Priced Discrete-Timed Petri Nets

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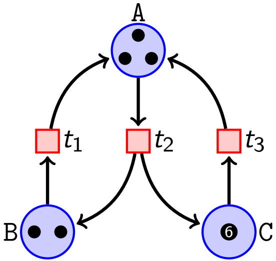
(Joint work with Richard Mayr)

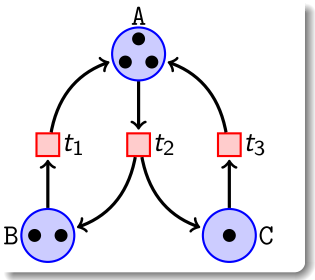
1. Petri Nets
2. Discrete-Timed Petri Nets
3. Priced Discrete-Timed Petri Nets (PTPNs)
4. From PTPNs to PNs
5. Algorithm
6. Further Results

Petri Nets



Petri Nets

Markings



$$M = [\mathtt{A}, \mathtt{A}, \mathtt{A}, \mathtt{B}, \mathtt{B}, \mathtt{C}]$$

$$M = [\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}]$$

$$M(\mathtt{A}) = 3 \quad M(\mathtt{B}) = 2 \quad M(\mathtt{C}) = 3$$

Petri Nets

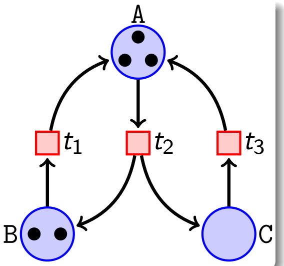
Q-Markings

M: Q-Marking

Tokens only in Q. $M(p) = 0$ if $p \notin Q$.

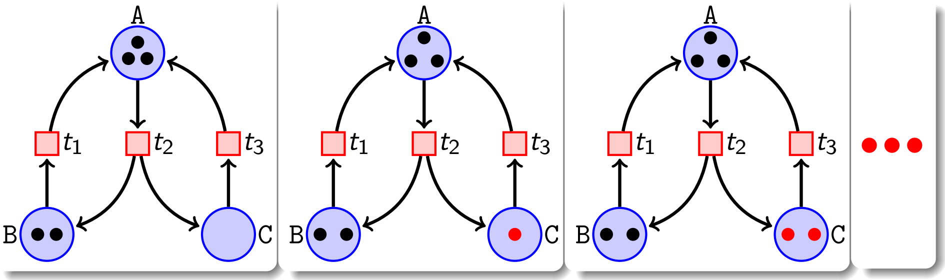
Example: $\{\mathtt{A}, \mathtt{B}\}$-Marking

$M = [\mathtt{A}^3, \mathtt{B}^2]$ $M(\mathtt{A}) = 3$ $M(\mathtt{B}) = 2$ $M(\mathtt{C}) = 0$



Petri Nets

ω-Markings



$$M = [A^3, B^2, C^\omega]$$

$$M = \{ [A^3, B^2] , [A^3, B^2, C] , [A^3, B^2, C^2] , [A^3, B^2, C^3] , [A^3, B^2, C^4] , \ldots \}$$

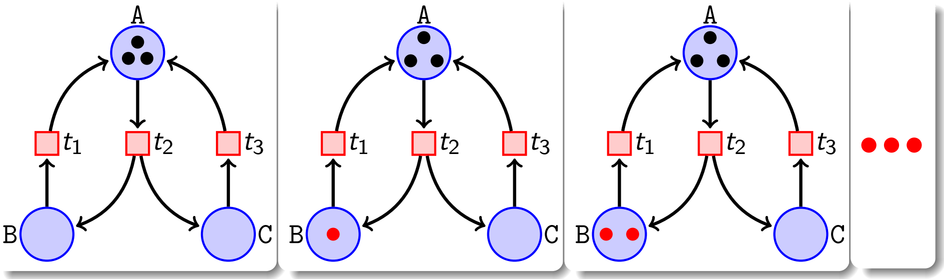
Petri Nets

Q-ω-Markings

Example: $\{A, B\}$-ω-Marking

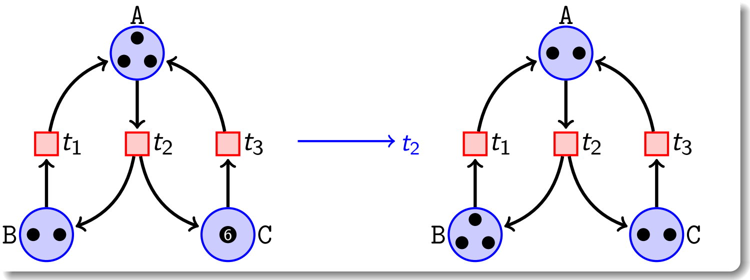
$$M = [A^3, B^\omega]$$

$$M = \{ [A^3], [A^3, B], [A^3, B^2], [A^3, B^3], [A^3, B^4], \ldots \}$$



Petri Nets

Transitions



$$[A^3, B^2, C] \longrightarrow_{t_2} [A^2, B^3, C^2]$$

Petri Nets

Computations

Computations

$[A^3, B^2, C] \longrightarrow_{t_2} [A^2, B^3, C^2] \longrightarrow_{t_1} [A^3, B^3, C^2] \longrightarrow_{t_3} [A^4, B^2, C]$

$[A^3, B^2, C] \xrightarrow{*} [A^4, B^2, C]$

Ordering

$[A^2, B] \leq [A^3, B^2, C] \leq [A^3, B^\omega, C]$

Monotonicity

$$\begin{array}{ccc} M_1 & \longrightarrow & M_2 \\ | \wedge & & \\ M_3 & & \end{array}$$

By Monotonicity:

- if F is upward closed
- then $\left(\xrightarrow{*} F\right)$ is upward closed

Petri Nets

Computations

Computations

$[\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}] \longrightarrow_{t_2} [\mathtt{A}^2, \mathtt{B}^3, \mathtt{C}^2] \longrightarrow_{t_1} [\mathtt{A}^3, \mathtt{B}^3, \mathtt{C}^2] \longrightarrow_{t_3} [\mathtt{A}^4, \mathtt{B}^2, \mathtt{C}]$

$[\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}] \stackrel{*}{\longrightarrow} [\mathtt{A}^4, \mathtt{B}^2, \mathtt{C}]$

Ordering

$[\mathtt{A}^2, \mathtt{B}] \leq [\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}] \leq [\mathtt{A}^3, \mathtt{B}^\omega, \mathtt{C}]$

Monotonicity

$$\begin{array}{ccc} M_1 & \longrightarrow & M_2 \\ \rotatebox{90}{$\leq$} & & \rotatebox{90}{$\leq$} \\ M_3 & \longrightarrow & M_4 \end{array}$$

By Monotonicity:

- if F is upward closed
- then $\left(\stackrel{*}{\longrightarrow} F\right)$ is upward closed

Petri Nets

Computations

Computations

$[A^3, B^2, C] \longrightarrow_{t_2} [A^2, B^3, C^2] \longrightarrow_{t_1} [A^3, B^3, C^2] \longrightarrow_{t_3} [A^4, B^2, C]$

$[A^3, B^2, C] \xrightarrow{*} [A^4, B^2, C]$

Ordering

$[A^2, B] \leq [A^3, B^2, C] \leq [A^3, B^\omega, C]$

Monotonicity

$$\begin{array}{ccc} M_1 & \xrightarrow{\quad * \quad} & M_2 \\ \mathrel{\rotatebox{90}{$\geq$}} & & \mathrel{\rotatebox{90}{$\geq$}} \\ M_3 & \xrightarrow{\quad * \quad} & M_4 \end{array}$$

By Monotonicity:

- if F is upward closed
- then $\left(\xrightarrow{*} F\right)$ is upward closed

Petri Nets

The Coverability Problem

Upward Closure

$M\uparrow := \{M' \mid M \leq M'\}$

$[\mathtt{A}^2, \mathtt{B}]\uparrow \;=\; \{\; [\mathtt{A}^3, \mathtt{B}] \;,\; [\mathtt{A}^2, \mathtt{B}^2, \mathtt{C}] \;,\; [\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}] \;,\; [\mathtt{A}^3, \mathtt{B}^2] \;,\; [\mathtt{A}^2, \mathtt{B}^2, \mathtt{C}^2] \;,\; [\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}^2] \;,\; \ldots \;\}$

DownwardClosure

$M\downarrow := \{M' \mid M' \leq M\}$ $[\mathtt{A}^2, \mathtt{B}]\downarrow \;=\; \{\; [\mathtt{A}^2] \;,\; [\mathtt{A}, \mathtt{B}] \;,\; [\mathtt{A}] \;,\; [\mathtt{B}] \;,\; [] \;\}$

Minimal Elements

- $min(P)$: minimal elements of P (w.r.t. $\leq$)
- P upward closed: $(min(P))\uparrow = P$

Petri Nets

The Coverability Problem

The Coverability Problem (I)

- Given:
    - *Init*: ω-marking
    - *Final*: finite set of markings
- $Init \xrightarrow{*} Final \uparrow$?

