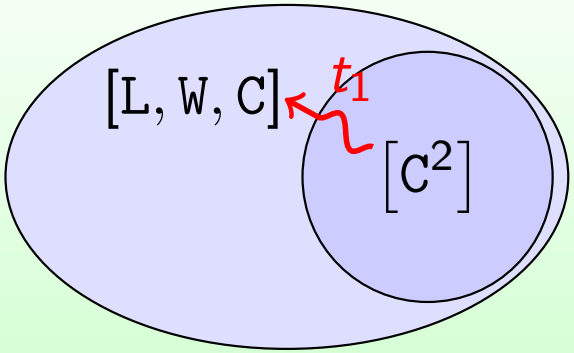

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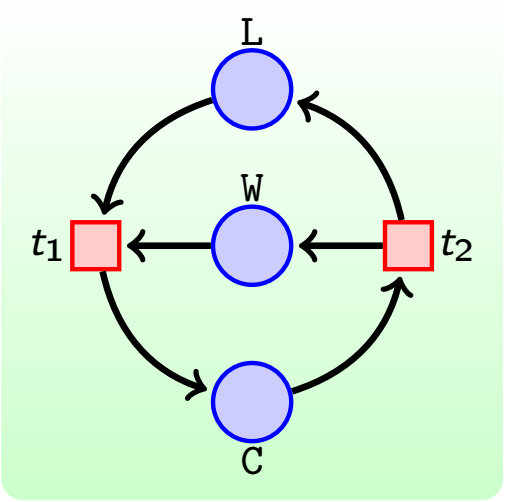

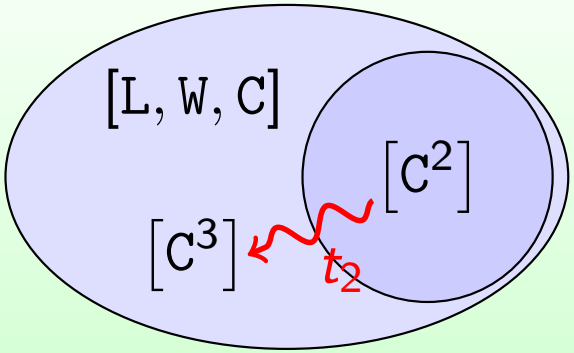

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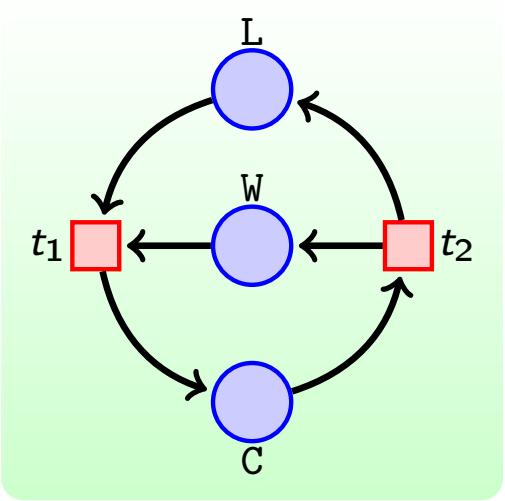

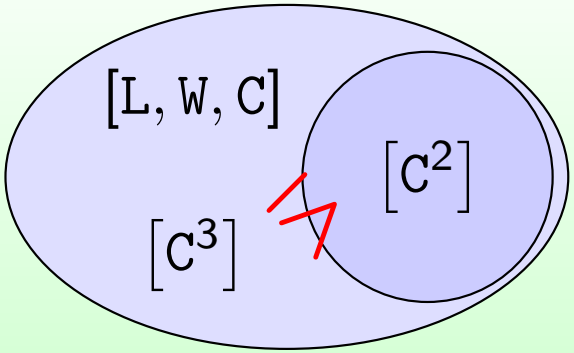

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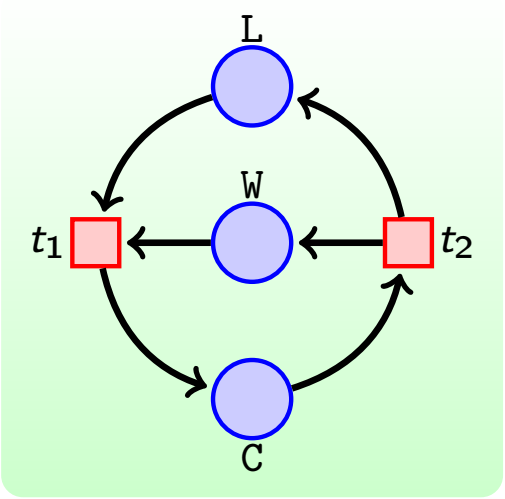

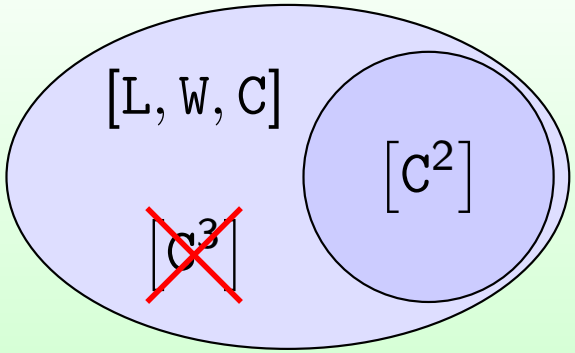

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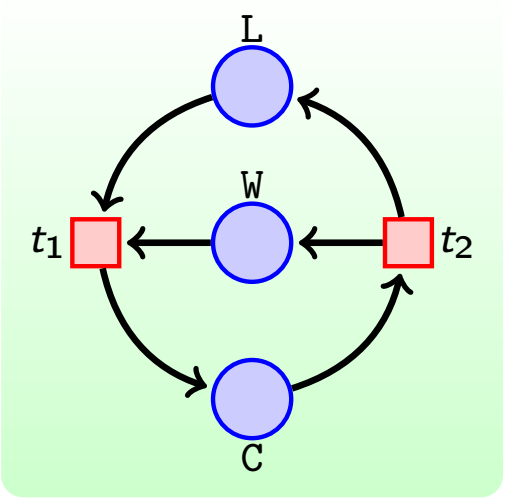

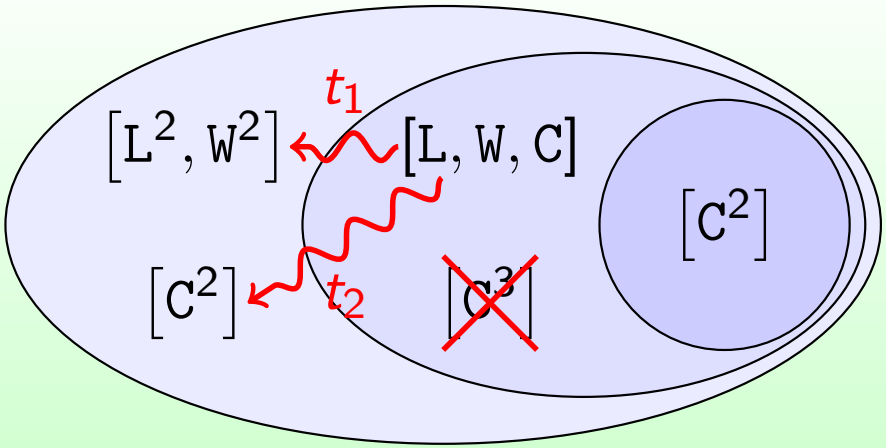

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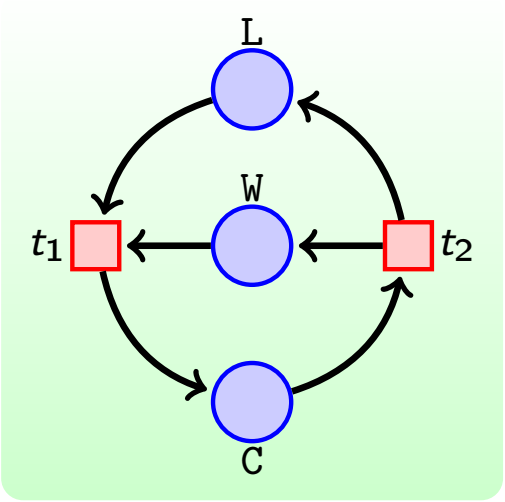

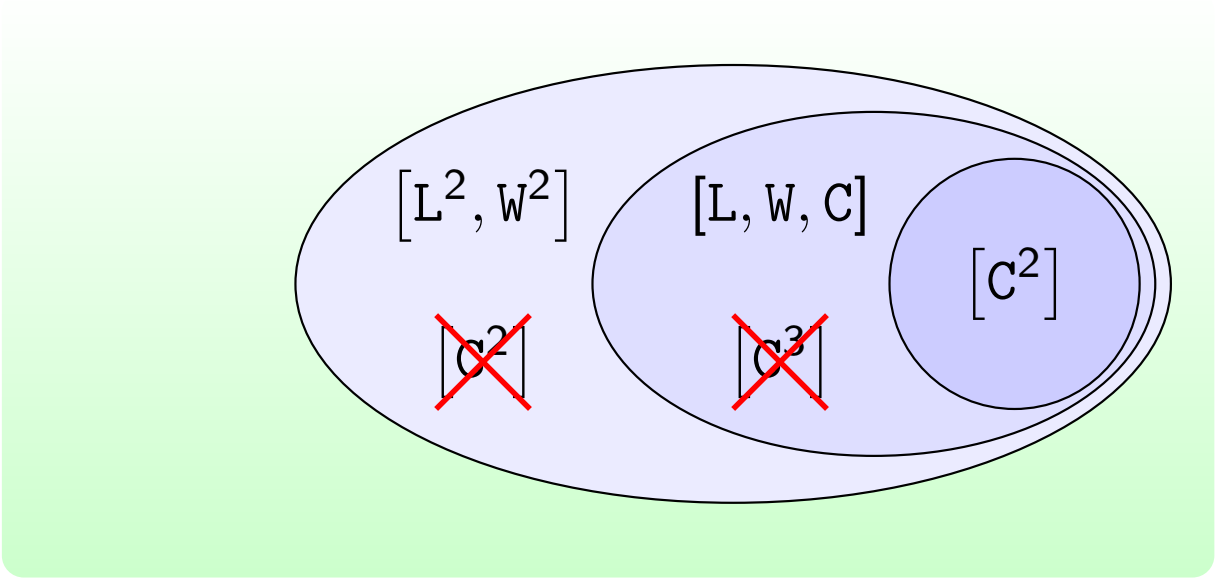

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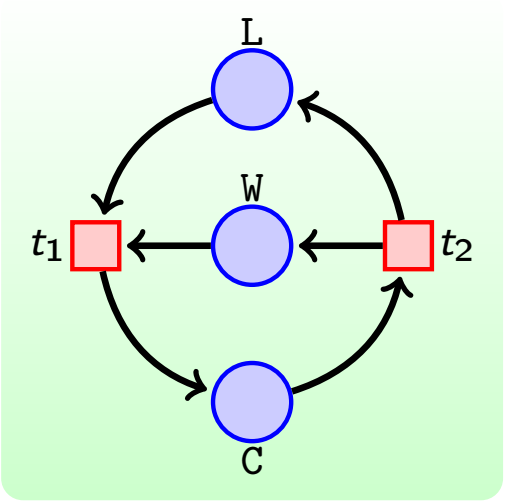

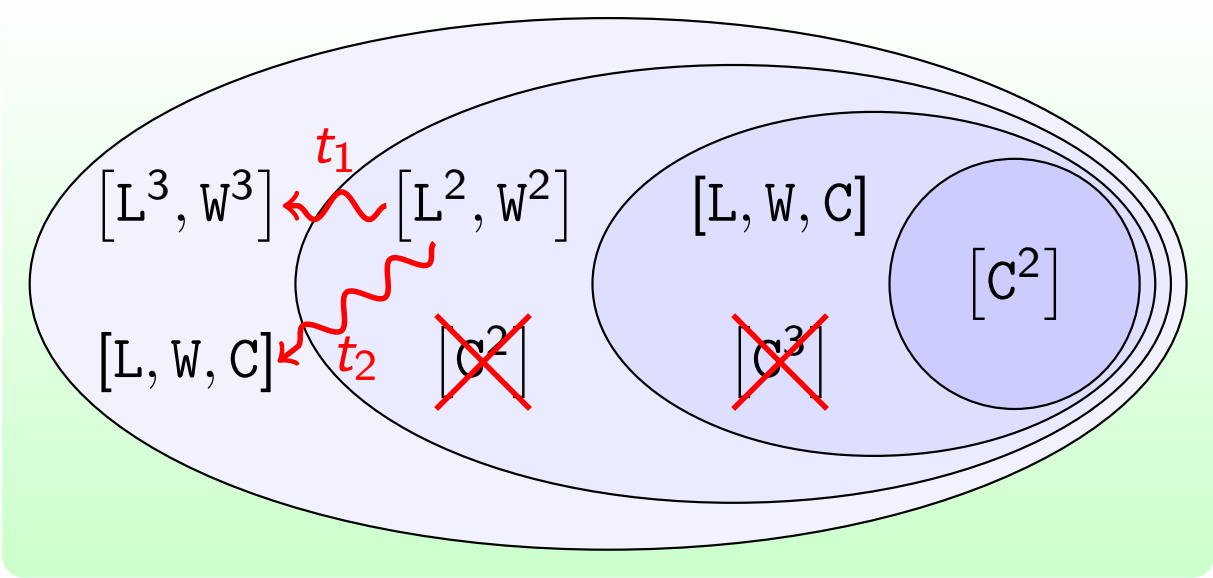

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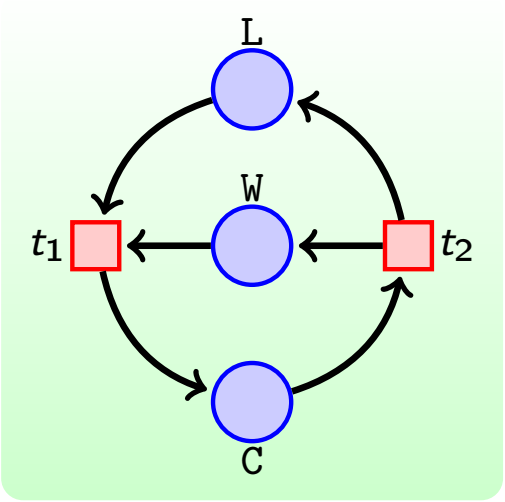

