Shape Analysis via Monotonic Abstraction

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Uppsala University

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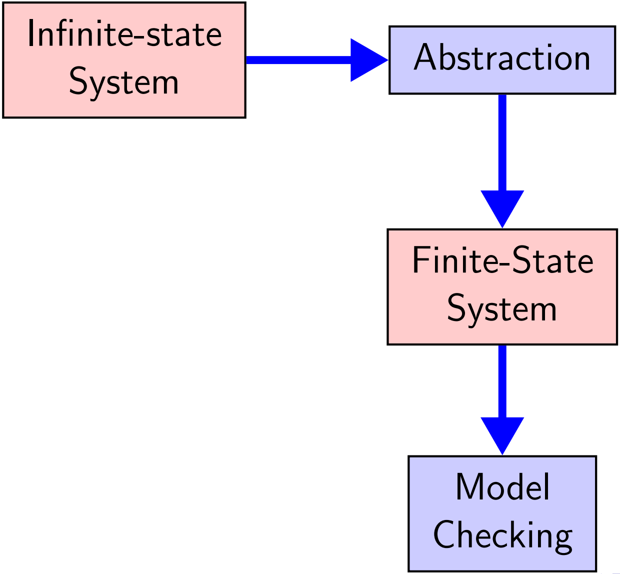
(Joint work with Ahmed Bouajjani, Jonathan Cederberg, Fédéric Haziza and Ahmed Rezine.)

Model Checking+Abstraction



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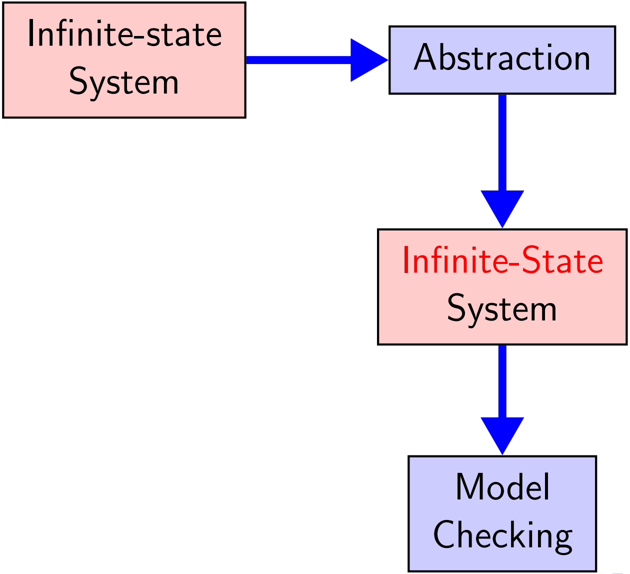

Monotonic Transition Systems

Monotonic Transition System

- $\mathcal{T} = (S, \longrightarrow, \preceq)$
- S: (infinite) set of configurations
- $\longrightarrow$: transition relation
- $\preceq$: preorder on S

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Monotonicity

$c_1 \longrightarrow c_2$

$\curlyvee$ (rotated $\preceq$)

c_3

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$$\begin{array}{ccc} c_1 & \longrightarrow & c_2 \\ \preceq & & \preceq \\ c_3 & \longrightarrow & c_4 \end{array}$$

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$\curlyvee\mid \qquad \curlyvee\mid$

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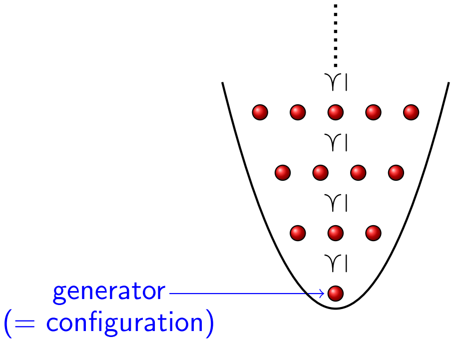
Examples

- Petri Nets.
- Lossy Channel Systems.
- Timed Petri Nets.
- Multiset Rewriting Systems.
- Broadcast Protocols.
- etc.