The Coverability Problem (II)

- Given:
    - *Final*: finite set of markings
- Compute $\min\left(\xrightarrow{*} Final \uparrow\right)$.

The Valk-Jantzen Theorem

The Valk-Jantzen Theorem

- $V \subseteq \mathbb{N}^k$: upward closed set
- if we can check
  - $(v\!\downarrow \cap V) = \emptyset$, for all $v \in \mathbb{N}_\omega^k$
- then we can compute
  - we can compute $\min(V)$.

Coverability (II) reducible to Coverability (I)

- $V := \left(\xrightarrow{*} \mathit{Final}\!\uparrow\right)$
- $v\!\downarrow \xrightarrow{*} \mathit{Final}\!\uparrow$ can be checked for all ω-markings v

Petri Nets

The Generalized Coverability Problem

Generalized Ordering

$[\mathtt{A}^2, \mathtt{B}, \mathtt{C}] \leq_{\{\mathtt{A},\mathtt{B}\}} [\mathtt{A}^3, \mathtt{B}^2, \mathtt{C}]$ $\qquad$ $[\mathtt{A}^2, \mathtt{B}] \leq_{\{\mathtt{A},\mathtt{B}\}} [\mathtt{A}^3, \mathtt{B}^3]$

Generalized Upward Closure

$M \uparrow Q := \{M' | \ M \leq_Q M'\}$

$[\mathtt{A}^2, \mathtt{B}] \uparrow \{\mathtt{A}, \mathtt{B}\} = \left\{ \begin{array}{l} [\mathtt{A}^3, \mathtt{B}], \ [\mathtt{A}^2, \mathtt{B}^3], [\mathtt{A}^3, \mathtt{B}^3], \\ [\mathtt{A}^3, \mathtt{B}^2], \ [\mathtt{A}^2, \mathtt{B}^4], [\mathtt{A}^3, \mathtt{B}^4], \ \ldots \end{array} \right\}$

(Generalized Definition of) Minimal Elements

- $min_Q(P)$: minimal elements of P (w.r.t. $\leq_Q$)
- P upward closed w.r.t. $\leq_Q$: $(min_Q(P)) \uparrow Q = P$

Petri Nets

The Generalized Coverability Problem

The Generalized Coverability Problem (I)

- Given:
  - (P_1, P_2): partitioning of set of places
  - $Init = (Init_1, Init_2)$
  - $Final = (Final_1, Final_2)$
- $Init \xrightarrow{*} Final{\uparrow}P_1$?

The Generalized Coverability Problem (I) is solvable:

using Petri Net reachability (even if $Init_1$ is an ω-marking)

$\left\{ M_1 | (M_1, M_2) \xrightarrow{*} Final{\uparrow}P_1 \right\}$ is upward closed w.r.t. $\leq_{P_1}$

Petri Nets

The Generalized Coverability Problem

The Generalized Coverability Problem (II)

- Given:
  - (P_1, P_2): partitioning of set of places
  - $Init_2$: P_2-marking
  - $Final = (Final_1, Final_2)$
- Compute $\min\left(\left\{ M_1 | (M_1, Init_2) \xrightarrow{*} Final{\uparrow}P_1 \right\}\right)$

Generalized Coverability (II) reducible to Generalized Coverability (I)

- Follows form the Valk-Jantzen Theorem
- Corollary: Generalized Coverability (II) solvable

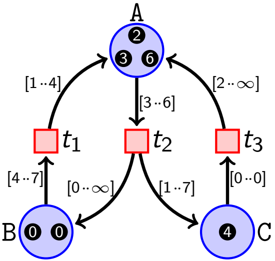
Petri Nets

The (even more) Generalized Coverability Problem

The (even more) Generalized Coverability Problem

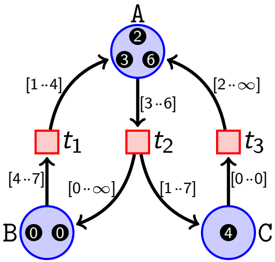
- Given:
  - (P_1, P_2): partitioning of set of places
  - $Init_2$: P_3-marking $(P_2 \subseteq P_3)$
  - $Final = (Final_1, Final_2)$
- Compute $\min\left(\left\{M_1 | (M_1, Init_2) \xrightarrow{*} Final{\uparrow}P_1\right\}\right)$

Discrete-Timed Petri Nets



Discrete-Timed Petri Nets

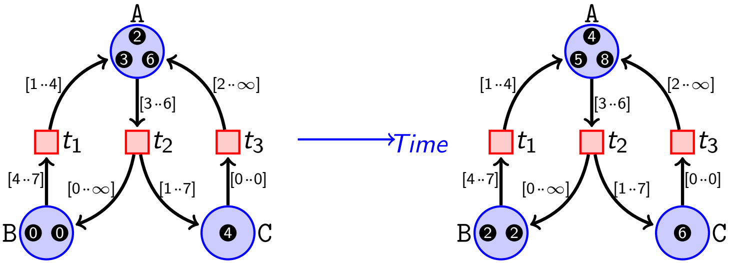
Markings



$[A(2), A(3), A(6), B^2(0), C(4)]$

Discrete-Timed Petri Nets

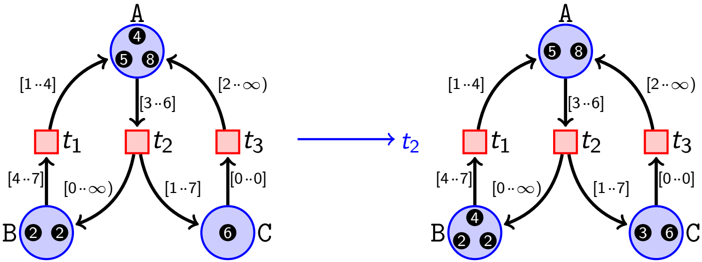
Timed Transitions



$[A(2), A(3), A(6), B^2(0), C(4)] \longrightarrow_{Time} [A(4), A(5), A(8), B^2(2), C(6)]$

Discrete-Timed Petri Nets

Discrete Transitions



$$[\mathtt{A}(4), \mathtt{A}(5), \mathtt{A}(8), \mathtt{B}^2(2), \mathtt{C}(6)] \longrightarrow_{t_2} [\mathtt{A}(5), \mathtt{A}(8), \mathtt{B}(4), \mathtt{B}^2(2), \mathtt{C}(3), \mathtt{C}(6)]$$