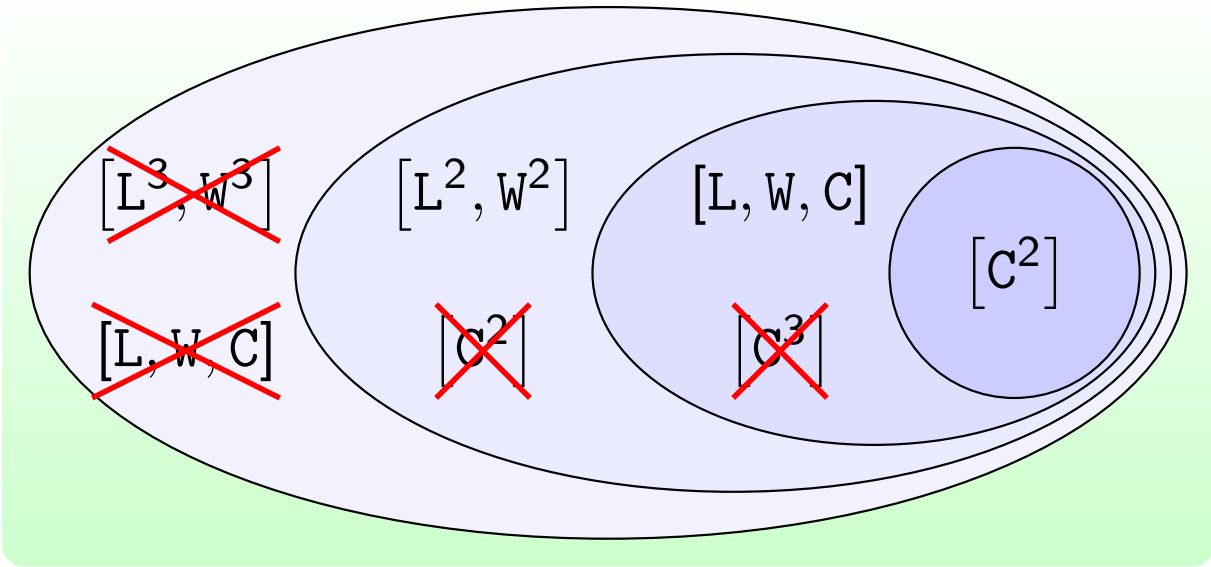

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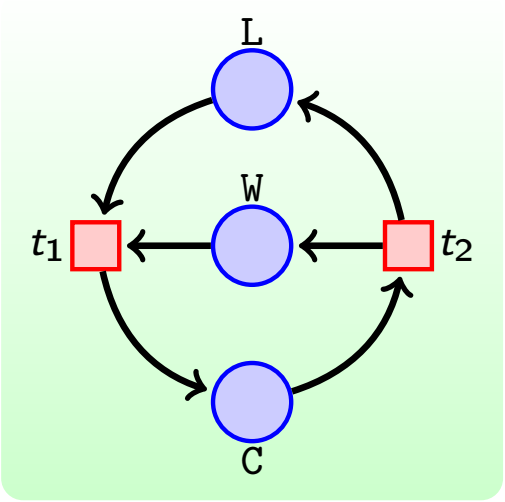
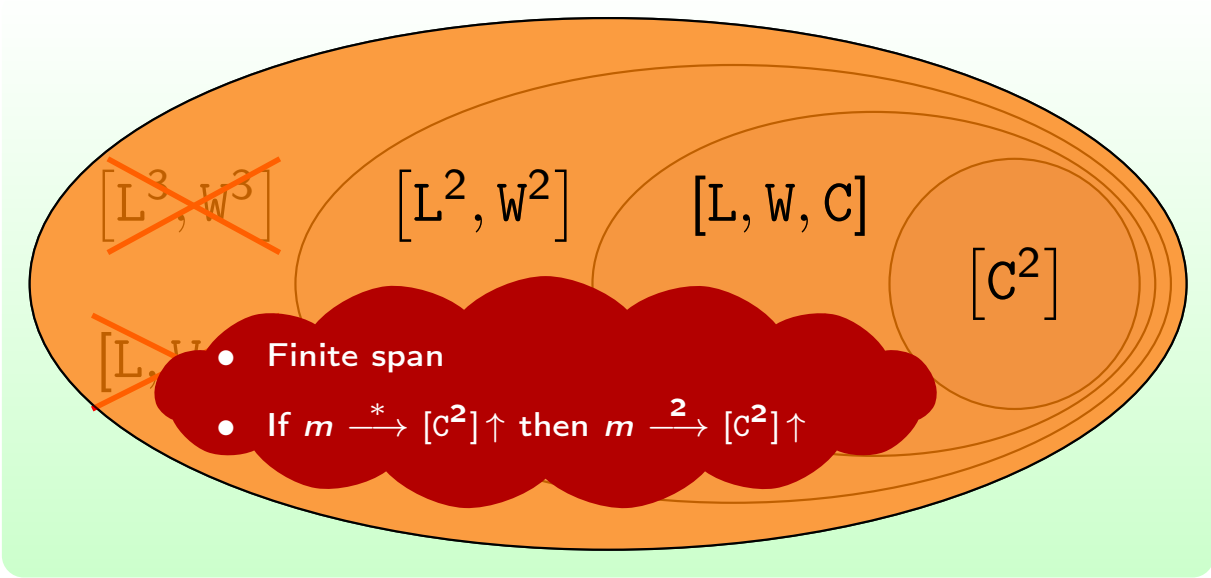
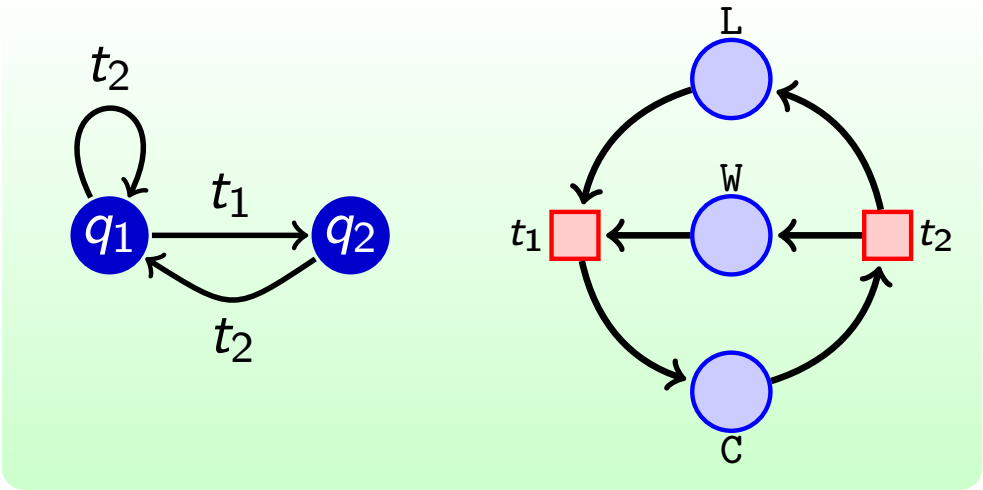




Vector Addition Systems with States (VASS)



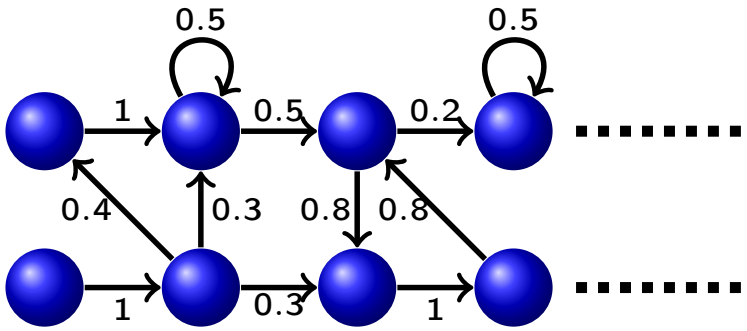
Configurations and Transition Relation

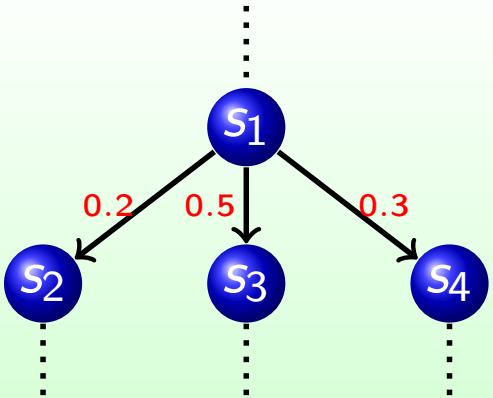
$$(q_1, [\mathtt{L}, \mathtt{W}^3]) \longrightarrow (q_2, [\mathtt{W}^2, \mathtt{C}])$$

The Control State Reachability Problem

- Instance: (q_1, m_1): configuration, q_2: control state
- Question: $\exists m_2.\ (q_1, m_1) \xrightarrow{*} (q_2, m_2)$?

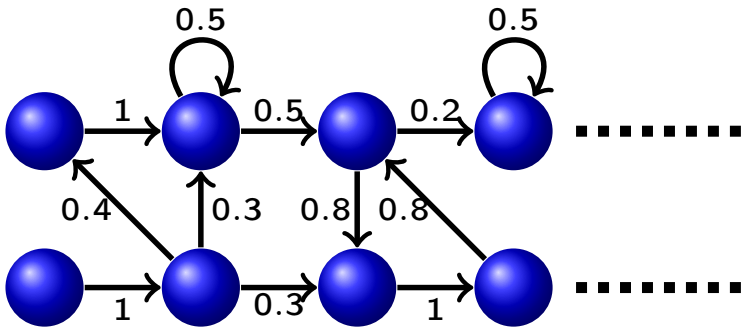
Infinite-State Markov Chains

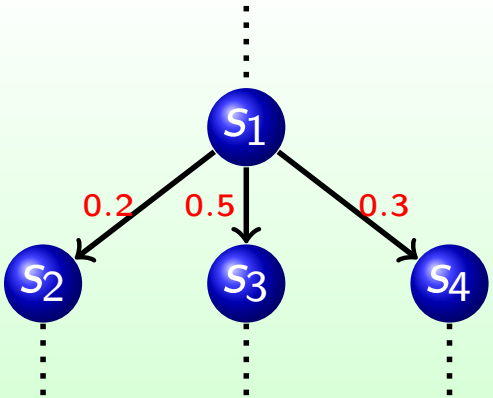




$$P(s_1, s_3) = 0.5$$

Infinite-State Markov Chains

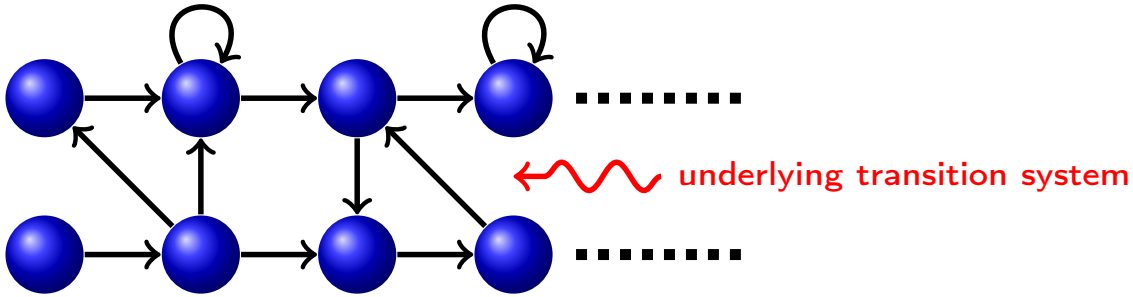


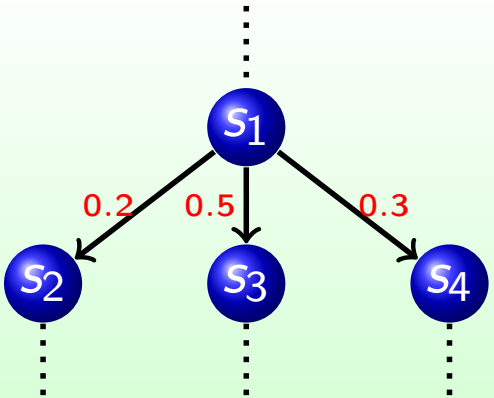


$P(s_1, s_3) = 0.5$

$Prob_s(\phi)$: Probability that a computation from s satisfies ϕ

Infinite-State Markov Chains





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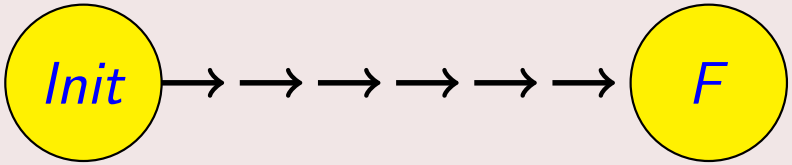
$Prob_s(\phi)$: Probability that a computation from s satisfies ϕ

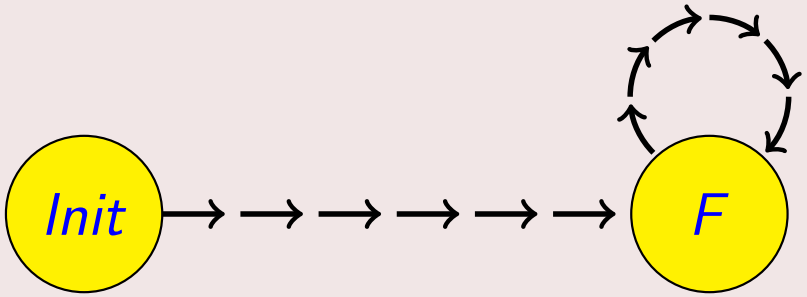
Qualitative Analysis

Qualitative Reachability Analysis



$$Prob_{Init}(\Diamond F) = 1\ ?$$

Qualitative Repeated Reachability Analysis



$$Prob_{Init}(\Box\Diamond F) = 1\ ?$$

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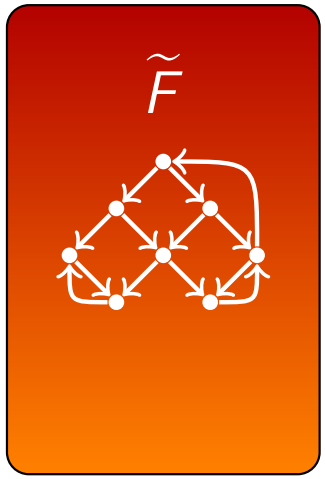
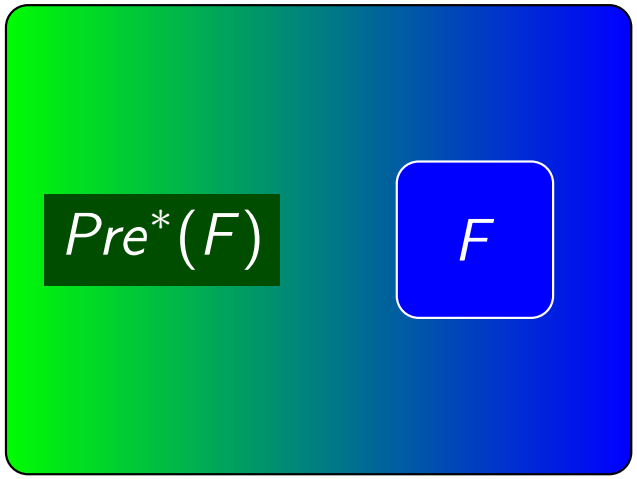
Decisive Markov Chains

Decisive Markov Chains

- Characterized by a simple property
- Covers a large class of systems:
  - Probabilistic Petri nets
  - Probabilistic lossy channel systems
  - Noisy Turing Machines
- Allows qualitative and quantitative analysis

Decisive Markov Chains





$\widetilde{F}$

- states from which F is not reachable
- $\widetilde{F} := \neg\exists\Diamond F$

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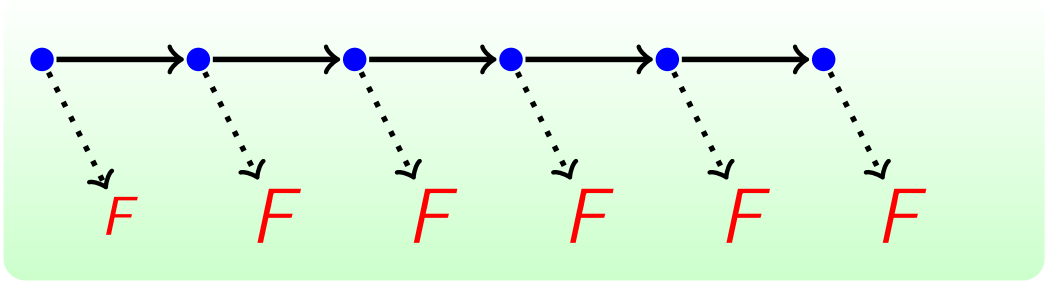
Decisive Markov Chains



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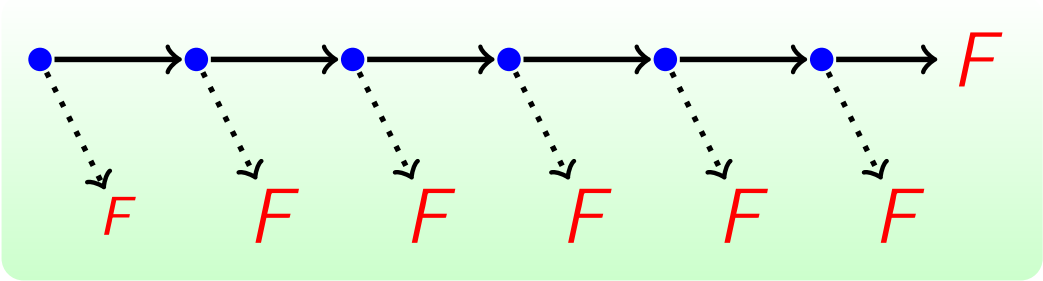

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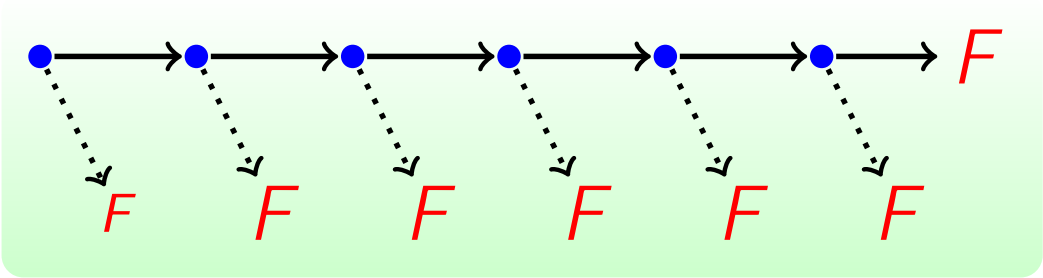