Upward-Closed Sets (UC)



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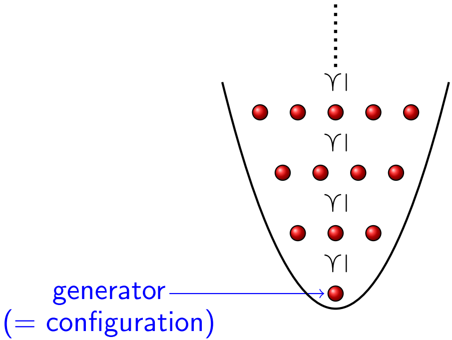

Why UC?

- Bad sets of states are UC
  - safety properties = reachability of UC
- Uniquely characterized by generator
  - simple representation = minimal element

Monotonicity and Upward Closedness

Monotonicity implies UC is closed under *Pre*

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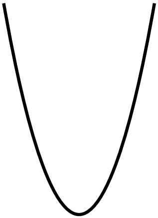
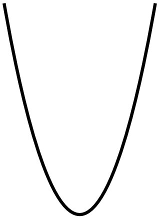


$Pre(U)$:Upward Closed?



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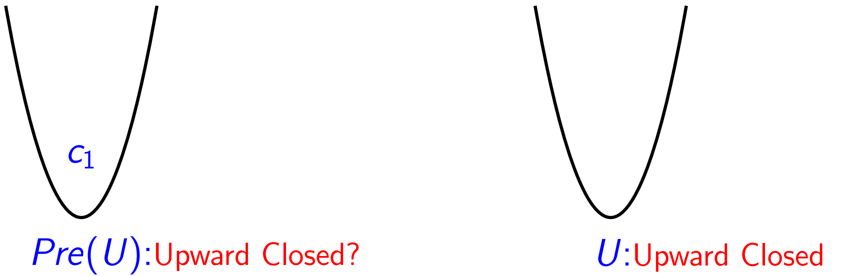

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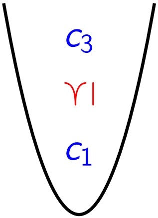

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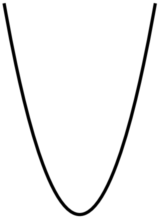

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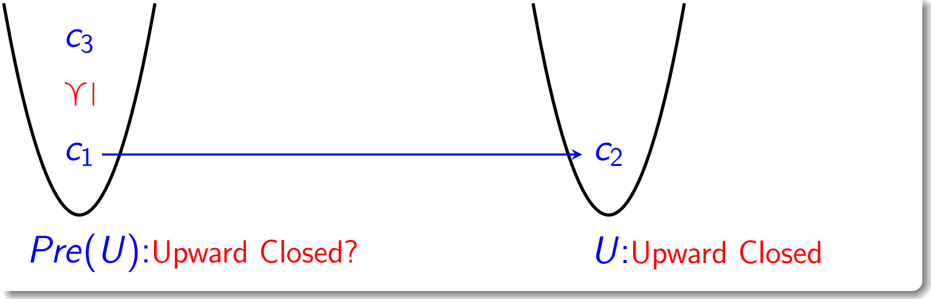

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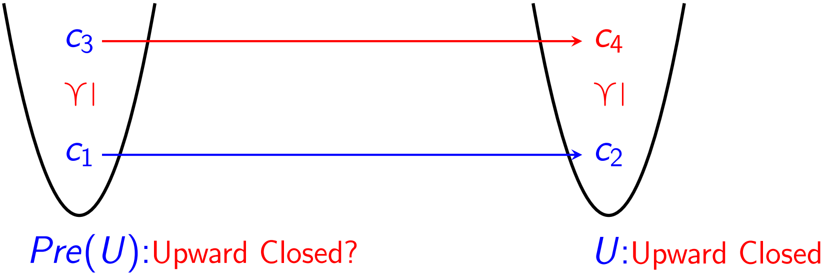

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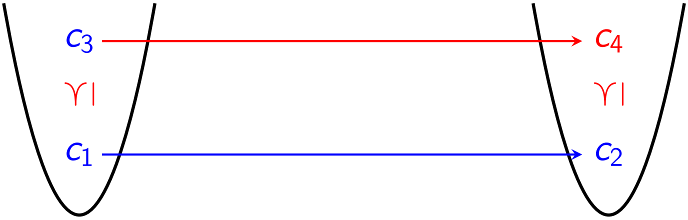