Discrete-Timed Petri Nets

Computation

$$[\mathtt{A}(2), \mathtt{A}(3), \mathtt{A}(6), \mathtt{B}^2(0), \mathtt{C}(4)] \longrightarrow_{Time}$$
$$[\mathtt{A}(4), \mathtt{A}(5), \mathtt{A}(8), \mathtt{B}^2(2), \mathtt{C}(6)] \longrightarrow_{t_2}$$
$$[\mathtt{A}(5), \mathtt{A}(8), \mathtt{B}(4), \mathtt{B}^2(2), \mathtt{C}(3), \mathtt{C}(6)] \longrightarrow_{Time}$$
$$[\mathtt{A}(7), \mathtt{A}(10), \mathtt{B}(6), \mathtt{B}^2(4), \mathtt{C}(5), \mathtt{C}(8)] \longrightarrow_{t_1}$$
$$[\mathtt{A}(1), \mathtt{A}(7), \mathtt{A}(10), \mathtt{B}(6), \mathtt{B}(4), \mathtt{C}(5), \mathtt{C}(8)]$$

$$[\mathtt{A}(2), \mathtt{A}(3), \mathtt{A}(6), \mathtt{B}^2(0), \mathtt{C}(4)] \xrightarrow{*} [\mathtt{A}(1), \mathtt{A}(7), \mathtt{A}(10), \mathtt{B}(6), \mathtt{B}(4), \mathtt{C}(5), \mathtt{C}(8)]$$

Discrete-Timed Petri Nets

Ordering

$[\mathtt{A}(2), \mathtt{A}(6), \mathtt{B}(0)] \leq [\mathtt{A}(2), \mathtt{A}(3), \mathtt{A}(6), \mathtt{B}(0), \mathtt{B}(0), \mathtt{C}(4)]$

Upward Closure

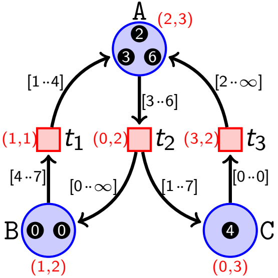
$M \uparrow := \{M' \mid M \leq M'\}$

$$[\mathtt{A}(2), \mathtt{A}(6), \mathtt{B}(0)] \uparrow \;=\; \left\{ \begin{array}{l} [\mathtt{A}(2), \mathtt{A}(3), \mathtt{A}(6), \mathtt{B}(0)] \;,\; [\mathtt{A}(2), \mathtt{A}(6), \mathtt{B}(0), \mathtt{B}(0), \mathtt{C}(4)] \;, \\ [\mathtt{A}(2), \mathtt{A}(3), \mathtt{A}(6), \mathtt{B}(0), \mathtt{B}(0), \mathtt{C}(4)] \;,\; \ldots \end{array} \right\}$$

The Coverability Problem

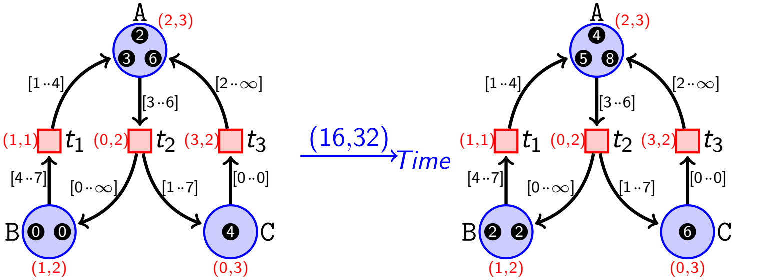
- Given:
  - *Init*: set of markings
  - *Final*: finite set of markings
- $Init \xrightarrow{*} Final \uparrow$?

Priced Discrete-Timed Petri Nets



Priced Discrete-Timed Petri Nets

Timed Transitions

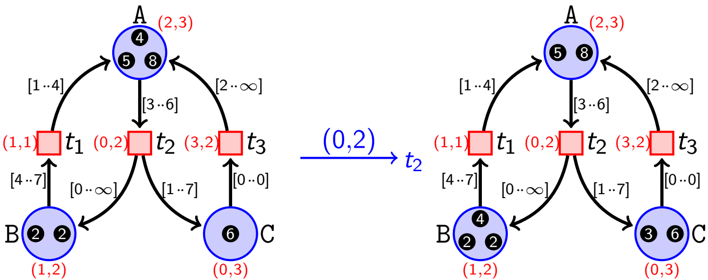


$$Cost = 2 \cdot (3 \cdot (2,3) + 2 \cdot (1,2) + 1 \cdot (0,3)) = (16,32)$$

$$[\mathtt{A}(2), \mathtt{A}(3), \mathtt{A}(6), \mathtt{B}(0), \mathtt{A}(0), \mathtt{C}(4)] \xrightarrow{(16,32)}_{\mathit{Time}} [\mathtt{A}(4), \mathtt{A}(5), \mathtt{A}(8), \mathtt{B}(2), \mathtt{B}(2), \mathtt{C}(6)]$$

Priced Discrete-Timed Petri Nets

Discrete Transitions



$$Cost = (0,2)$$

$$[\mathtt{A}(4), \mathtt{A}(5), \mathtt{A}(8), \mathtt{B}^2(2), \mathtt{C}(6)] \xrightarrow{(0,2)}_{Disc} [\mathtt{A}(5), \mathtt{A}(8), \mathtt{B}(4), \mathtt{B}^2(2), \mathtt{C}(3), \mathtt{C}(6)]$$

Priced Discrete-Timed Petri Nets

Computation

$$[A(2), A(3), A(6), B^2(0), C(4)] \xrightarrow{(16,32)}_{Time}$$
$$[A(4), A(5), A(8), B^2(2), C(6)] \xrightarrow{(0,2)}_{t_2}$$
$$[A(5), A(8), B(4), B^2(2), C(3), C(6)] \xrightarrow{(7,18)}_{Time}$$
$$[A(7), A(10), B(6), B^2(4), C(5), C(8)] \xrightarrow{(1,1)}_{t_1}$$
$$[A(1), A(7), A(10), B(6), B(4), C(5), C(8)]$$

Total Cost=

$$(16,32) + (0,2) + (7,18) + (1,1) \; = \; (24,53)$$

$$[A(2), A(3), A(6), B^2(0), C(4)] \xrightarrow{(24,53)} [A(1), A(7), A(10), B(6), B(4), C(5), C(8)]$$

Discrete-Timed Petri Nets

The Threshold Priced Coverability Problem

- Given:
  - *Init*: set of markings
  - *Final*: finite set of markings
  - v: price vector.
- $\exists u \leq v.\ Init \xrightarrow{u} Final \uparrow$?

The Optimal Priced Coverability Problem

- Given:
  - *Init*: set of markings
  - *Final*: finite set of markings
- Find minimal v such that $Init \xrightarrow{v} Final \uparrow$

Discrete-Timed Petri Nets

The Optimal Priced Coverability Problem is reducible to
The Threshold Priced Coverability Problem

- $\left\{ v | \exists u \leq v.\ Init \xrightarrow{u} Final \uparrow \right\}$ is upward closed
- The Valk-Jantzen Theorem.