Decisiveness wrt. F

- F always reachable implies F almost certainly reached
- $\forall s.\ Prob_s(\Diamond F \mid \Box\exists\Diamond F) = 1$
- $\forall s.\ Prob_s(\Diamond F \ \vee \ \Diamond\widetilde{F}) = 1$

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Decisive Markov Chains

- All finite-state Markov chains are decisive.
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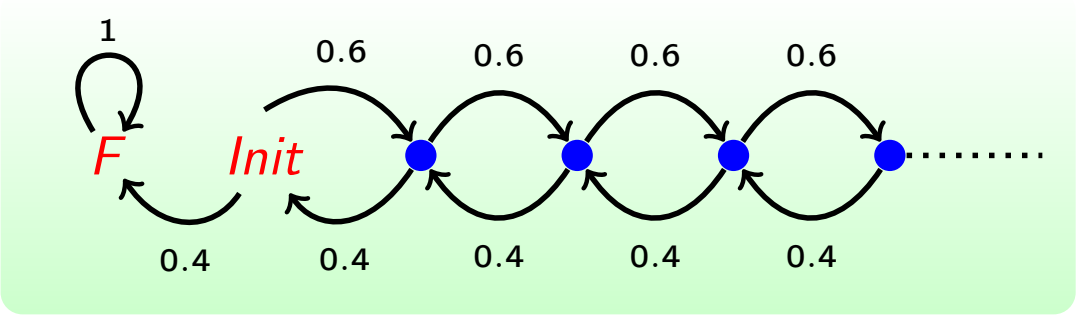

- $\widetilde{F} = \emptyset$
- $Prob_{Init}(\Diamond F) = \frac{2}{3}$
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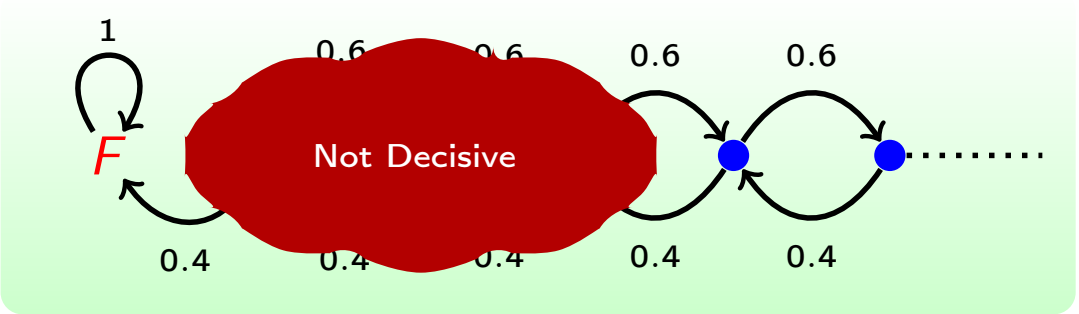

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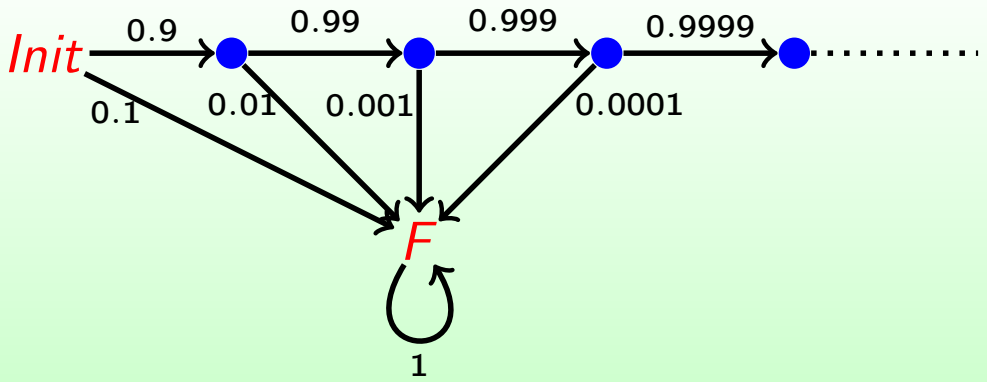

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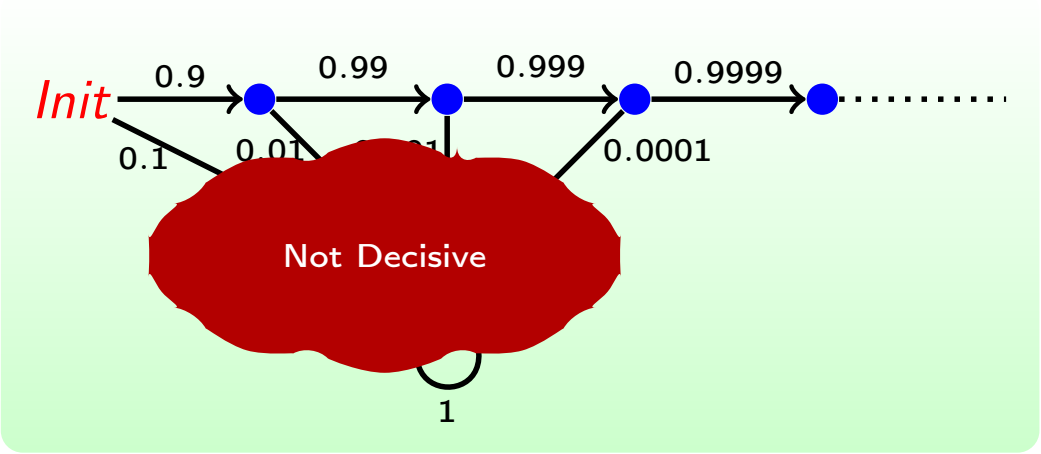

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Decisive Markov Chains

Decisiveness – Sufficient Condition I

- coarseness + finite span:
  - Probabilistic Petri nets
  - Noisy Turing machines

Decisiveness – Sufficient Condition II

- existence of finite attractor:
  - Probabilistic lossy channel systems

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Decisiveness: Sufficient Condition I

Coarseness and Finite Span

α-coarseness (for $\alpha > 0$)

$P(s_1, s_2) > 0$ implies $P(s_1, s_2) \geq \alpha$

K-span (wrt. F)

$s \xrightarrow{*} F$ implies $s \xrightarrow{K} F$

Decisiveness: Sufficient Condition I

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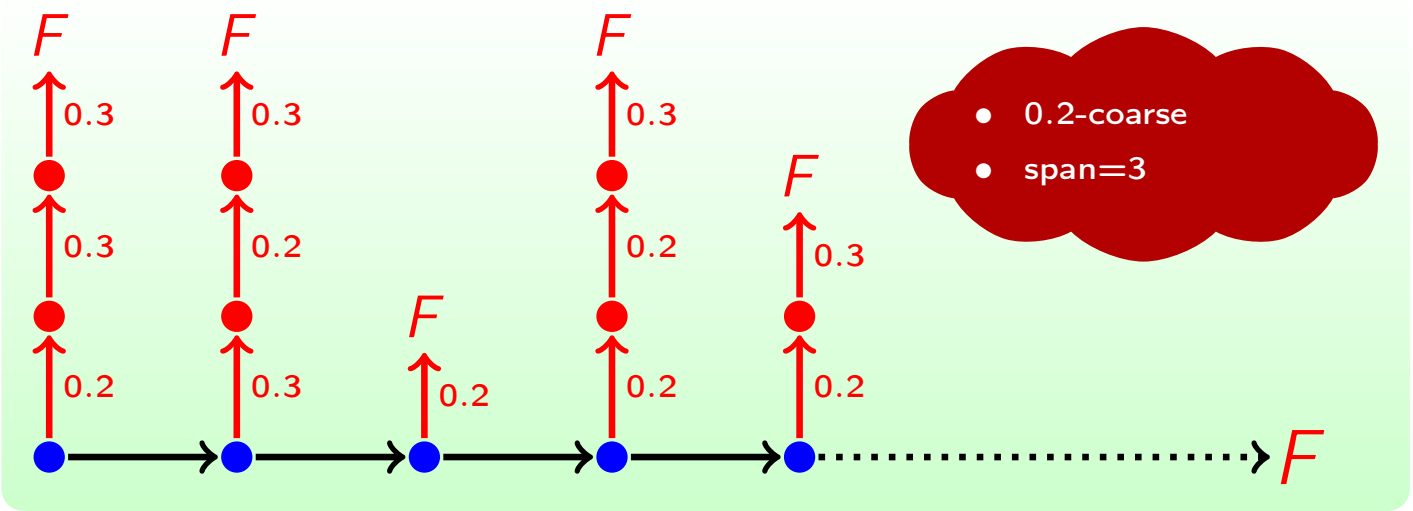
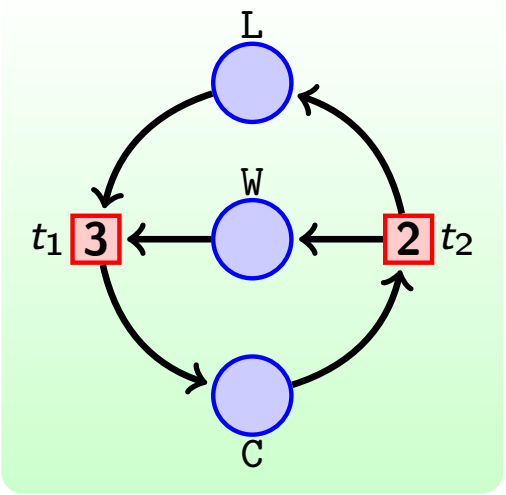


Probabilistic Petri Nets (PPNs)



Weights

- Each transition has a weight
- $P(c_1, c_2)$ decided by:
  - relative weights of enabled transitions

Example

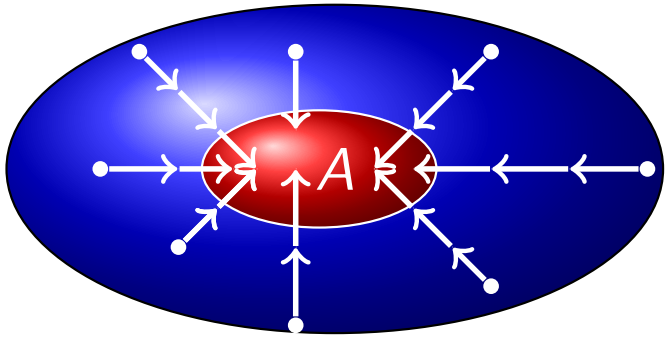
$P([\mathtt{L}, \mathtt{C}^2], [\mathtt{L}^2, \mathtt{W}, \mathtt{C}]) = 1$ $\qquad$ $P([\mathtt{L}, \mathtt{W}, \mathtt{C}^2], [\mathtt{L}^2, \mathtt{W}^2, \mathtt{C}]) = \frac{2}{5}$

finite set of weights $\Longrightarrow$ coarse

PPN $\Longrightarrow$ coarse, finite span $\Longrightarrow$ decisive

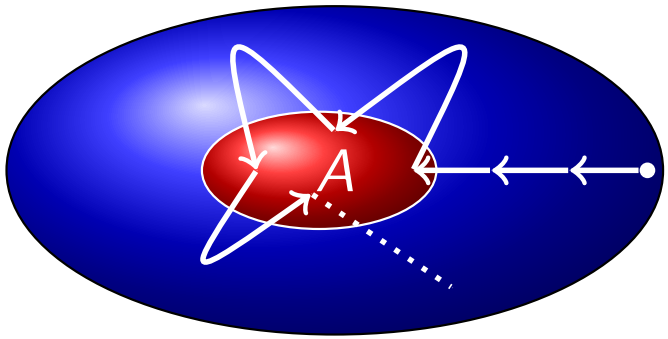
Decisiveness: Sufficient Condition II

Existence of Finite Attractor



$\forall s.\ Prob_s(\Diamond A) = 1$

implies



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Decisiveness: Sufficient Condition II

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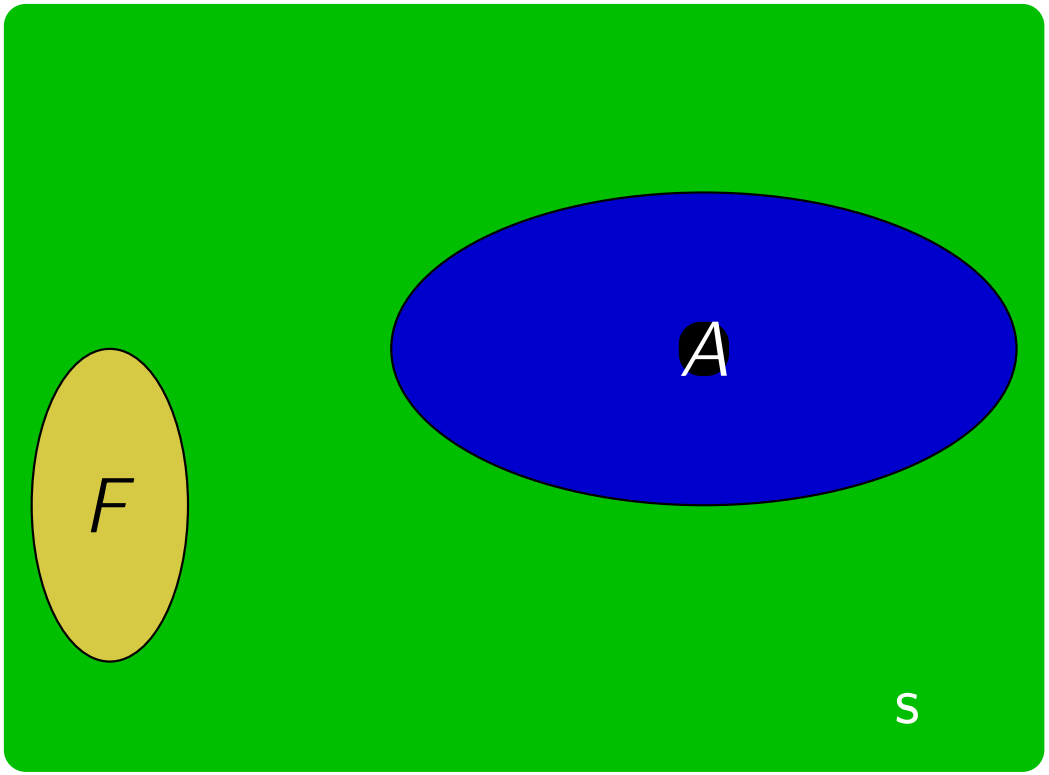
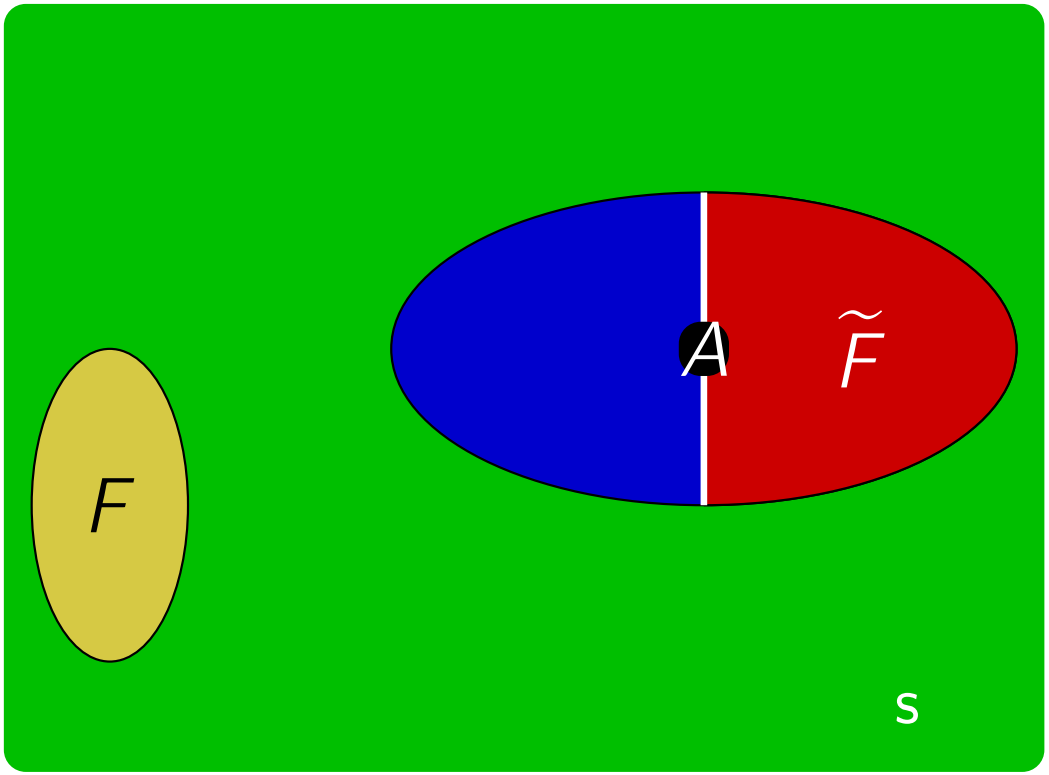