$Pre(U)$: Upward Closed? Yes

U: Upward Closed

Monotonic Abstraction

Problem

- When transition system not monotonic

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Solution: Monotonic Abstraction

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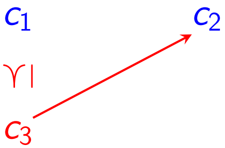

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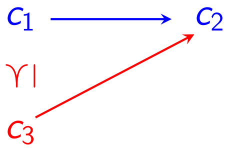

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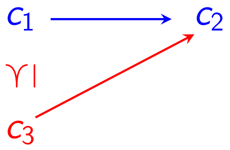


Examples

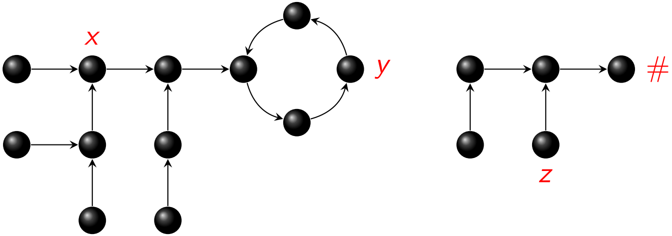
- Parameterized Systems.
- Shape Analysis.

Shape Analysis: Singly Linked Lists

Transition System $= (S, \longrightarrow, \preceq)$

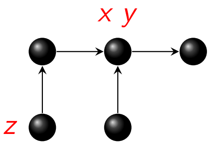
Configuration

- graph
  - node: cell
  - edge: successor
  - pointers: x, y, z, $\#$



Transitions

$x = y?$



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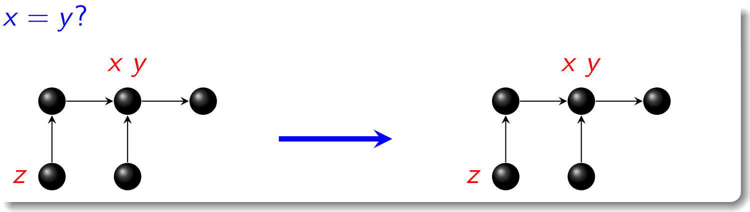

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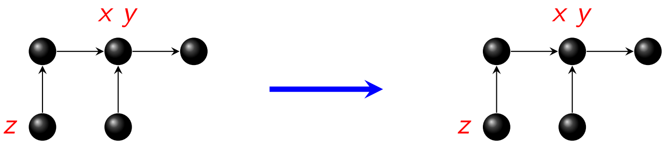

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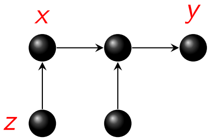

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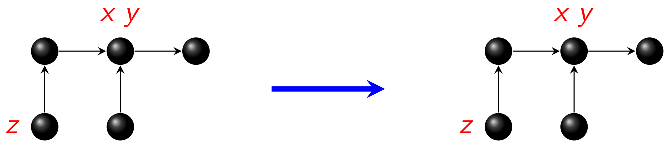

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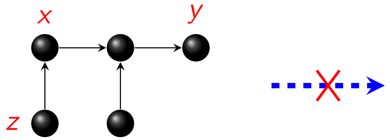

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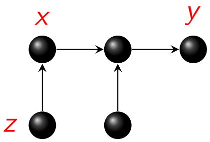
$x \neq y$?