From PTPNs to PNs

Translation Scheme

Solving The Cost Threshold Problem

- Reduce The Cost Threshold Problem to The Generalized Coverability Problem for PNs
- Translation Scheme - Encoding of Places
  - marking M in PTPN $\rightarrow$ marking $Encoding(M)$ in PN.
  - Differentiate between free and priced places.
  - Number of tokens in priced places bounded during timed transitions.
- Translation Scheme - Encoding of Transitions
  - Simulate discrete and timed transitions.
- Keep track of remaining allowed costs

From PTPNs to PNs

Free Places

max $= 2$ $v = (5{,}5)$

PTPN

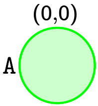
PN

From PTPNs to PNs

Free Places

$\max = 2 \qquad v = (5,5)$

PTPN

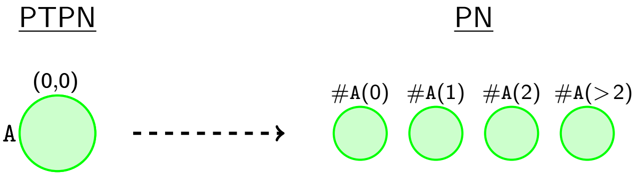


PN

From PTPNs to PNs

Free Places

$max = 2 \qquad v = (5,5)$



From PTPNs to PNs

Free Places

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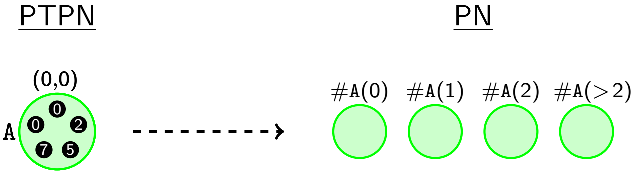
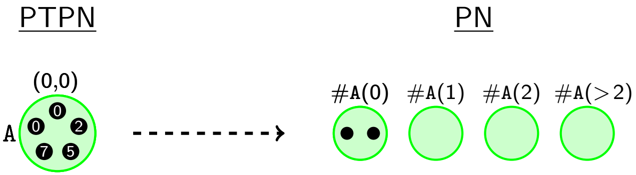


From PTPNs to PNs

Free Places

$\max = 2 \qquad v = (5,5)$



From PTPNs to PNs

Free Places

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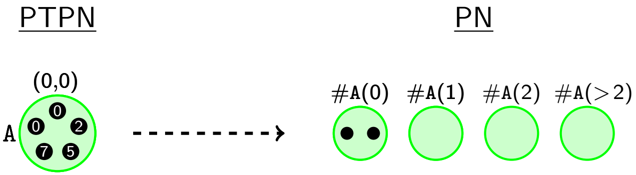
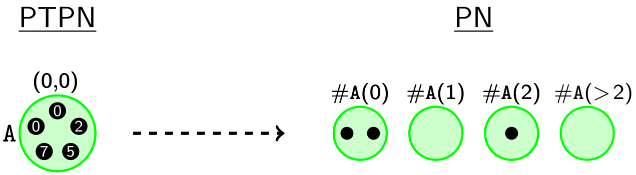


From PTPNs to PNs

Free Places

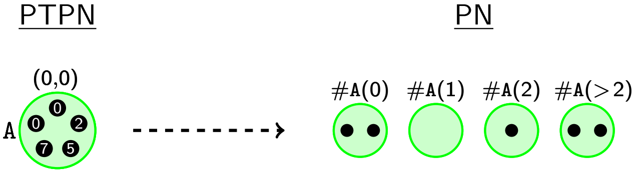
$\max = 2 \qquad v = (5{,}5)$



From PTPNs to PNs

Free Places

max = 2 $v = (5,5)$



From PTPNs to PNs

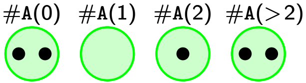
Free Places

max $= 2$ $v = (5{,}5)$

PTPN

PN

$[A^2(0), A(2), A(5), A(7)]$ - - - - - - - - - ->



From PTPNs to PNs

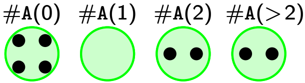
Free Places

$\max = 2 \qquad v = (5,5)$

PTPN

PN

[$\mathtt{A}^4(0)$, $\mathtt{A}^2(2)$, $\mathtt{A}(5)$, $\mathtt{A}(7)$] - - - - - - - - - →



From PTPNs to PNs

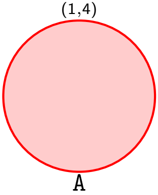
Priced Places

PTPN $\max = 2$ $v = (5,5)$ PN

From PTPNs to PNs

Priced Places

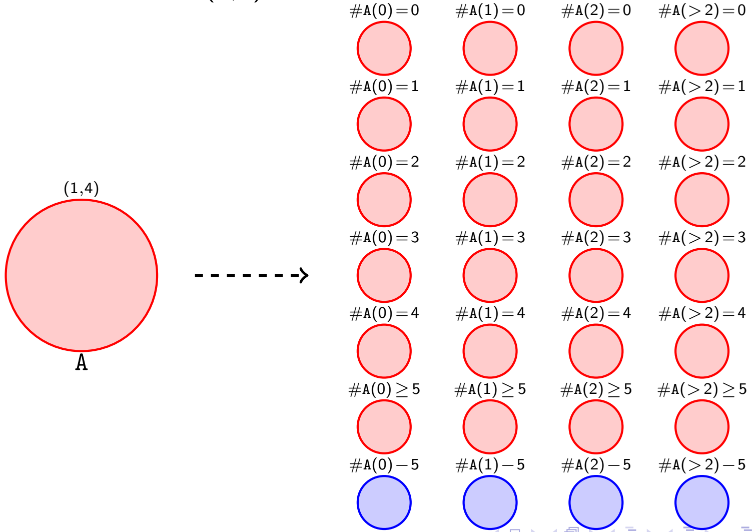
PTPN max $= 2$ $v = (5,5)$ PN



From PTPNs to PNs

Priced Places

PTPN   max = 2 $v = (5,5)$ PN



From PTPNs to PNs

Priced Places

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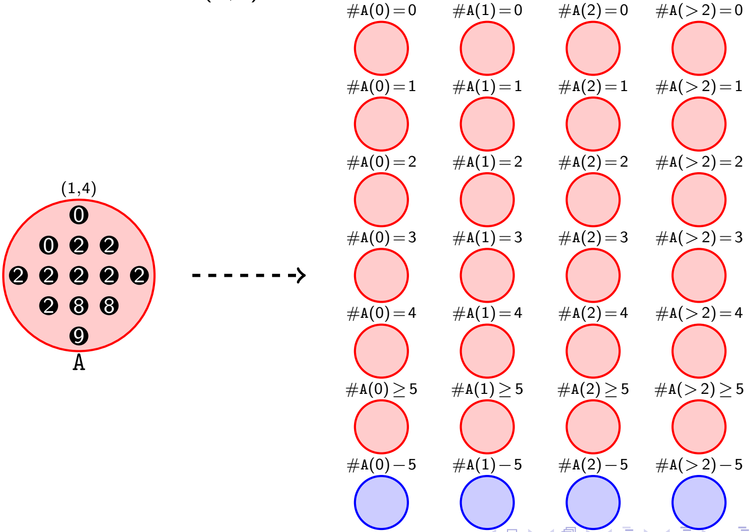
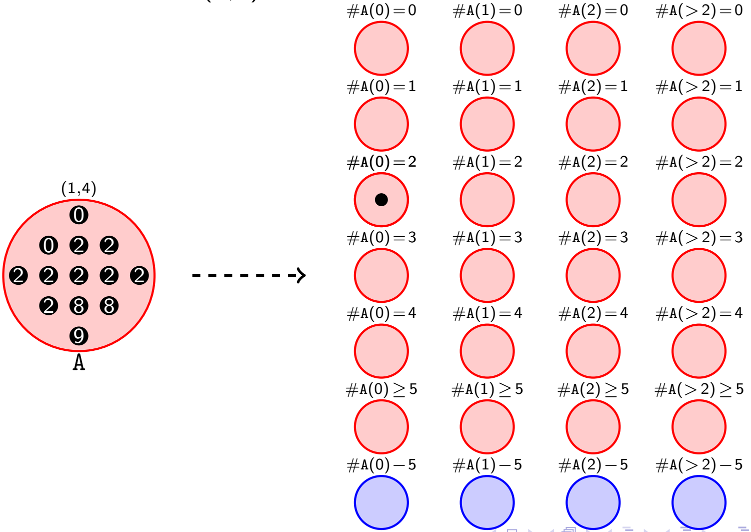