Finite Attractor → implies → Decisiveness

Decisiveness: Sufficient Condition II

Existence of Finite Attractor



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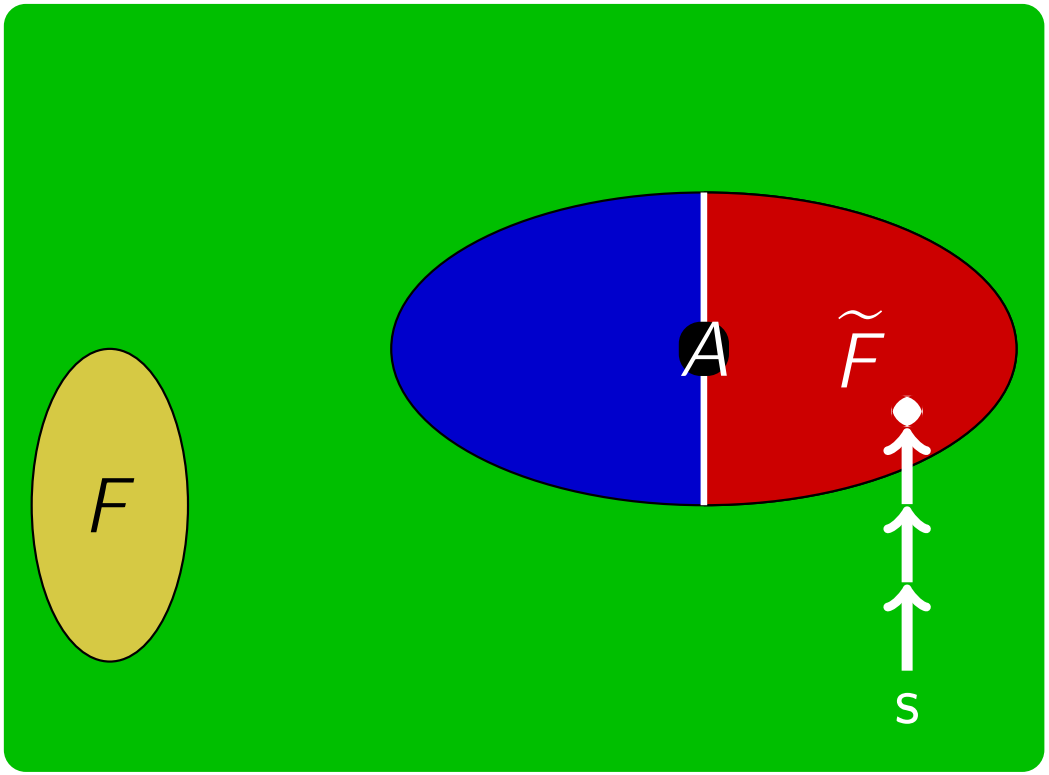

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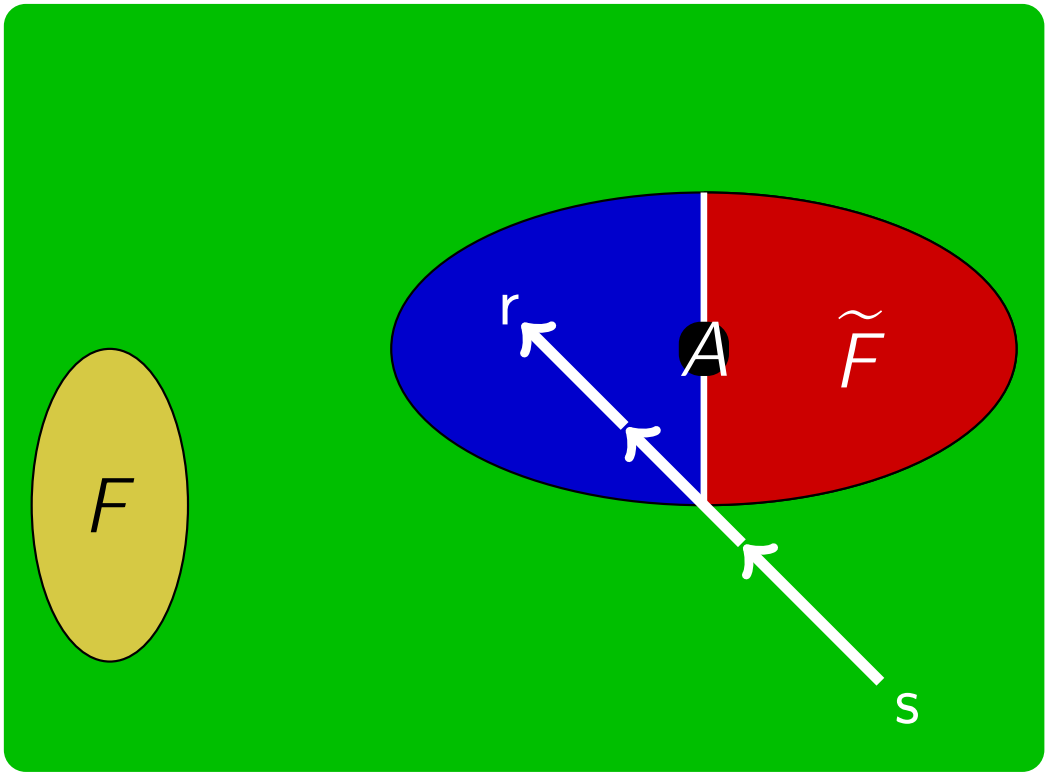

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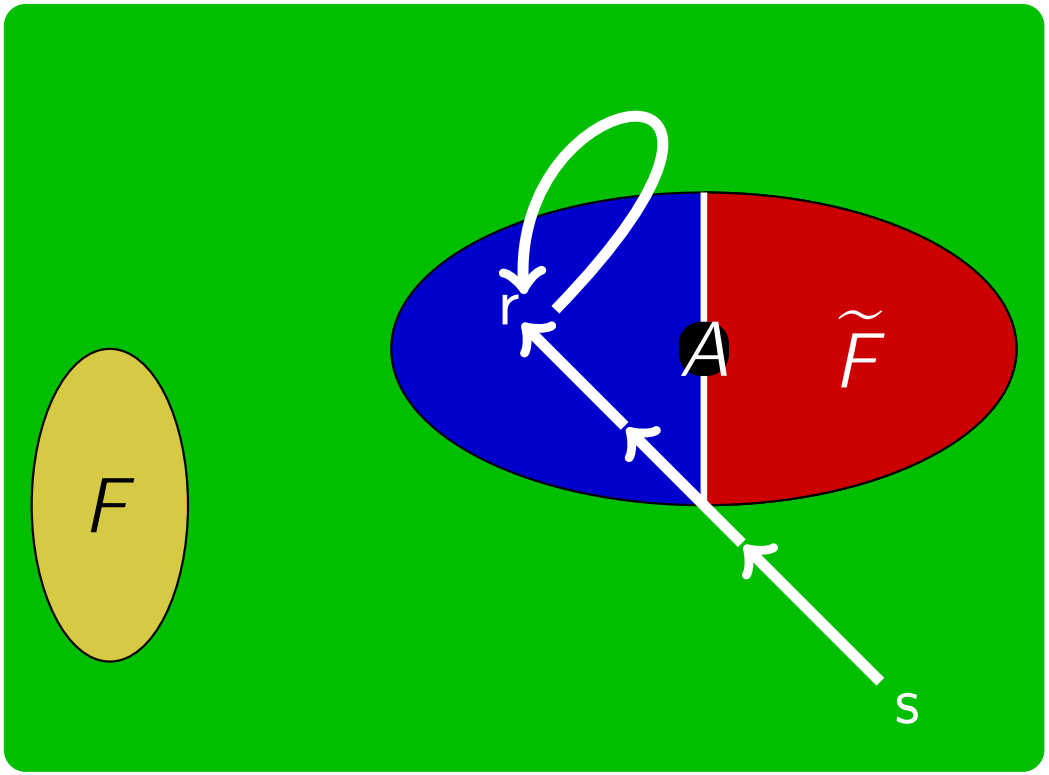

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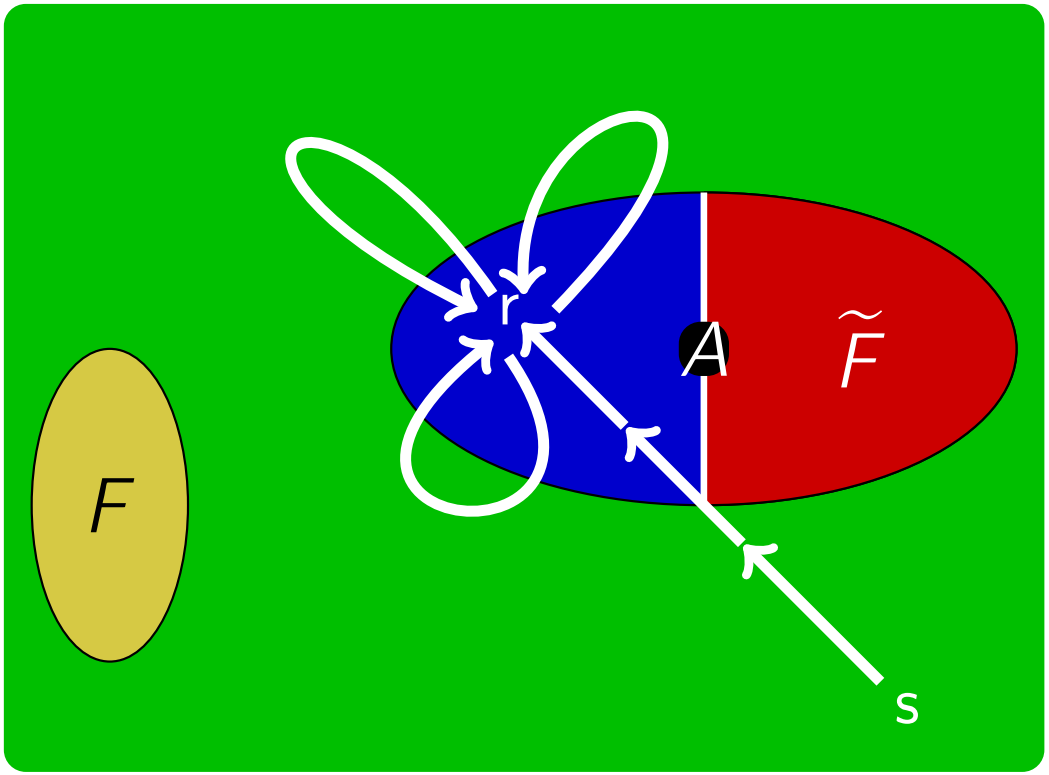

Finite Attractor → implies → Decisiveness

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Decisiveness: Sufficient Condition II

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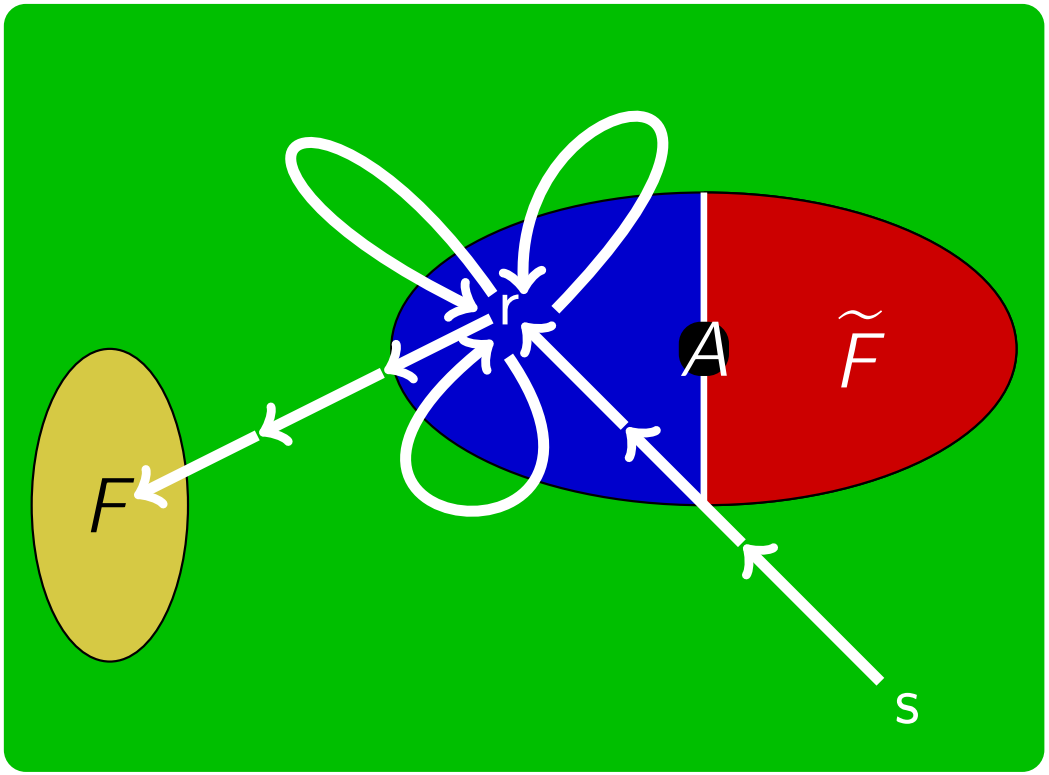
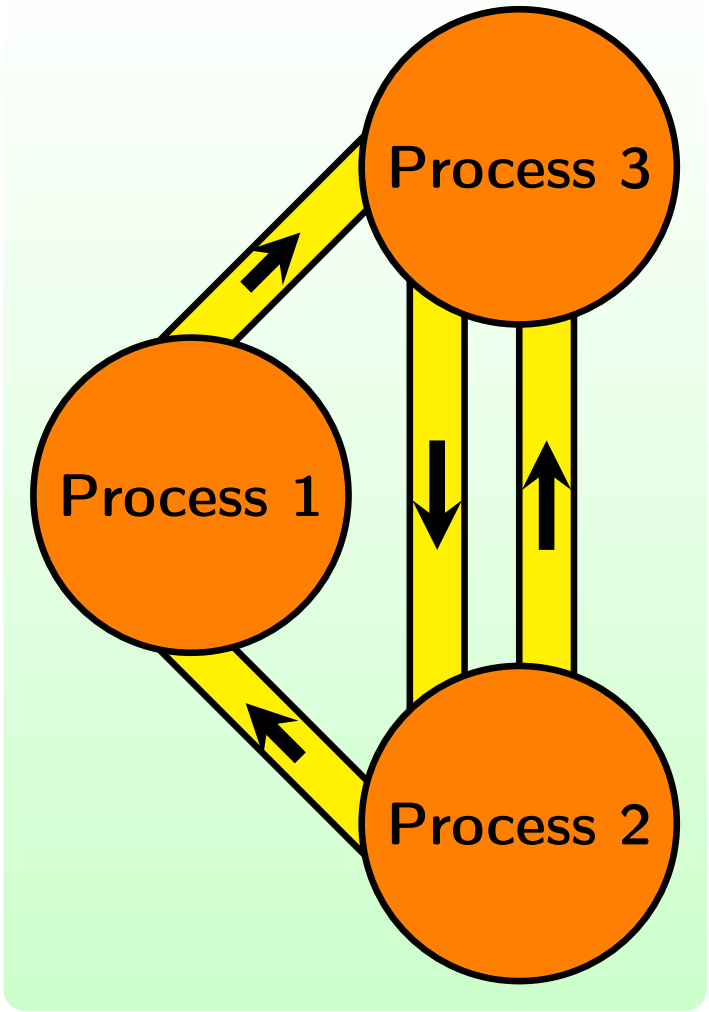
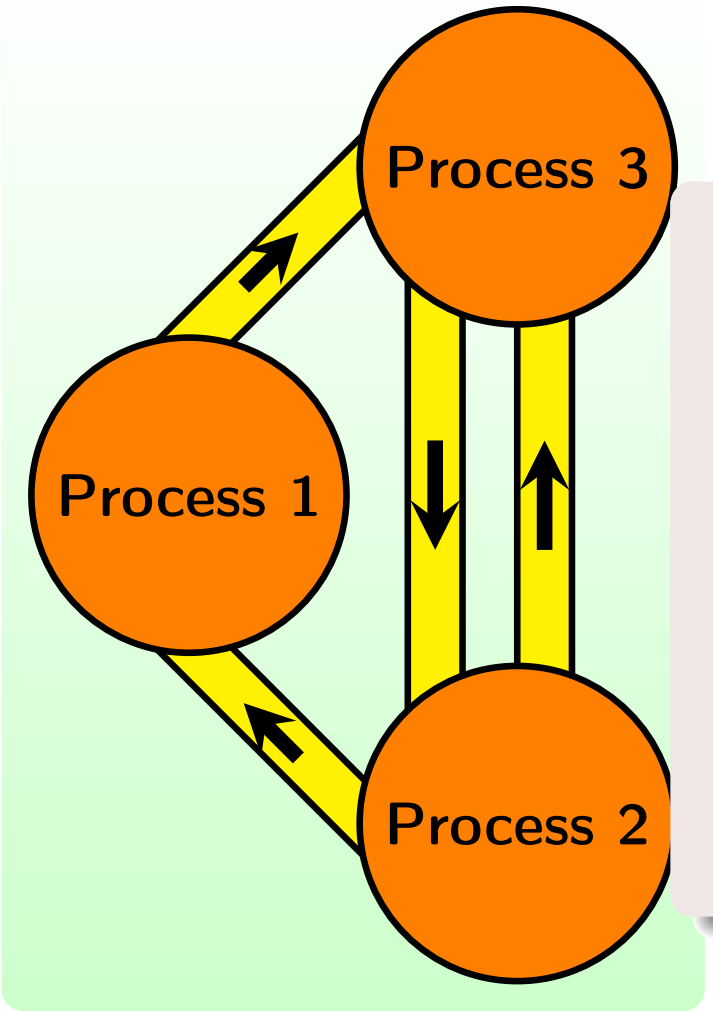