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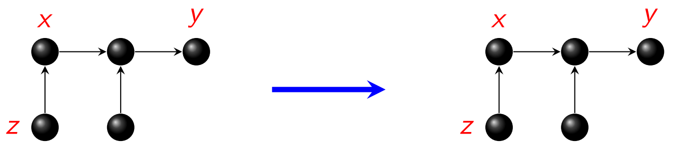

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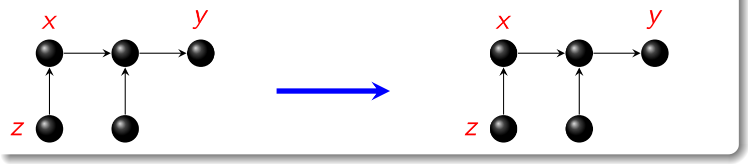

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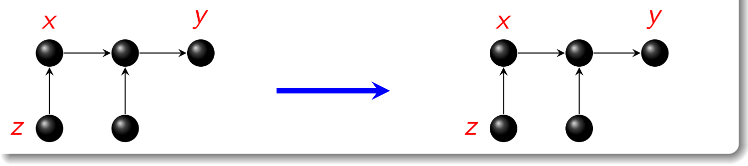

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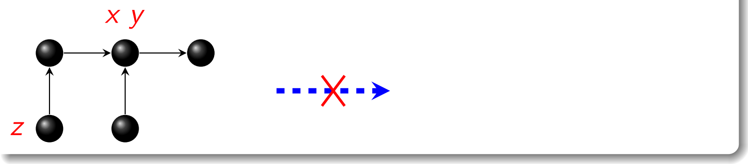

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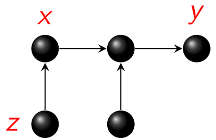

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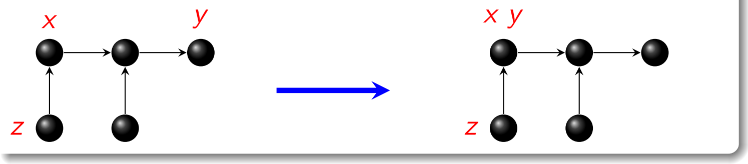

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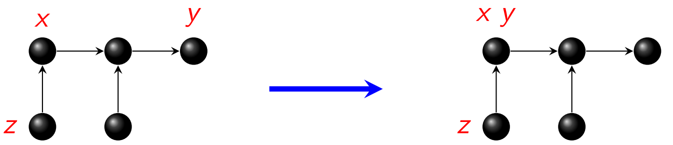

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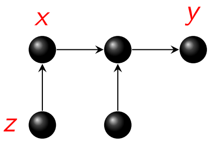

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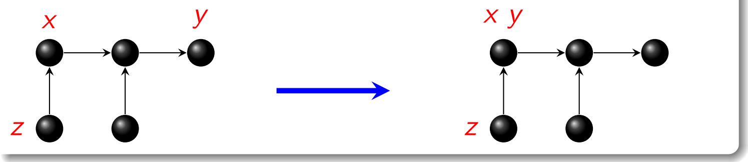

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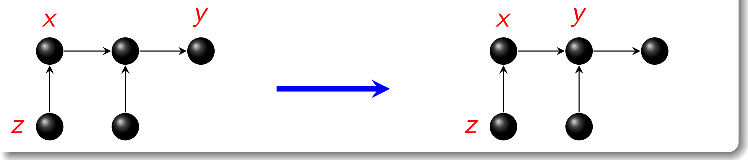

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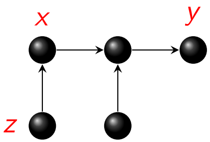

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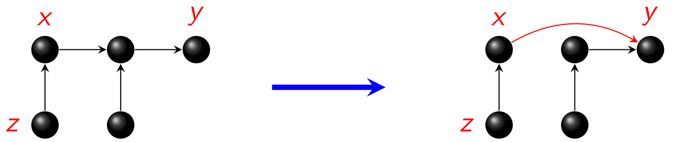

Ordering on Graphs

Variable Deletion

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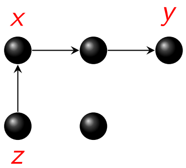

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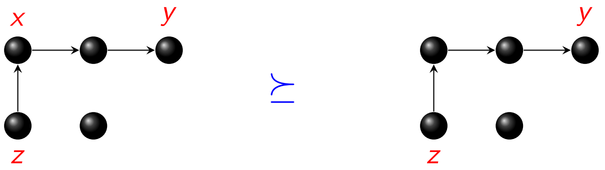

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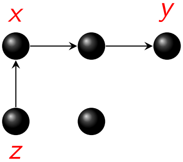