From PTPNs to PNs

Priced Places

PTPN max $= 2$ $v = (5,5)$ PN



From PTPNs to PNs

Priced Places

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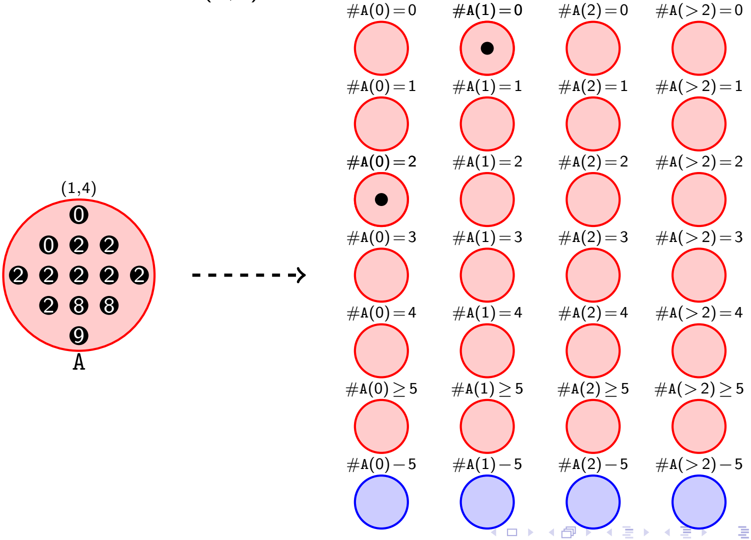
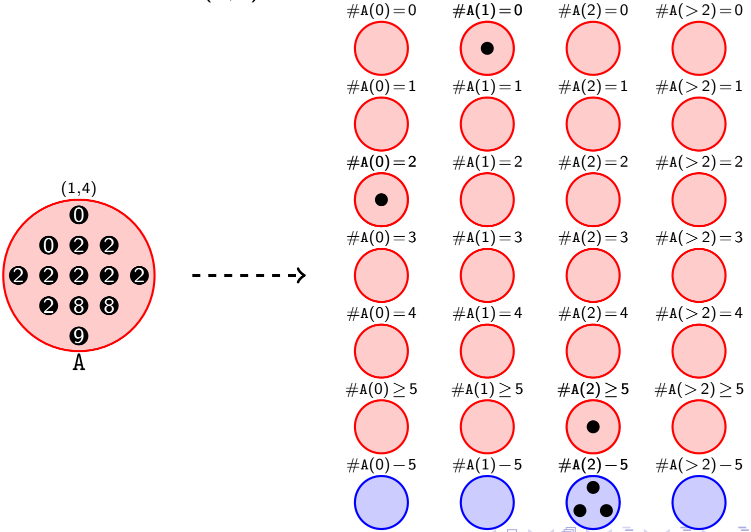