Finite Attractor → implies → Decisiveness

Probabilistic Lossy Channel Systems (PLCS)



Probabilistic Lossy Channel Systems (PLCS)



- Finite-state processes
- Unbounded lossy channels
- Send and receive transitions
- Each transition:
  - each message lost with a fixed probability $\lambda > 0$
- Finite attractors:
  - set of configurations with empty channels

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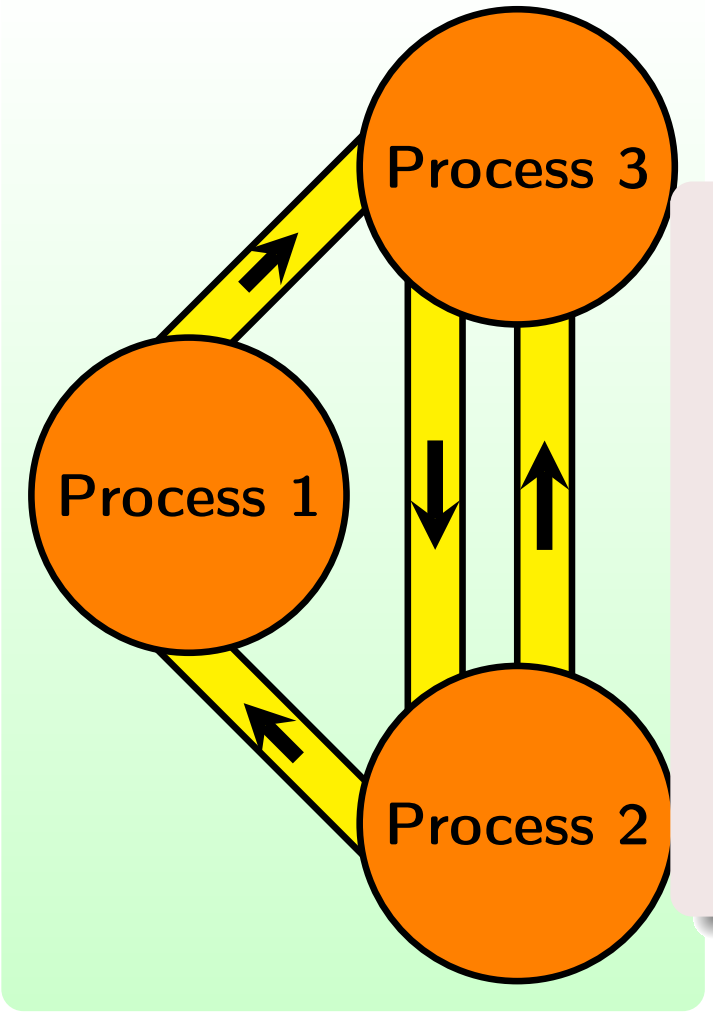

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PLCS $\Longrightarrow$ Finite Attractor $\Longrightarrow$ Decisive

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Qualitative Reachability Analysis

$$Prob_{Init}(\Diamond F) = 1$$

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Qualitative Reachability Analysis

$Prob_{Init}(\Diamond F) = 1$

- analyze underlying transition system
- structural properties: reachability of F and $\widetilde{F}$

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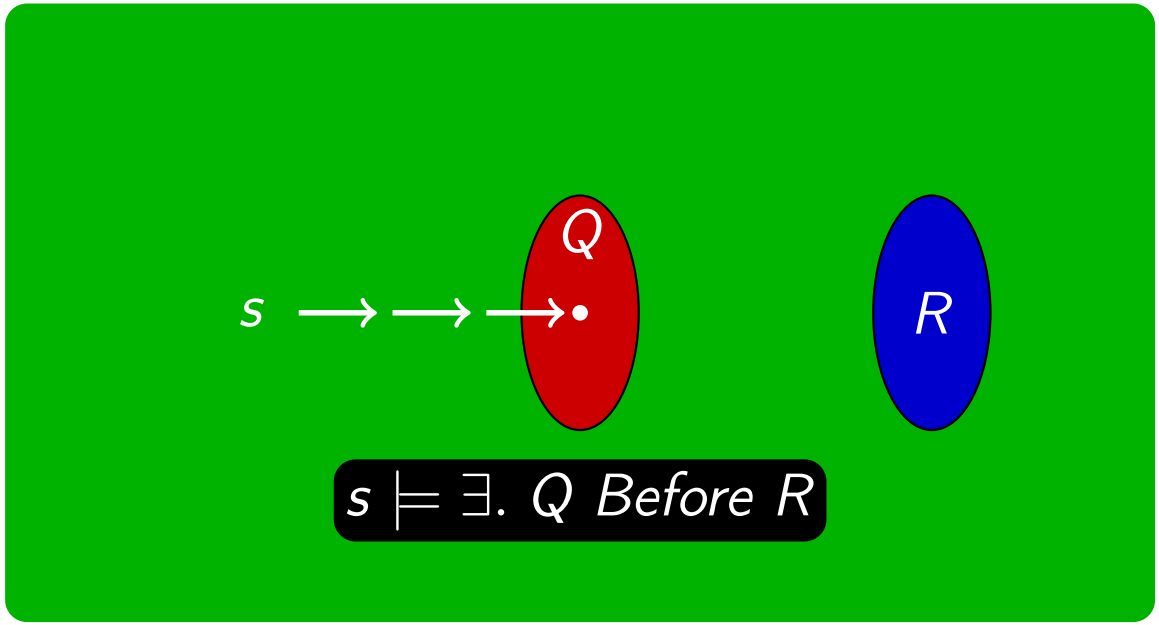
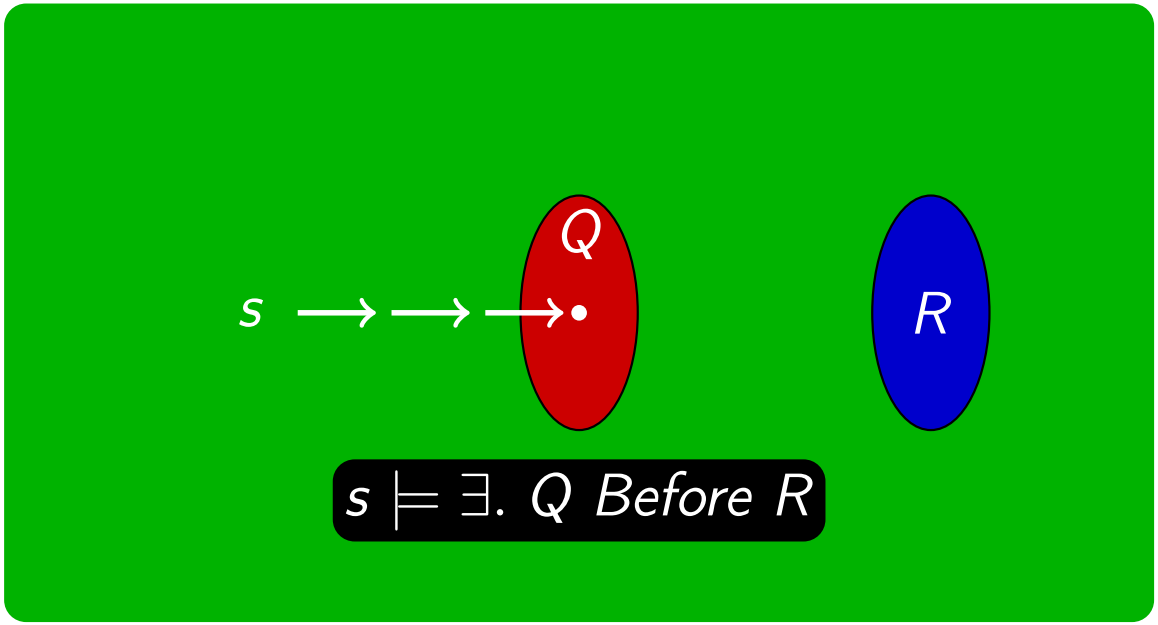


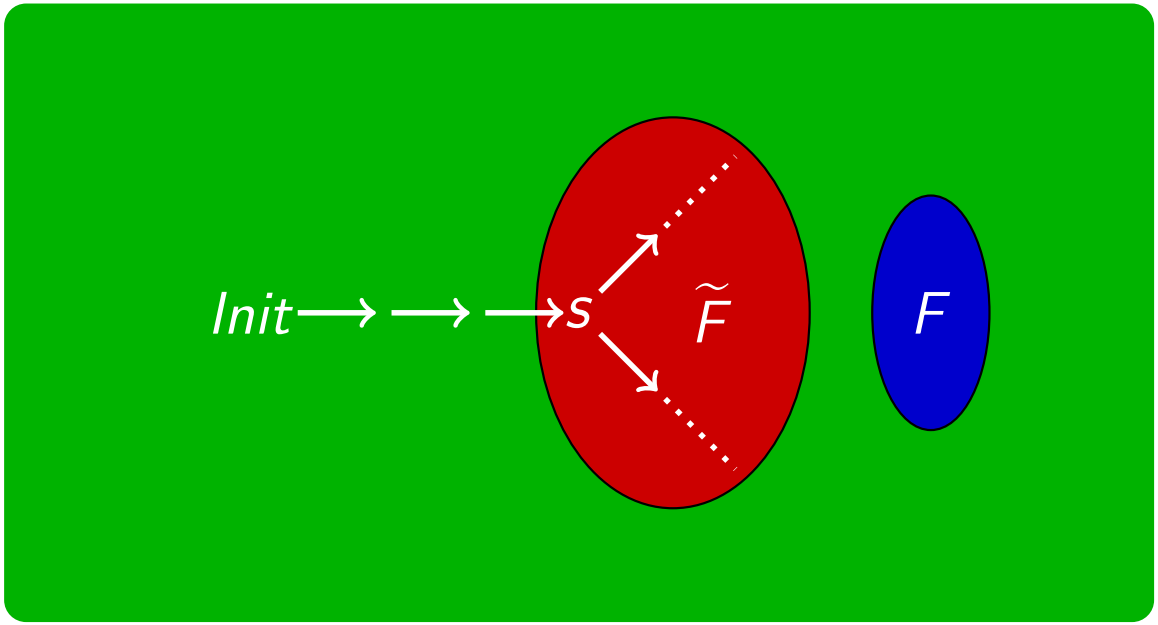
Qualitative Reachability Analysis

$Prob_{Init}(\Diamond F) = 1 \iff Init \not\models \exists.\ \widetilde{F}\ Before\ F$



Qualitative Reachability Analysis

$Prob_{Init}(\Diamond F) < 1 \quad \Longleftarrow \quad Init \models \exists.\ \widetilde{F}\ Before\ F$



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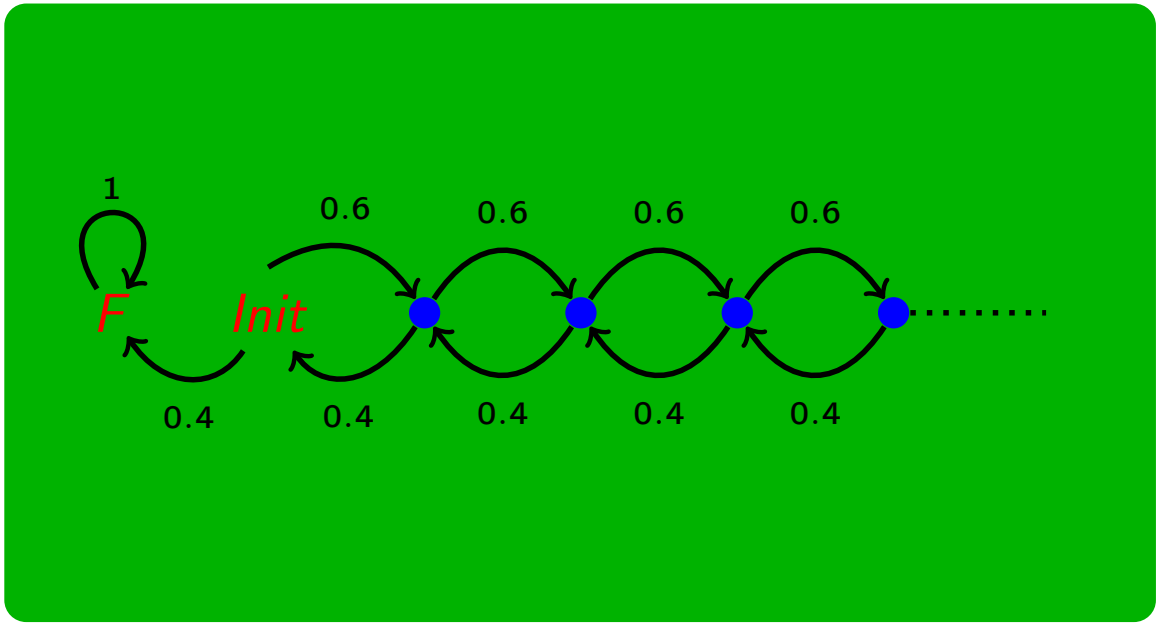

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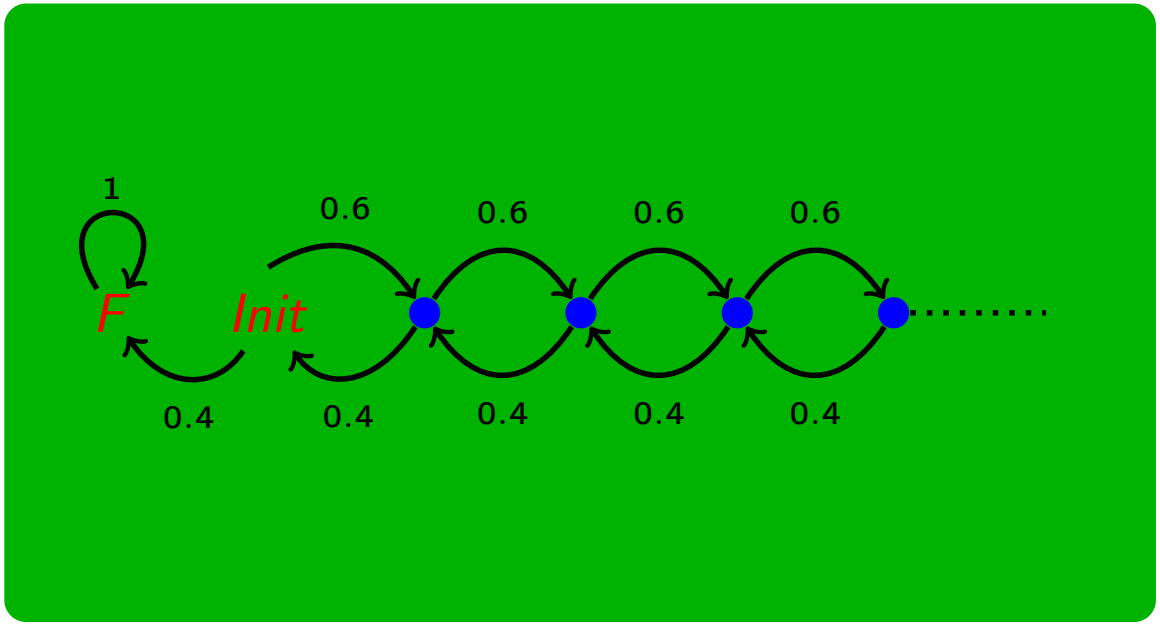