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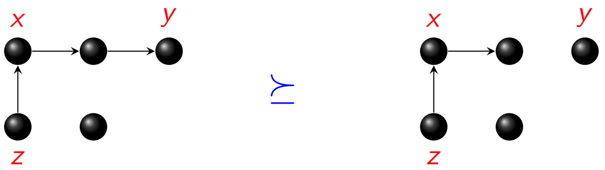

Ordering on Graphs

Vertex Deletion

Isolated Vertex

- no label
- no incoming/outgoing arcs

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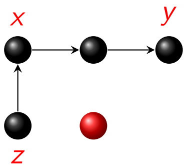

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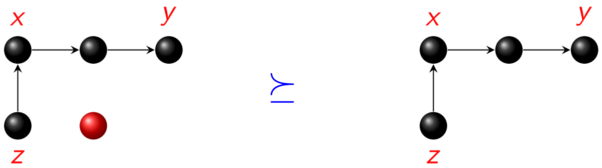

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Contraction

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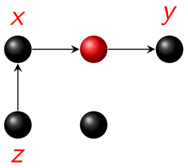

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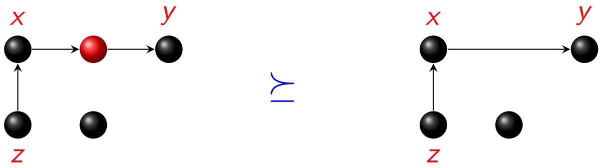
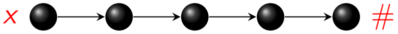


Bad Configurations

Well-formed Lists

Well-Formed List:



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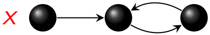

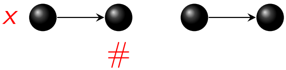

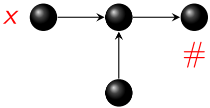

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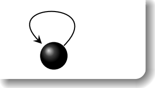
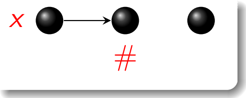
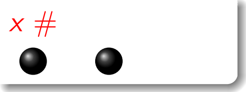
Bad Patetrns:

- minimal elements
- finitely many
- upward closure = all badly-formed lists







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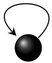
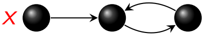
Bad pattern



$\preceq$

Bad configuration



Bad Configurations

Well-formed Lists

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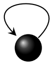

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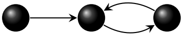

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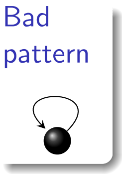

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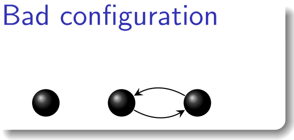

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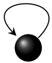

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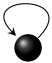

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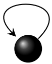

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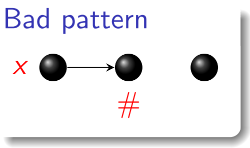
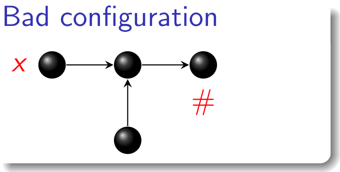
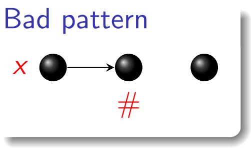


$\preceq$



Bad Configurations

Well-formed Lists



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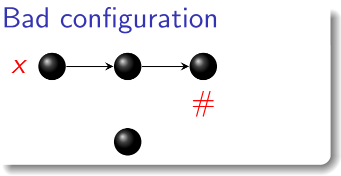

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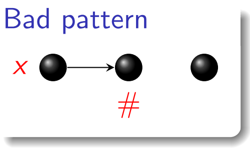

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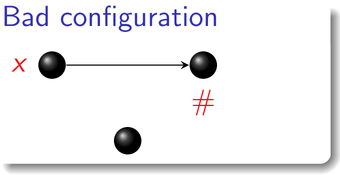
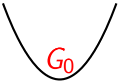