From PTPNs to PNs

Priced Places

PTPN $\max = 2$ $v = (5,5)$

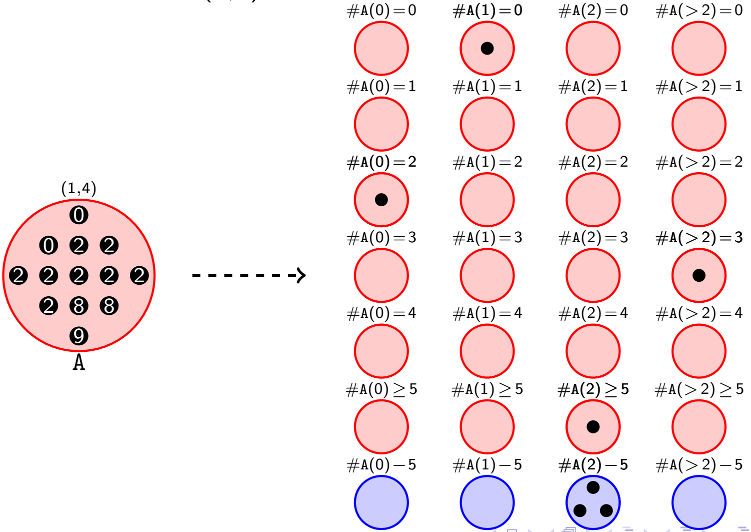
PN



From PTPNs to PNs

Priced Places

PTPN   max = 2 $v = (5,5)$ PN

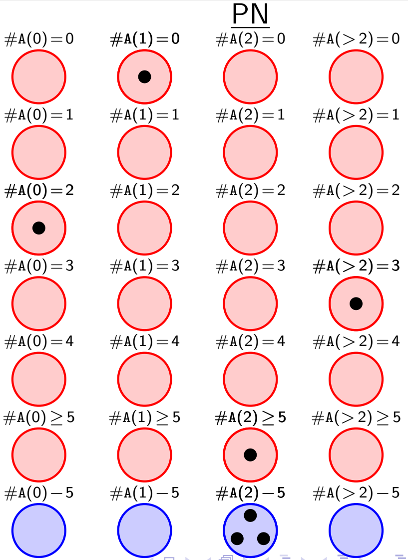


From PTPNs to PNs

Priced Places

PTPN $\quad max = 2 \quad v = (5,5)$

$[A^2(0), A^7(2)], A^2(8), , A(9)]]$ - - - - - - ->



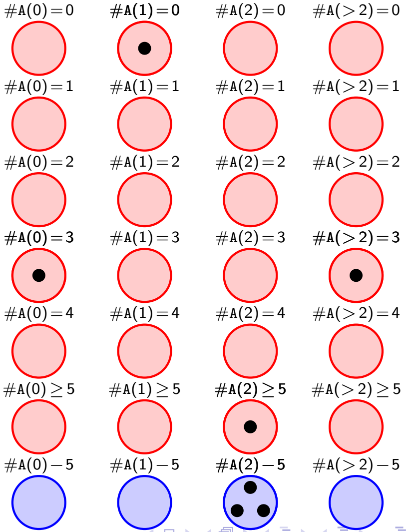
From PTPNs to PNs

Priced Places

PTPN $\max = 2$ $v = (5{,}5)$

PN

$[A^3(0), A^7(2)], A^2(8), , A(9)]]$



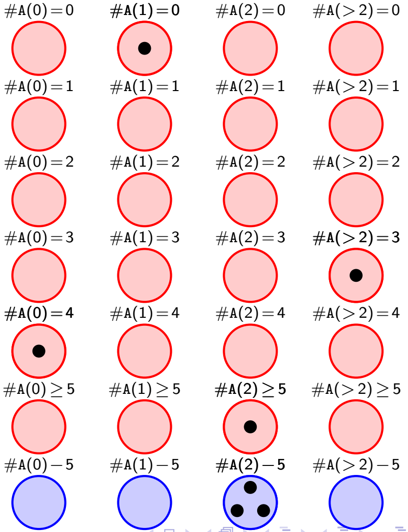
From PTPNs to PNs

Priced Places

PTPN   max = 2 $v = (5{,}5)$

PN

$[A^4(0), A^7(2)], A^2(8), , A(9)]]$ - - - - - - ->



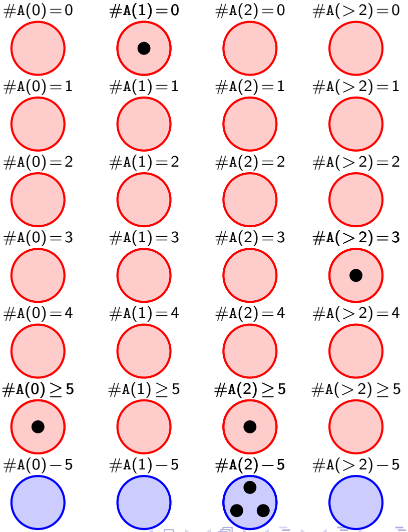
From PTPNs to PNs

Priced Places

PTPN $\max = 2$ $v = (5,5)$

PN

$[A^5(0), A^7(2)], A^2(8), , A(9)]]$ - - - - - - ->



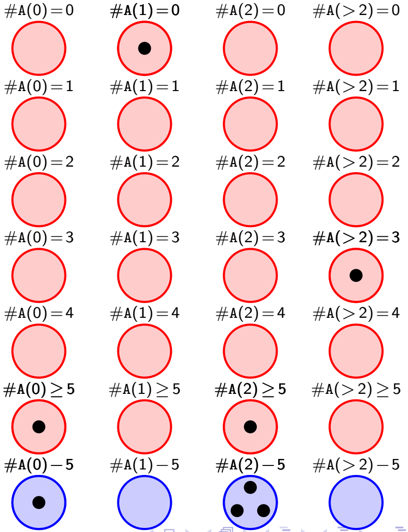
From PTPNs to PNs

Priced Places

PTPN $\max = 2 \quad v = (5{,}5)$

PN

$[A^6(0), A^7(2)], A^2(8), , A(9)]]$ - - - - - - →



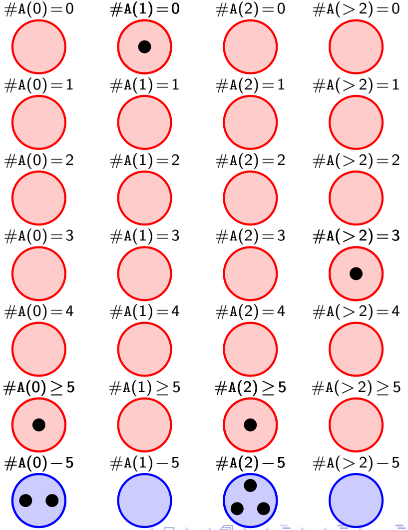
From PTPNs to PNs

Priced Places

PTPN $\max = 2$ $v = (5{,}5)$

PN

$[\mathtt{A}^7(0), \mathtt{A}^7(2)], \mathtt{A}^2(8), , \mathtt{A}(9)]]$ - - - - - - ->

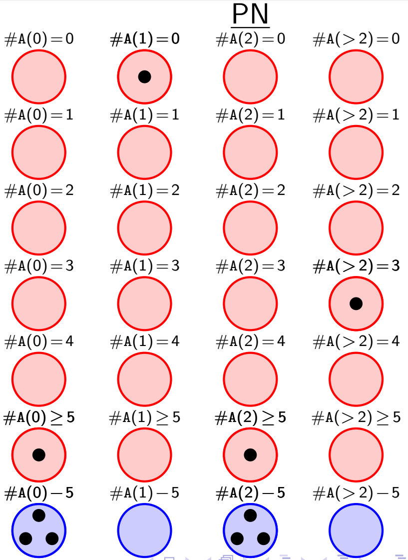


From PTPNs to PNs

Priced Places

PTPN $max = 2$ $v = (5,5)$

[A^8(0), A^7(2)], A^2(8), , A(9)]] - - - - - - ->



From PTPNs to PNs

"Remaining Cost" Places

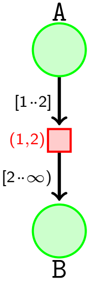
- keep track of remaining allowed costs.
- if $v = (5,5)$ then, a place $\#R(i,j)$ for $0 \leq i,j \leq 5$.
- token in $\#R(i,j)$ indicates remaining cost $= (i,j)$.