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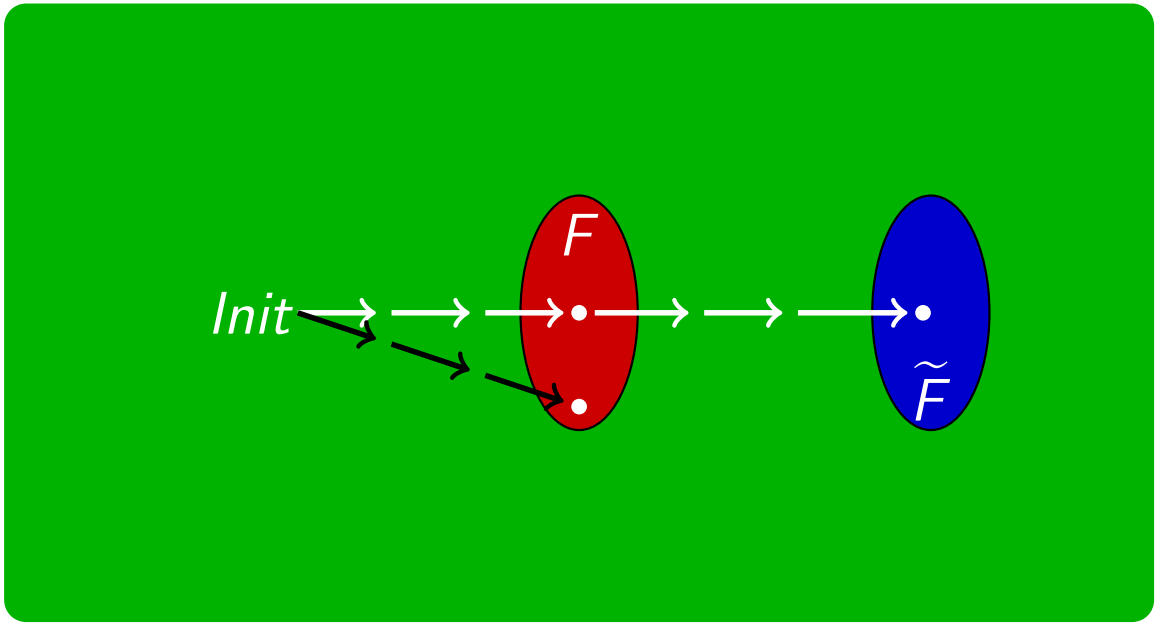

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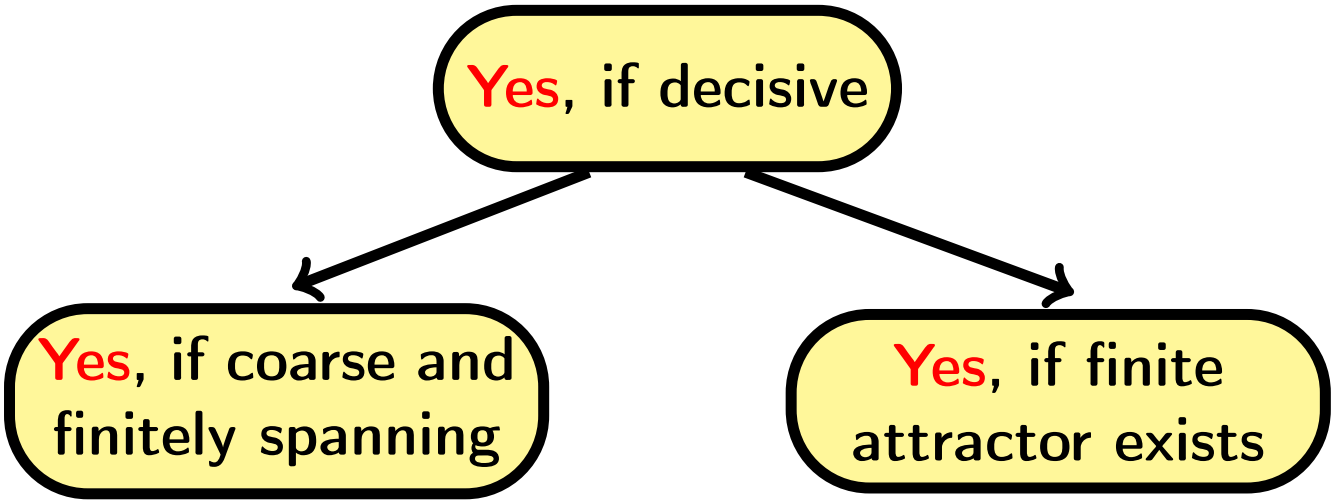

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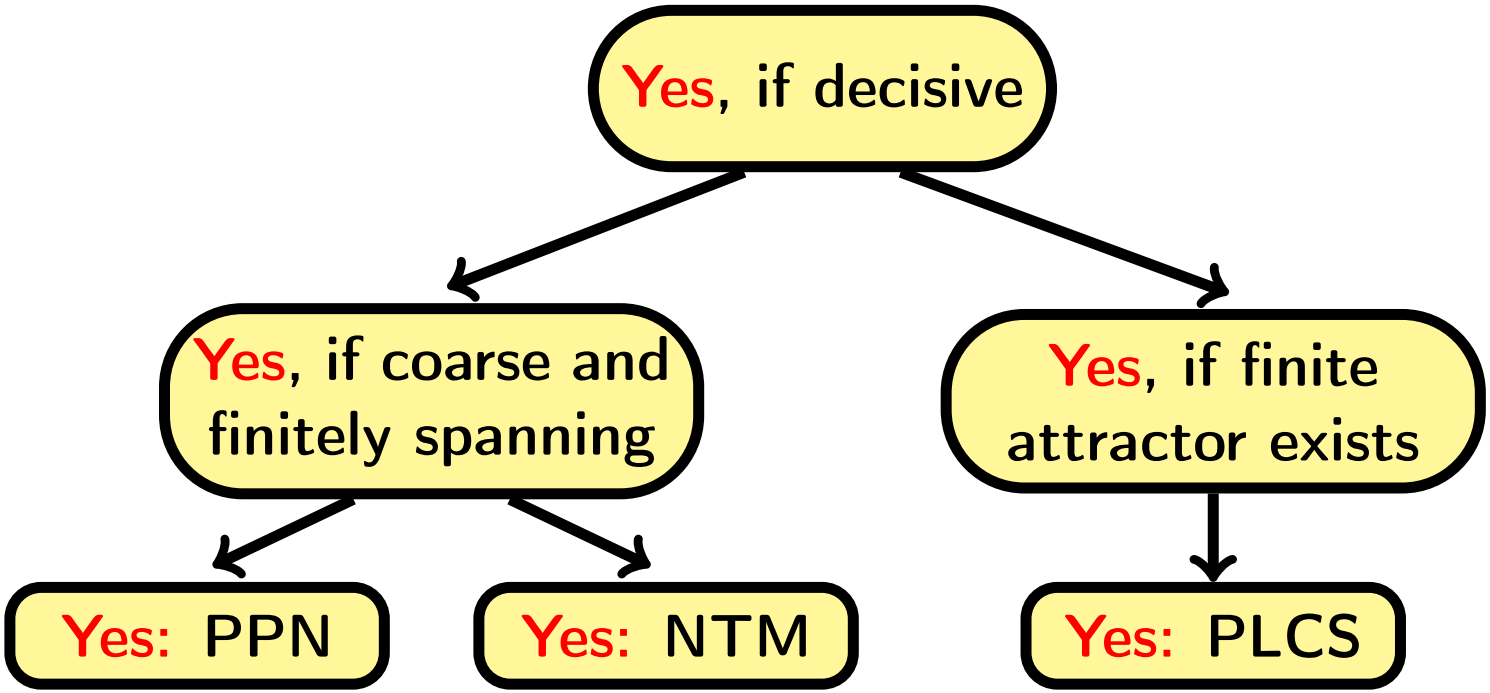

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Qualitative Reachability Analysis

Can we check $\mathit{Init} \models \exists.\ \widetilde{F}\ \mathit{Before}\ F$?

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Yes: PPN – F set of control states

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Yes: : NTM

Yes: : PLCS

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Qualitative Repeated Reachability Analysis

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$$Prob_{Init}(\Box\Diamond F) = 1 \iff Init \models \forall\Box\,\exists\Diamond F$$

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$Prob_{Init}(\Box\Diamond F) < 1$ $\Longleftarrow$ $Init \not\models \forall\Box\,\exists\Diamond F$

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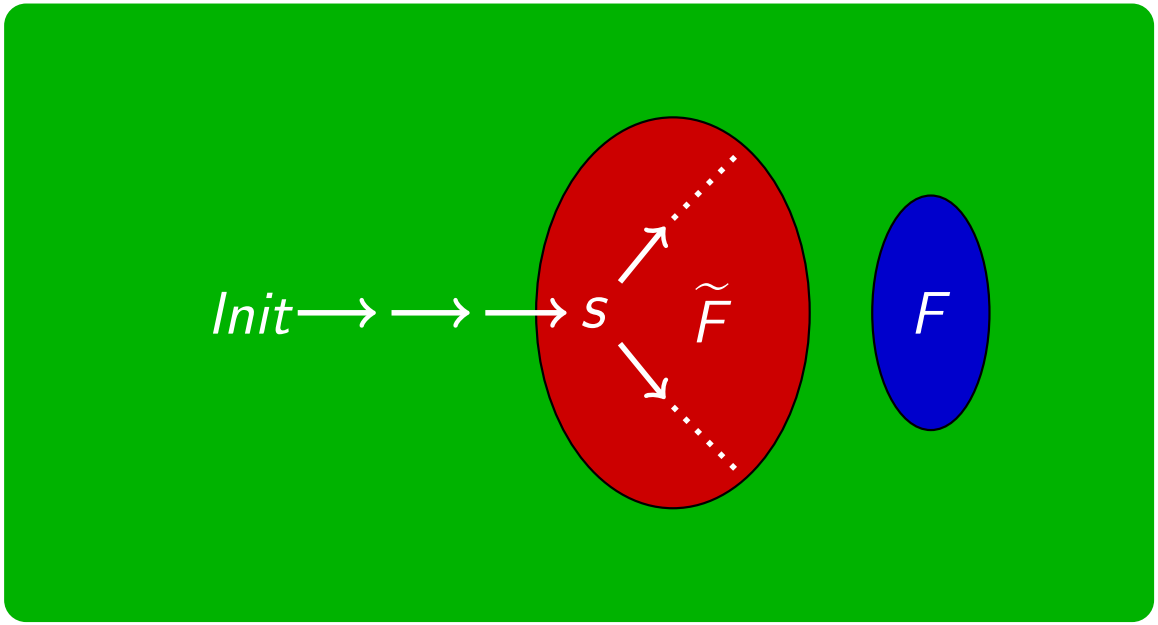

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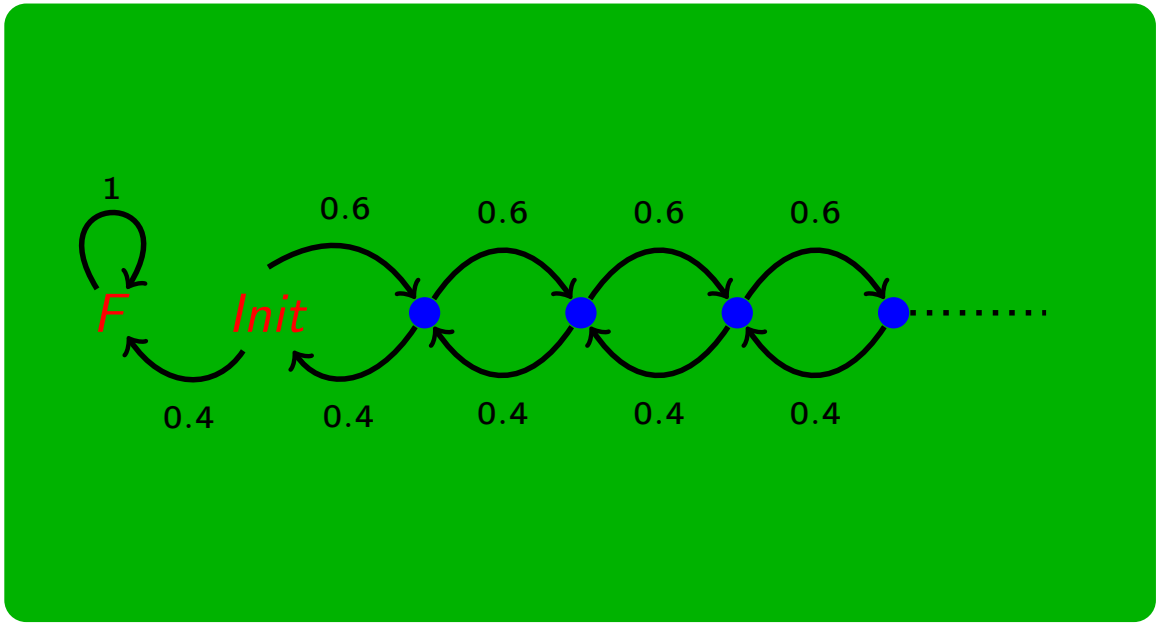

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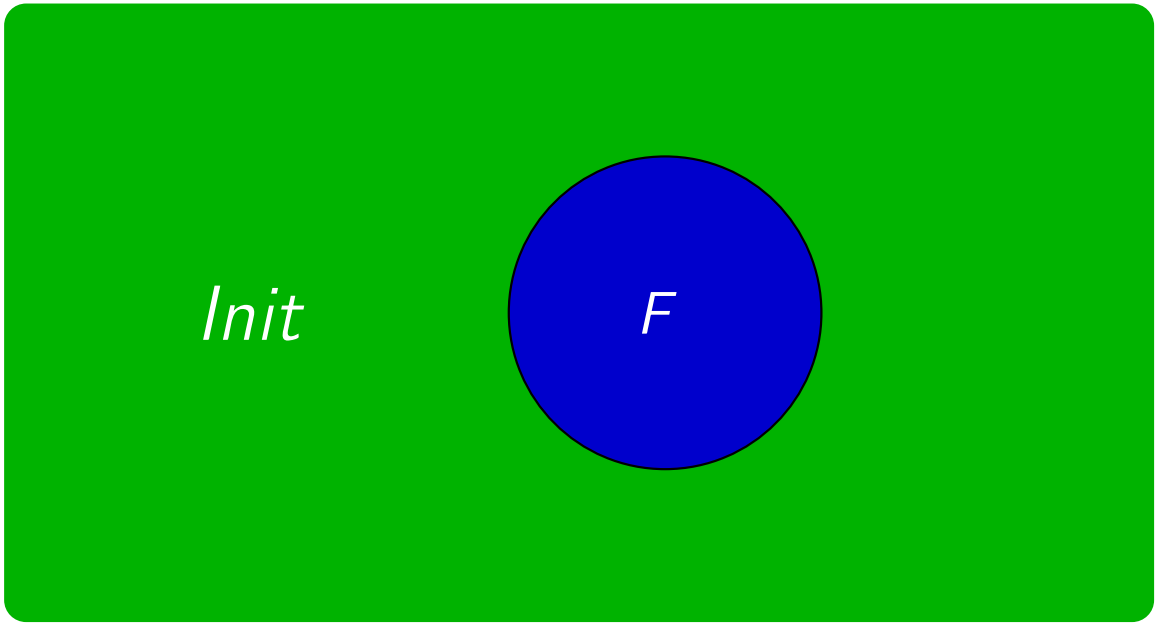

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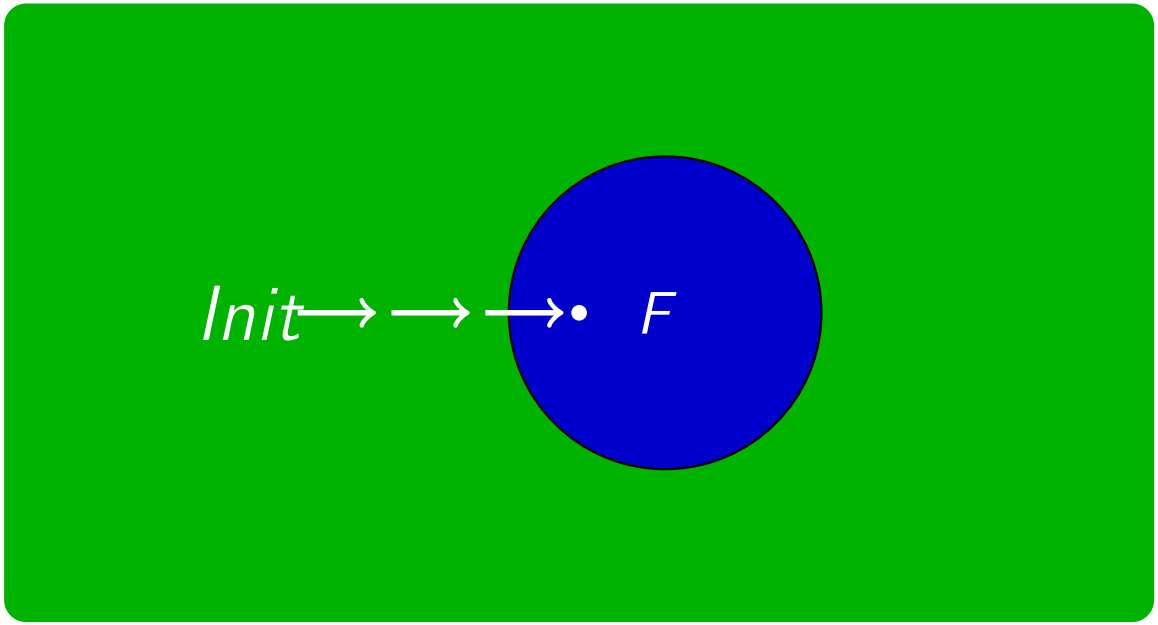