Backward Reachability Analysis



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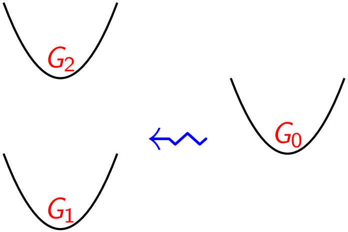

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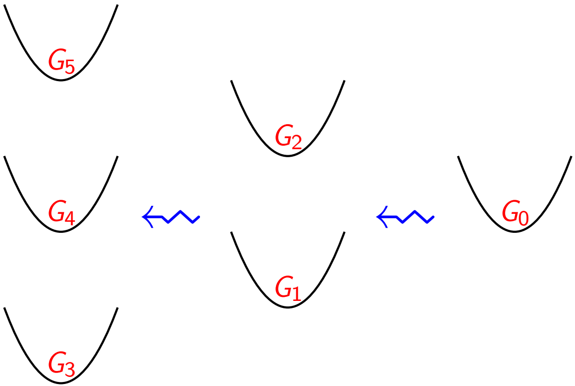

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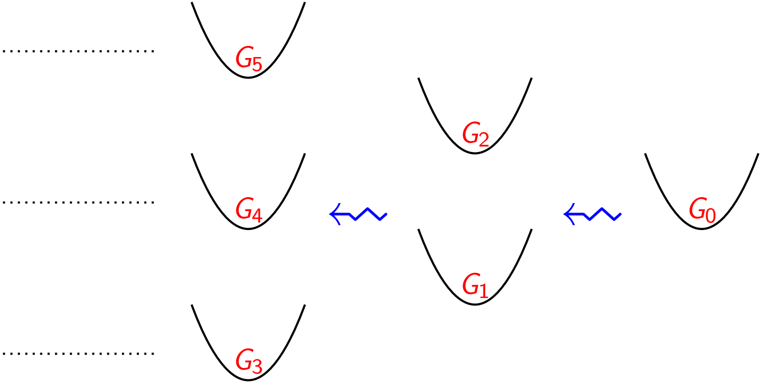

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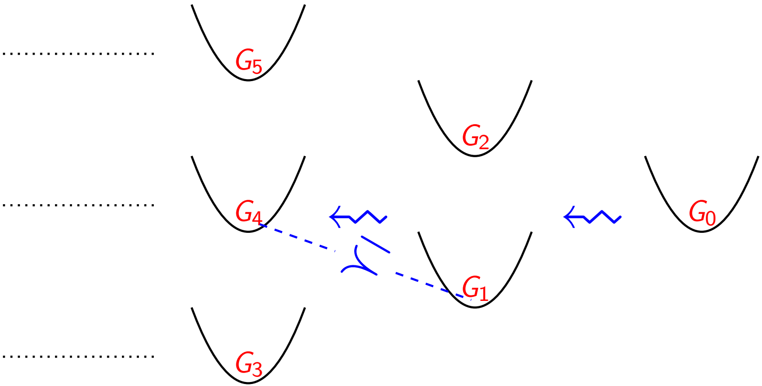

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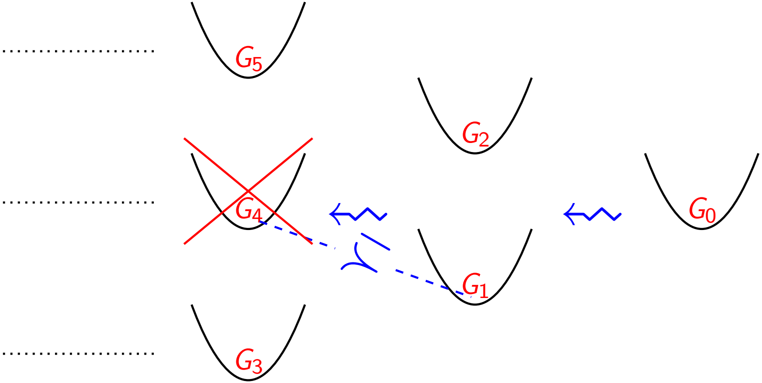