From PTPNs to PNs

Discrete Transitions

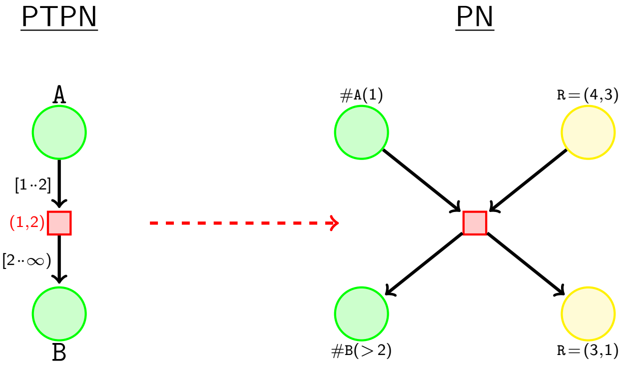
PTPN

PN



From PTPNs to PNs

Discrete Transitions

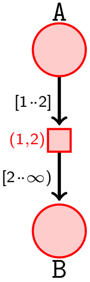


From PTPNs to PNs

Discrete Transitions

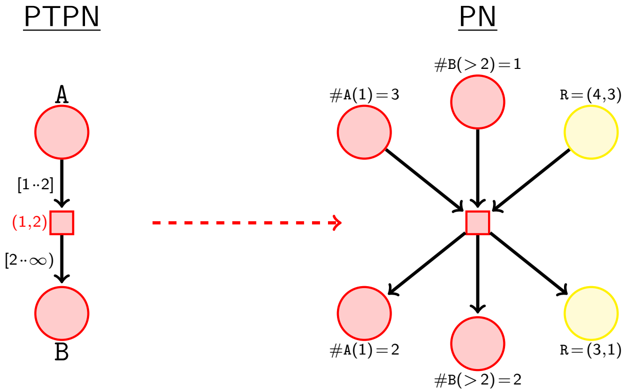
PTPN

PN



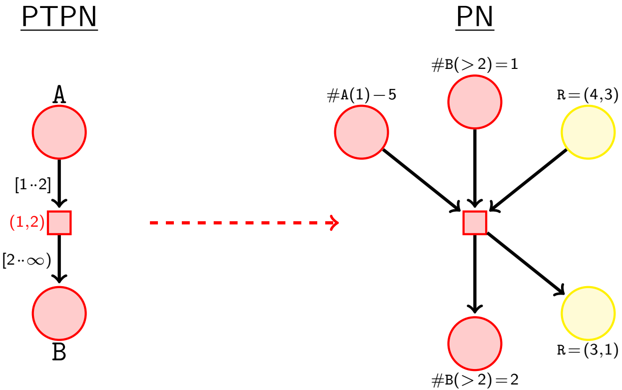
From PTPNs to PNs

Discrete Transitions



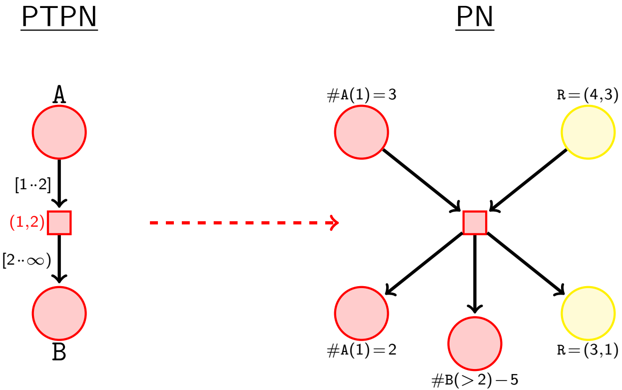
From PTPNs to PNs

Discrete Transitions



From PTPNs to PNs

Discrete Transitions



From PTPNs to PNs

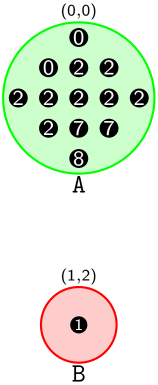
Timed Transitions

PTPN

From PTPNs to PNs

Timed Transitions

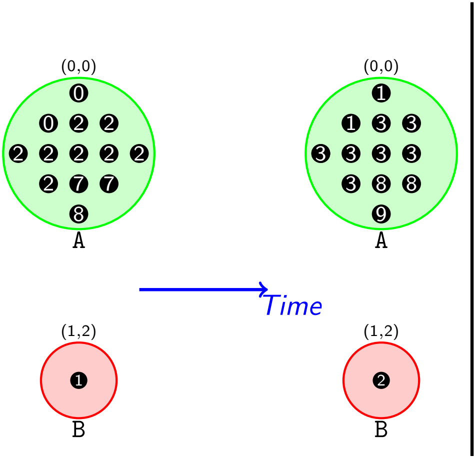
PTPN



From PTPNs to PNs

Timed Transitions

PTPN

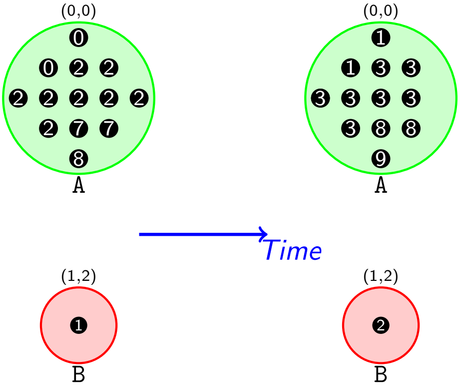


From PTPNs to PNs

Timed Transitions

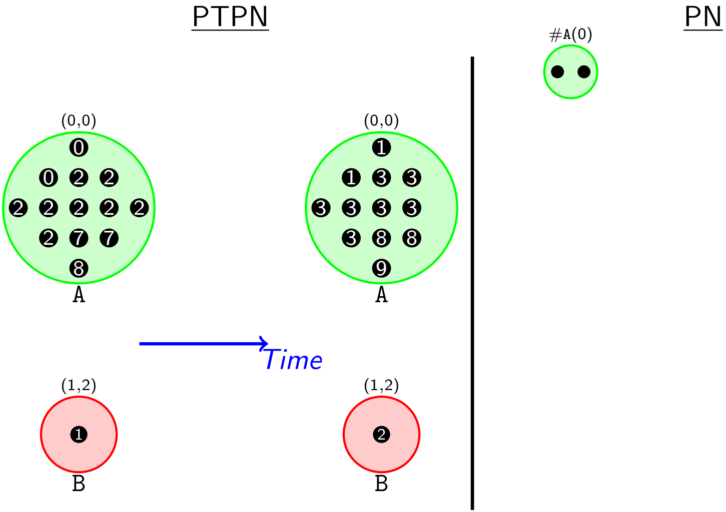
PTPN

PN



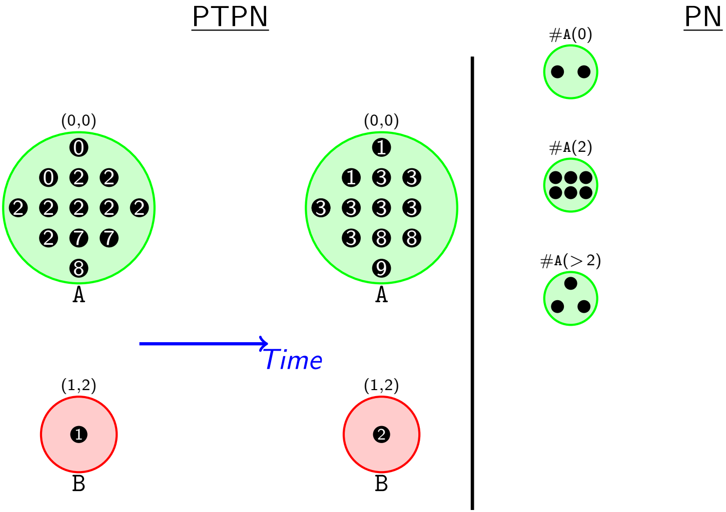
From PTPNs to PNs

Timed Transitions



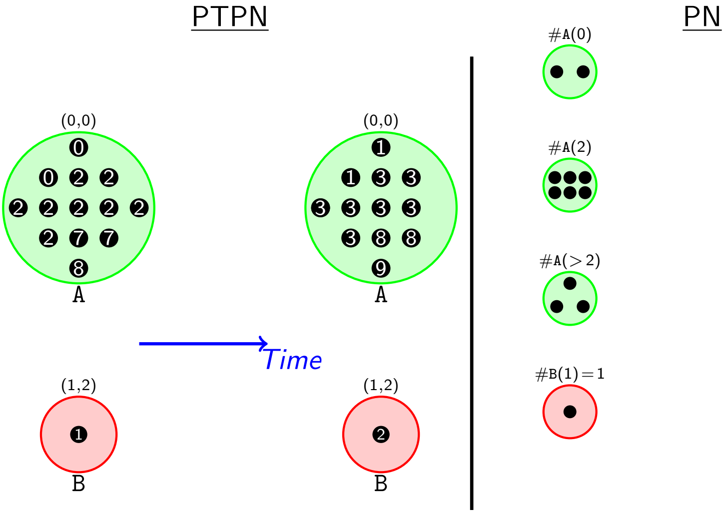
From PTPNs to PNs

Timed Transitions



From PTPNs to PNs

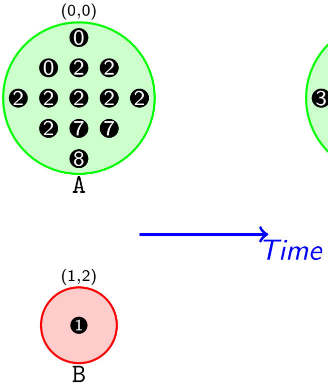
Timed Transitions



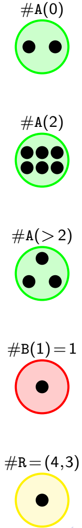
From PTPNs to PNs

Timed Transitions

PTPN

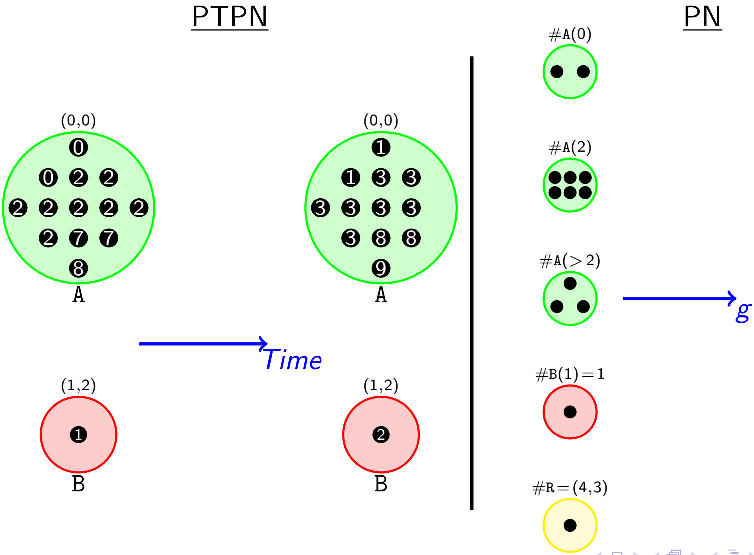


PN



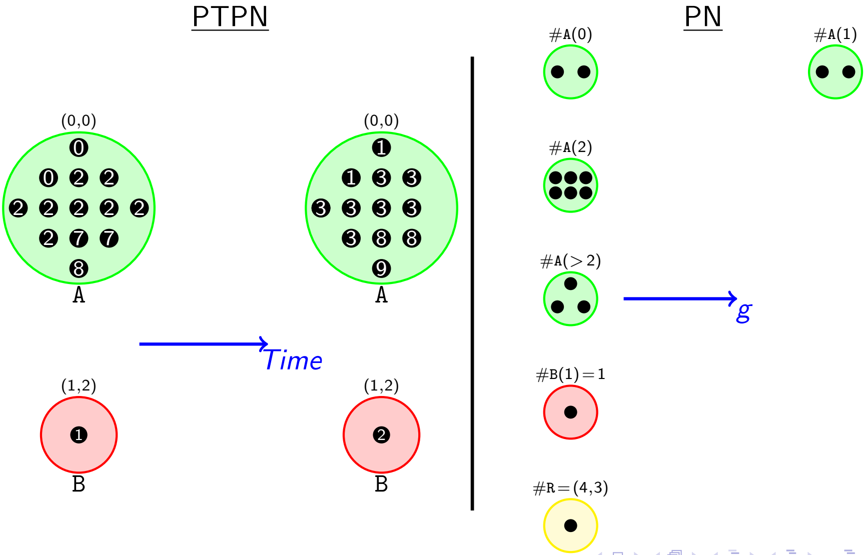
From PTPNs to PNs

Timed Transitions



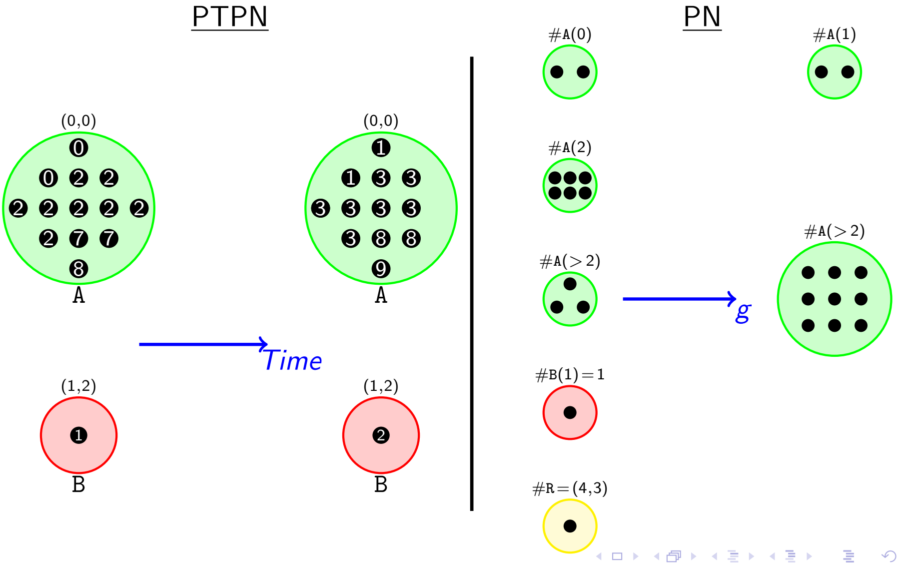
From PTPNs to PNs

Timed Transitions



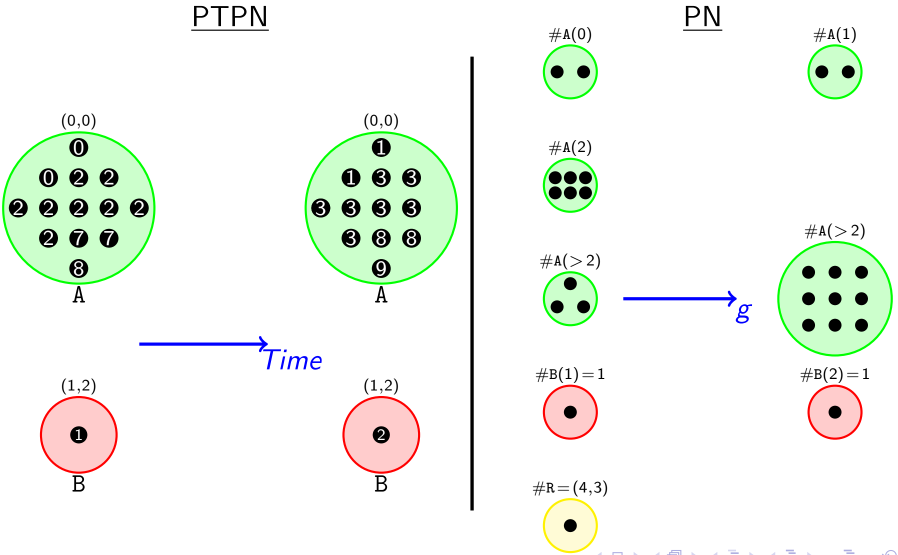
From PTPNs to PNs

Timed Transitions



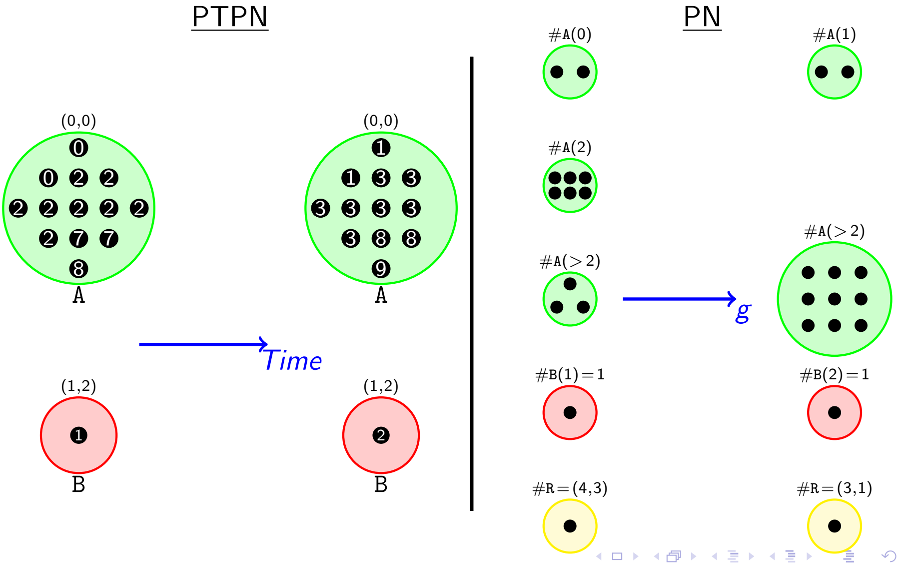
From PTPNs to PNs

Timed Transitions



From PTPNs to PNs

Timed Transitions



From PTPNs to PNs

g-Transitions

$M_1 \longrightarrow_g M_2$

- Free Places:
  - $M_2\,(\#\mathtt{A}(0)) \;=\; 0$