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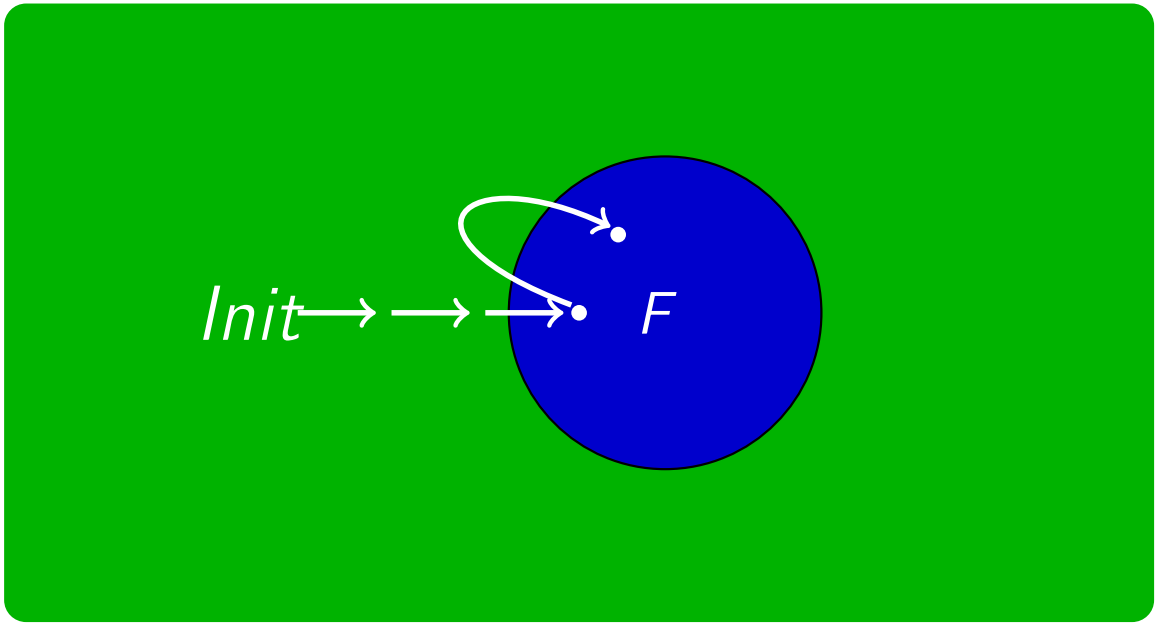

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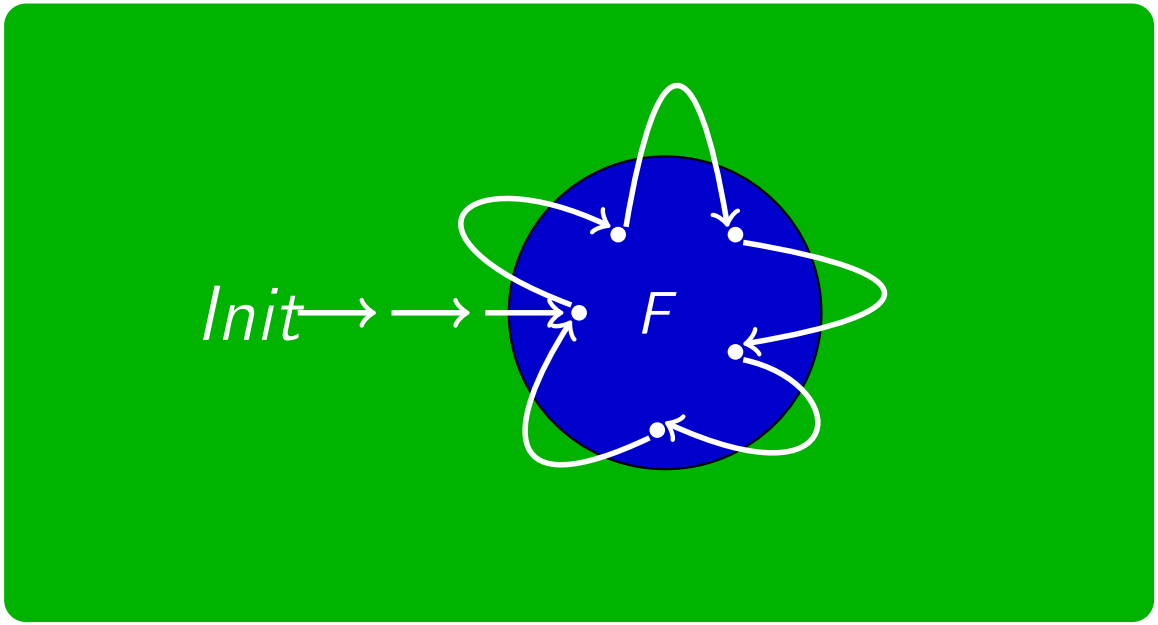
Yes, if decisive wrt. F



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Qualitative Repeated Reachability Analysis

$$Prob_{Init}(\Box\Diamond F) = 1 \iff Init \models \forall\Box\,\exists\Diamond F$$

Qualitative Repeated Reachability Analysis

$Prob_{Init}(\Box\Diamond F) = 1 \iff Init \models \forall\Box\,\exists\Diamond F$

Yes, if decisive

Qualitative Repeated Reachability Analysis

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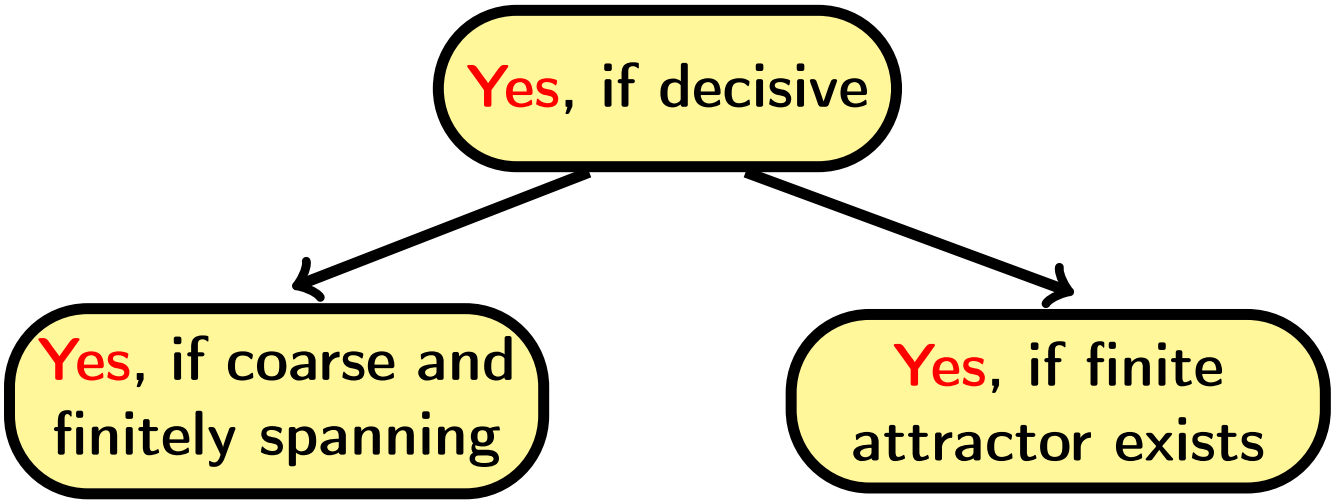
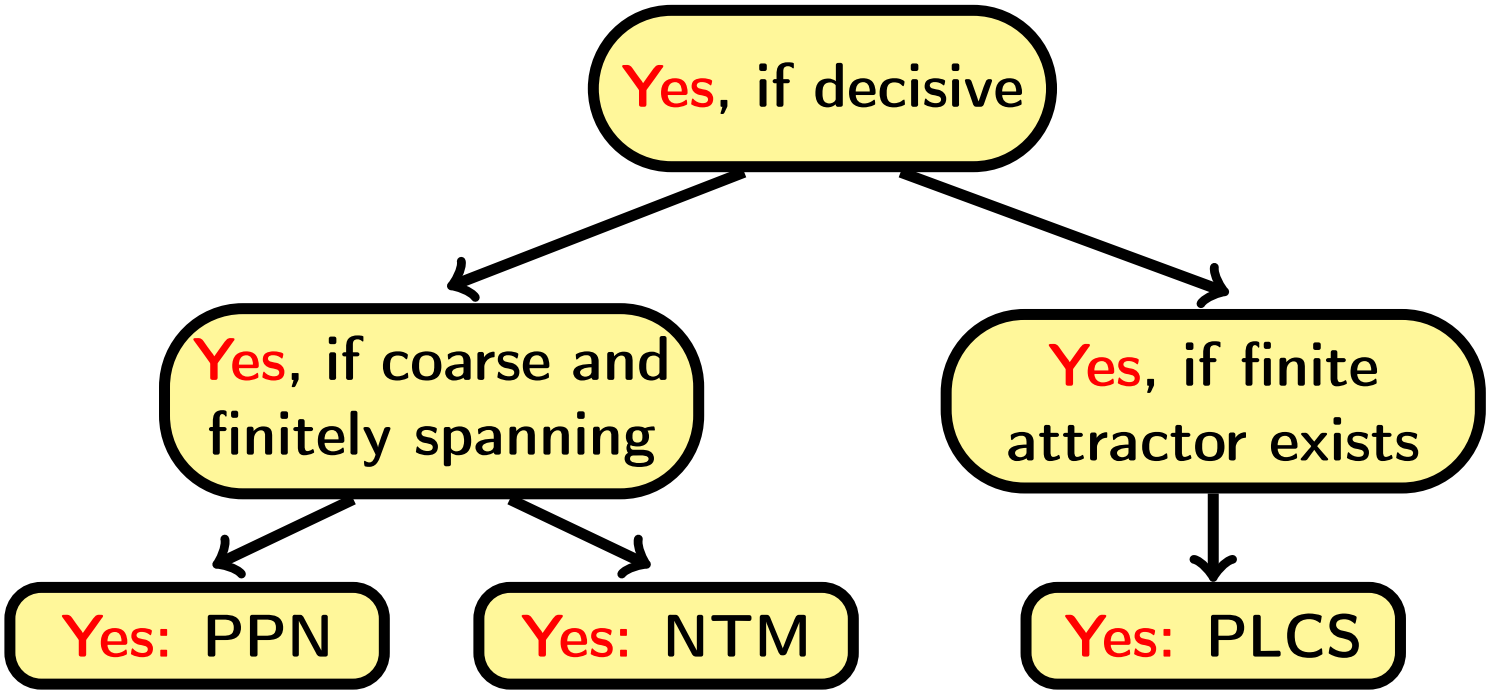


Qualitative Repeated Reachability Analysis

$Prob_{Init}(\Box\Diamond F) = 1 \iff Init \models \forall\Box\,\exists\Diamond F$



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Qualitative Repeated Reachability Analysis

Can we check $\mathit{Init} \models \forall\Box\,\exists\Diamond F$?

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Qualitative Repeated Reachability Analysis

Can we check $\mathit{Init} \models \forall\Box\,\exists\Diamond F$?

Yes: PPN – F set of control states

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Qualitative Repeated Reachability Analysis

Can we check $\mathit{Init} \models \forall\Box\,\exists\Diamond F$?

Yes: PPN – F set of control states

Yes: PPN – F upward set ⟵ decidable

Qualitative Repeated Reachability Analysis

Can we check $\mathit{Init} \models \forall\Box\,\exists\Diamond F$?

Yes: PPN – F set of control states

Yes: PPN – F upward set ⟵ decidable

Yes: : NTM

Yes: : PLCS

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Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Diamond F) \leq \rho + \epsilon$

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Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Diamond F) \leq \rho + \epsilon$

Yes

No

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Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
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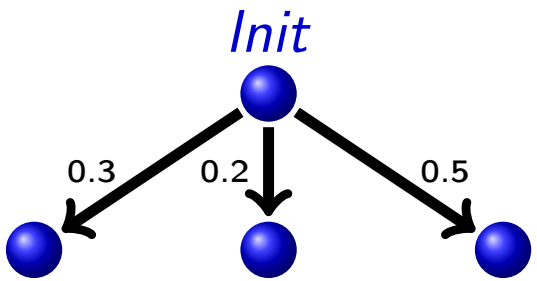

Yes

No

Quantitative Reachability Analysis

Problem

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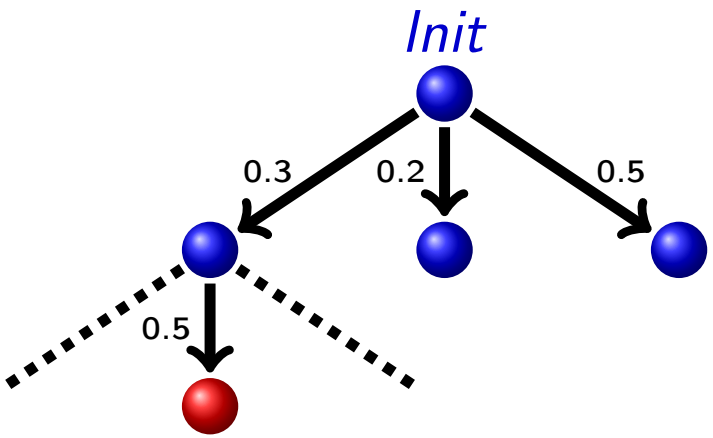

Yes

No

Quantitative Reachability Analysis

Problem

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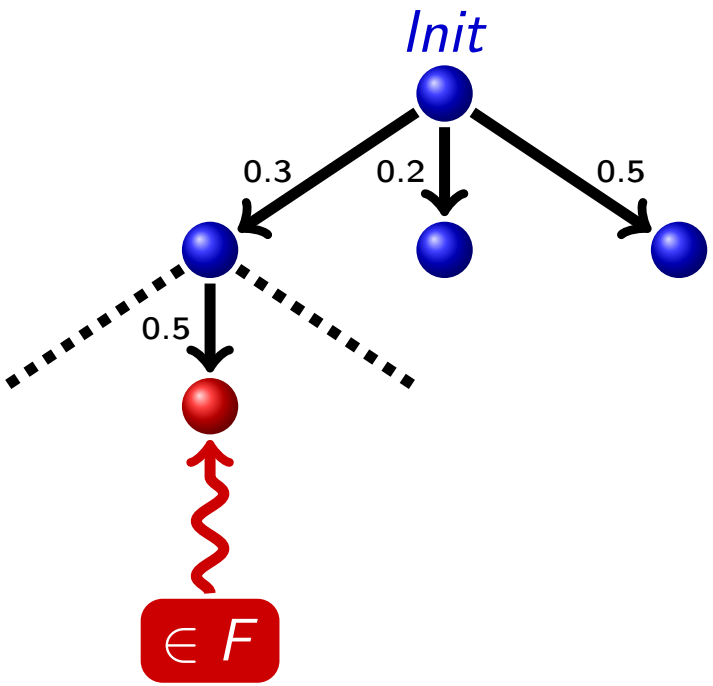