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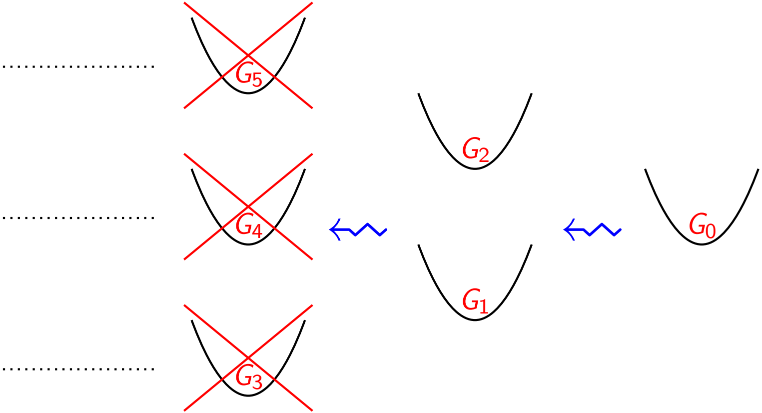

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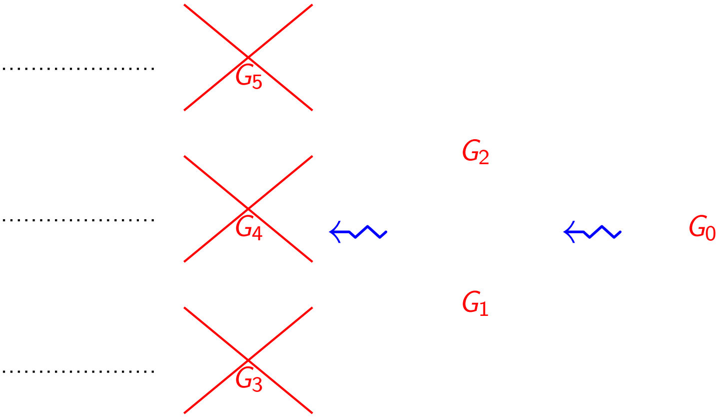
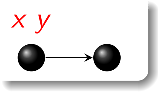


- symbolic representation = graphs
- $\preceq$ WQO implies termination

Computing predecessors

Testing Equality: $x = y$?



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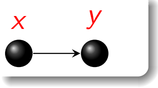

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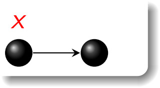

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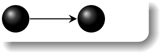

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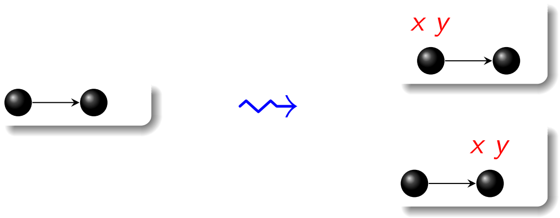

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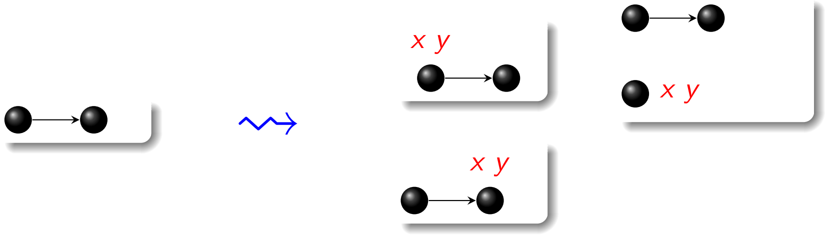

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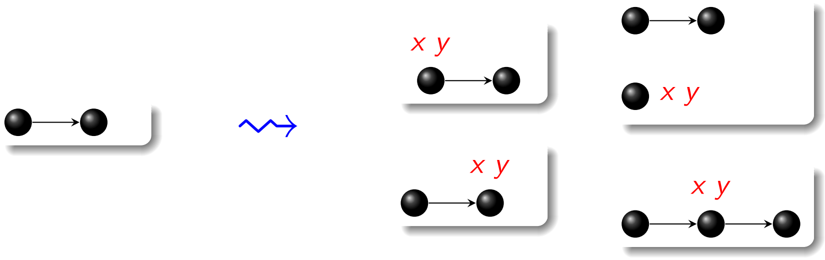
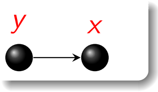