  - $M_2\,(\#\mathtt{A}(j+1)) \;=\; M_1\,(\#\mathtt{A}(j)), \quad 0 \leq j < \max$
  - $M_2\,(\#\mathtt{A}(>\max)) \;=\; M_1\,(\#\mathtt{A}(>!\max)) + M_1\,(\#\mathtt{A}(\max))$
- Priced Places:
  - $M_2\,(\#\mathtt{A}(0)=0) \;=\; 1$
  - $M_2\,(\#\mathtt{A}(0)=i) \;=\; 0, \quad 0 < i \leq \max.$
  - $M_2\,(\#\mathtt{A}(i+1)=j) \;=\; M_2\,(\#\mathtt{A}(i)=j), \quad 0 \leq i < \max, \quad j \leq 0 \leq R-1$
  - $M_2\,(\#\mathtt{A}(i+1)\geq R) \;=\; M_2\,(\#\mathtt{A}(i)\geq R), \quad 0 \leq i < \max$
  - $M_2\,(\#\mathtt{A}(i+1)\geq R) \;=\; M_2\,(\#\mathtt{A}(i)\geq R), \quad 0 \leq i < \max$ .....

Algorithm

The Threshold Priced Coverability Problem:

$\exists u \leq v.\ Init \xrightarrow{u} Final\!\uparrow$?

Algorithm:

$F_0, G_0, F_1, G_1, F_2, G_2, \ldots$

- $F_0' := encoding(Final)$.
- F_0 is upward closure of F_0' w.r.t. the ordering on PN.
- F_i: set of PN-markings M that
  - can reach F_0 via a computation starting with a g-transition.
  - M is zero on all cost places.
- G_i: defined analogously, except that we start with a normal transition.

Algorithm

Algorithm:

$F_0, G_0, F_1, G_1, F_2, G_2, \ldots$

- Since F_0 is upward closed, all sets F_i and G_i are upward closed on the free places.
  - follows from monotonicity of Petri nets and monotonicity of g-transitions.
- Define $H_\ell := \bigcup_i F_i$.
- Markings in H^ℓ are
  - upward closed on free places.
  - have empty cost places
- The sequence converges at some ℓ by Dickson's lemma.
- Check whether $Encoding(Init)$ can reach H_ℓ via PN-transitions.

Further Results

Further Results

- Coverability for Petri Nets with one-inhibitor arc.
- Priced Reachability for Priced (untimed) Petri nets.
- The Dense-Time case.