Yes:= Yes + 0.15

No

Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
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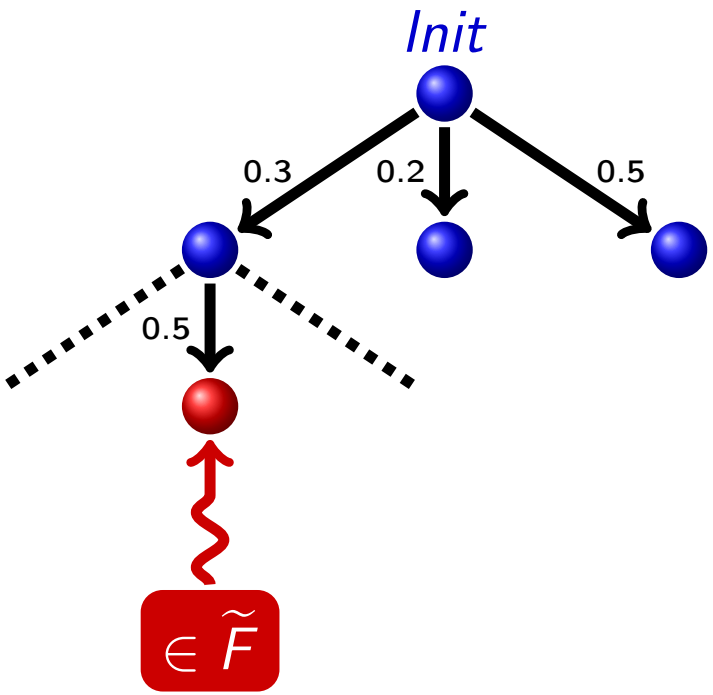


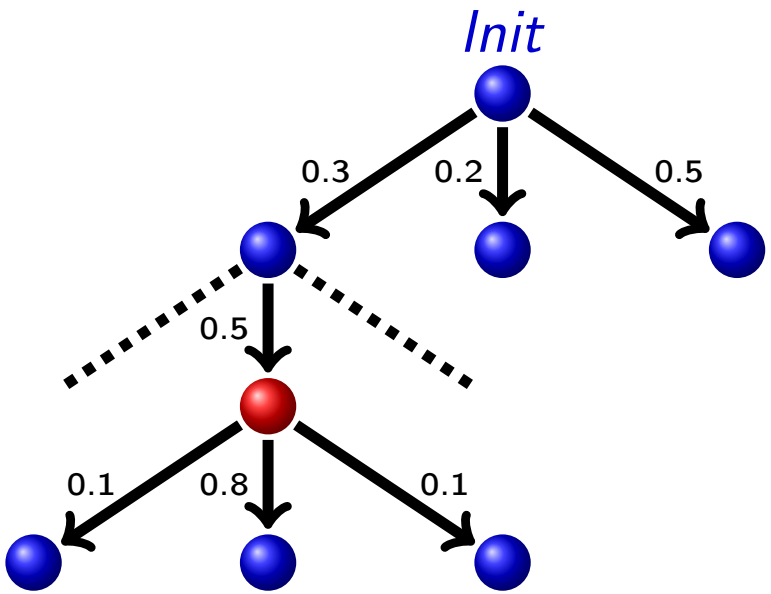
Yes

No:=No+0.15

Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Diamond F) \leq \rho + \epsilon$



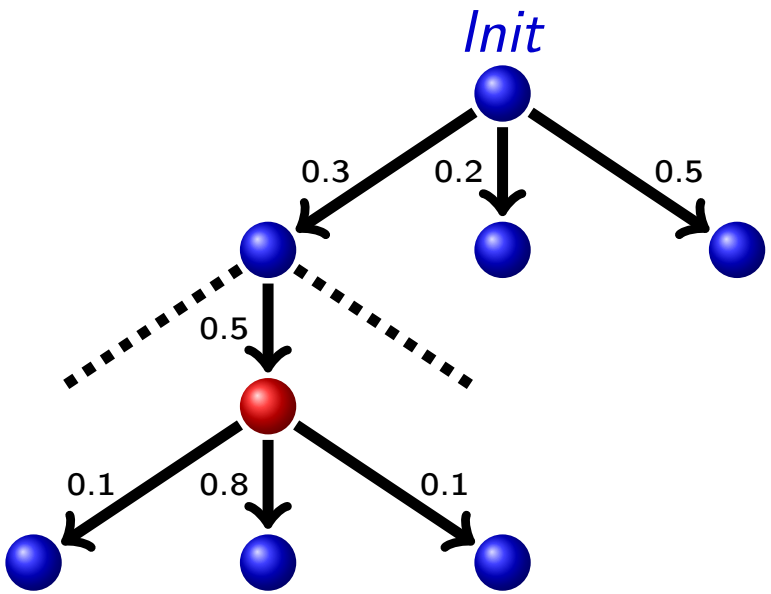
Yes

No

Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Diamond F) \leq \rho + \epsilon$

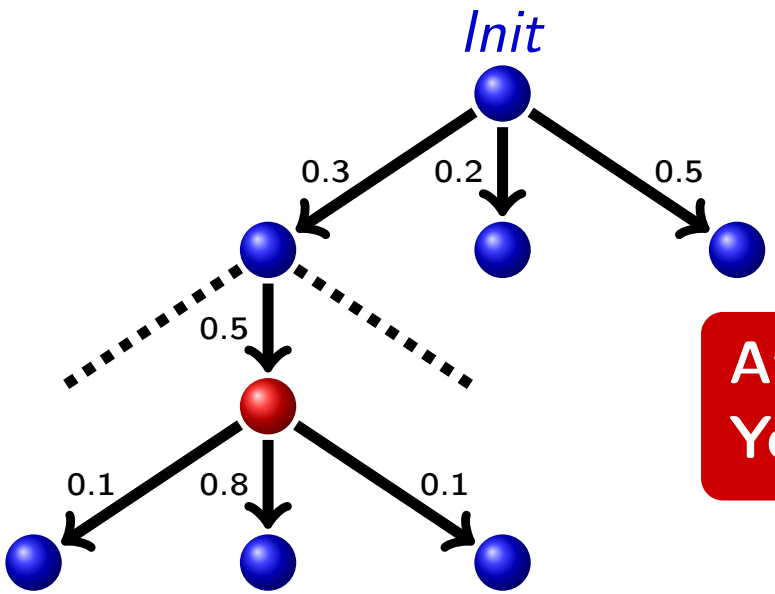


Until Yes + No $> 1 - \epsilon$

Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Diamond F) \leq \rho + \epsilon$

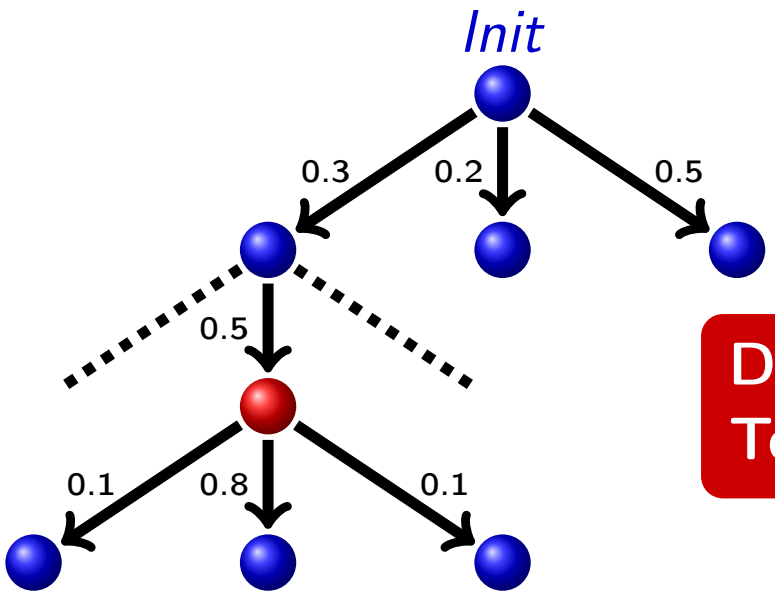


After termination:
Yes $\leq Prob_{Init}(\Diamond \mathbf{F}) \leq$ **Yes** $+ \epsilon$

Quantitative Reachability Analysis

Problem

- Compute $Prob_{Init}(\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Diamond F) \leq \rho + \epsilon$



Decisiveness:
Termination guaranteed

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Quantitative Repeated Reachability Analysis

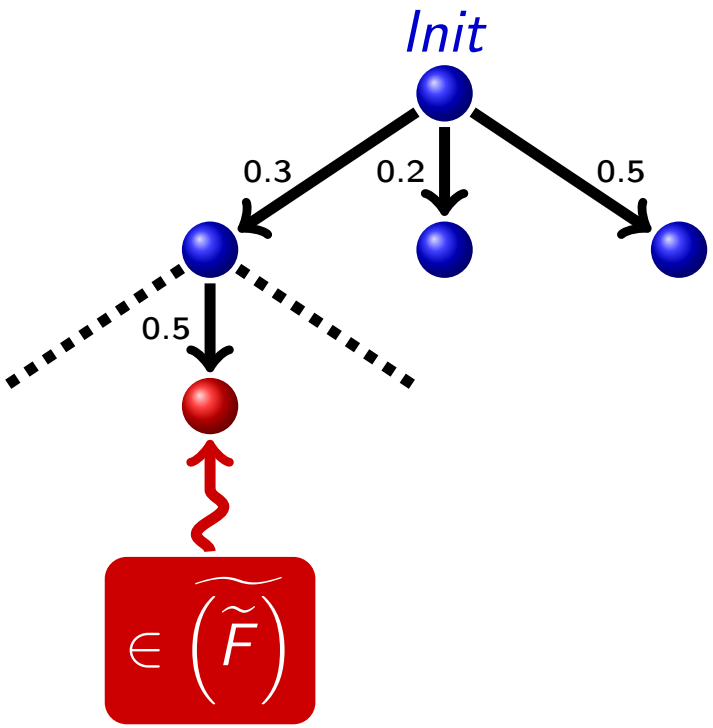
Problem

- Compute $Prob_{Init}(\Box\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Box\Diamond F) \leq \rho + \epsilon$

Quantitative Repeated Reachability Analysis

Problem

- Compute $Prob_{Init}(\Box\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Box\Diamond F) \leq \rho + \epsilon$

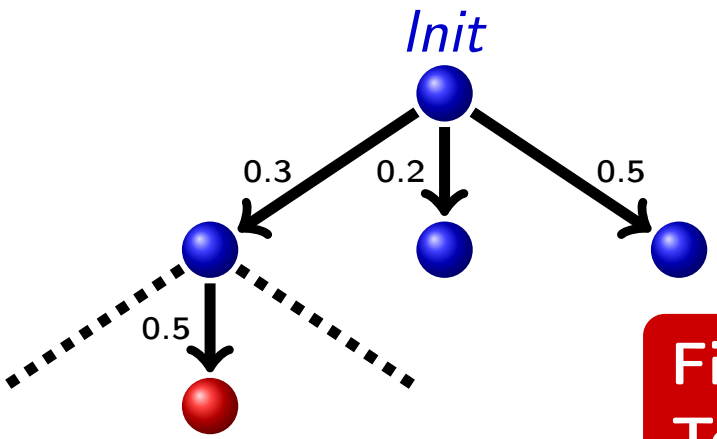


$\mathbf{Yes} := \mathbf{Yes} + \mathbf{0.15}$

Quantitative Repeated Reachability Analysis

Problem

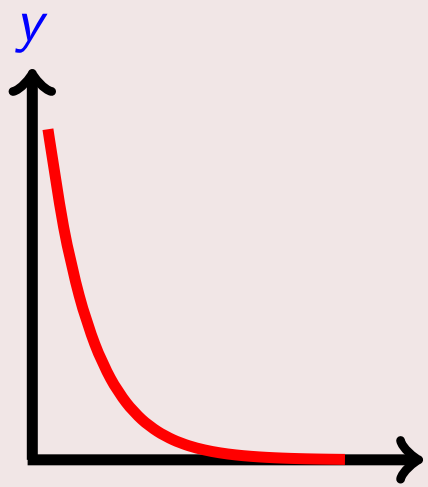
- Compute $Prob_{Init}(\Box\Diamond F)$
- Approximation (with arbitrary precision ϵ): compute ρ s.t. $\rho \leq Prob_{Init}(\Box\Diamond F) \leq \rho + \epsilon$



Finite attractor:
Termination guaranteed

Extensions

Eager Markov Chains (wrt. F)

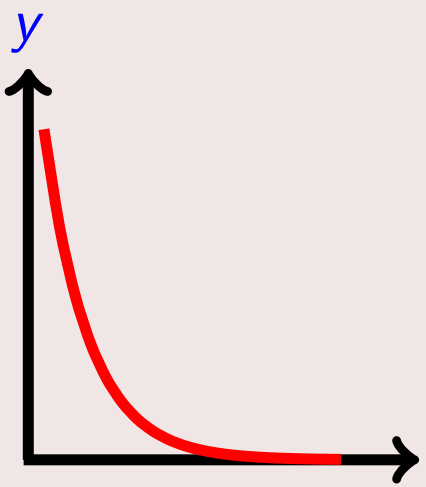


x: number of steps of computation

y: probability of reaching F in x or more steps

Extensions

Eager Markov Chains (wrt. F)



x: number of steps of computation

y: probability of reaching F in x or more steps

Examples

- Probabilistic Petri Nets
- Probabilistic Lossy Channel Systems
- Noisy Turing Machines

Property

- Expected Cost of runs

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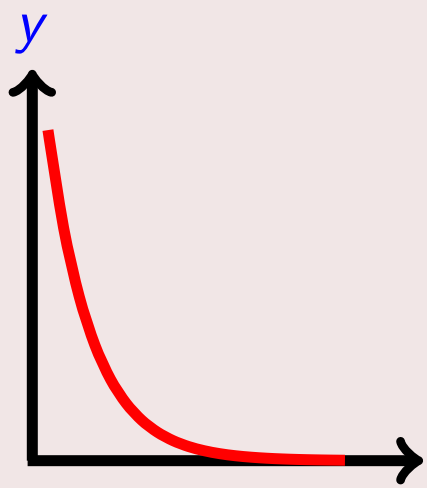
Extensions

Eager Attractors



x: number of steps of computation

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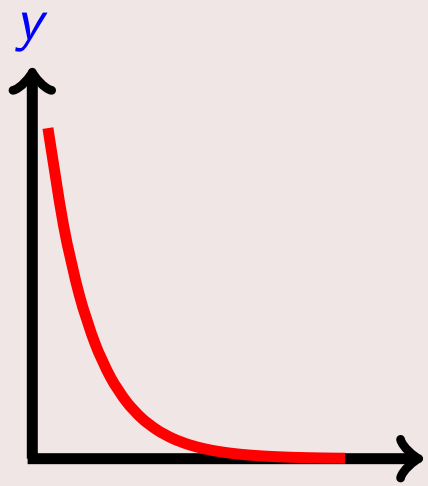
Extensions

Eager Attractors



x: number of steps of computation

y: probability of avoiding attractor

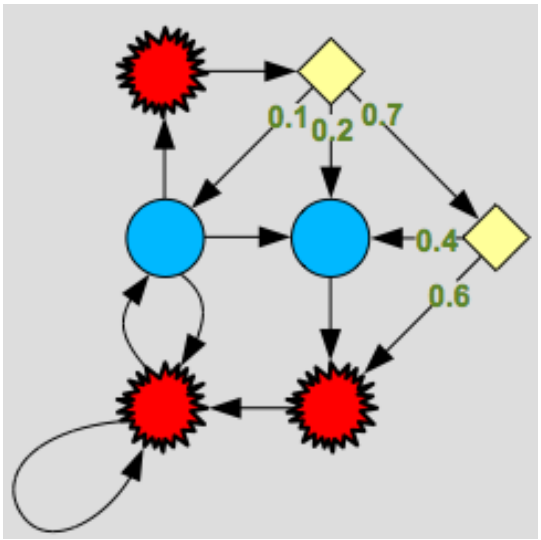
Examples

- Probabilistic Lossy Channel Systems

Properties

- Steady-state distribution
- Expected residence time.
- Limiting average expected cost.

Stochastic Games



$2\frac{1}{2}$-Games

- Three types of states
- Models: lossy channel systems, Petri nets, etc
- Infinite-state
- Winning conditions: Büchi, Co-Büchi, Parity, . . .

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Conclusions and Current Challenges

Decisive Markov Chains

- Simple characterization.
- Examples: Petri nets, lossy channel systems, noisy Turing machines, ...
- Analysis: qualitative, quantitative analysis, expected reward, limiting behaviors, stochastic games, ...
- Unexpected behaviors:
  - Decidability of repeated reachability vs. undecidability of simple reachability (for PPN).
  - Decidability of simple reachability for control states vs. undecidability for upward closed sets (for PPN).
  - Decidability of repeated reachability for probabilistic lossy channel systems vs. undecidability in the non-probabilistic case.

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Conclusions and Current Challenges

Challenges:

- Open problems:
  - Approximated quantitative repeated reachability analysis for PPNs.
  - Exact quantitative analysis.
  - Qualitative analysis for PPNs wrt. reachability problems.
- Relation to recently studied models: e.g., energy games.
- Implementation effort:
  - Probabilistic parameterized systems: mutex protocols, distributed algorithms, ...
  - Communication protocols