Computing predecessors

$x := y \cdot next$



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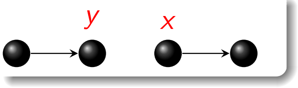

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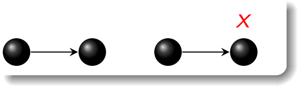

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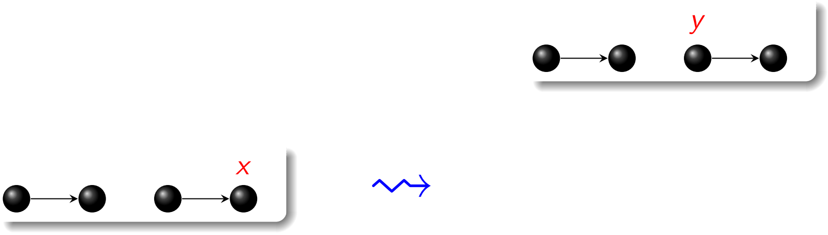

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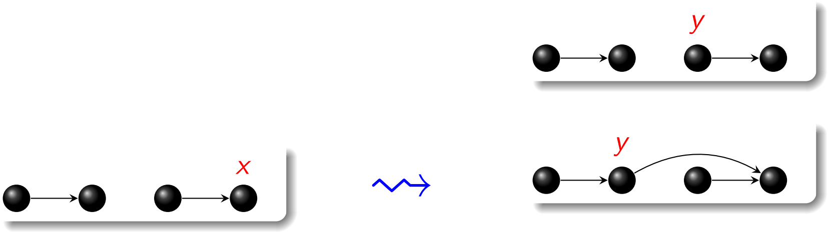

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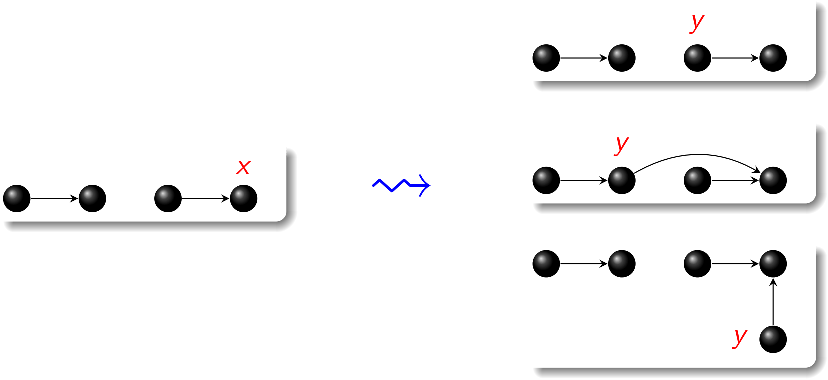

WQO

Degree

Degree

$deg(G) :=$ # unlabeled leafs

WQO

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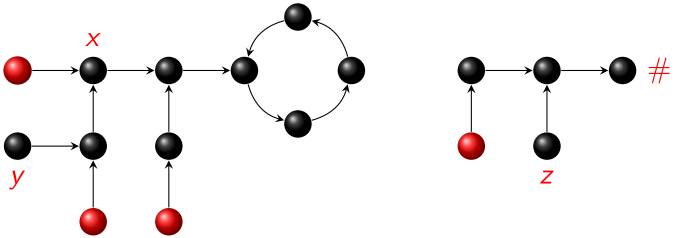
Example: $deg(G) = 4$



WQO

Block

Block

maximal subgraph which is connected

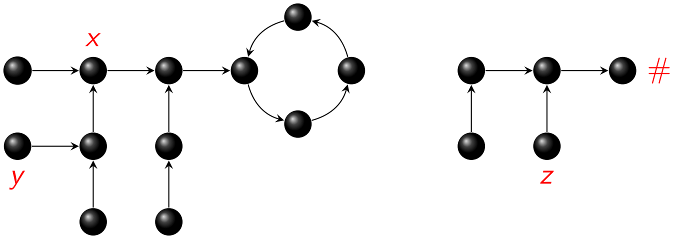
WQO

Block

Block

maximal subgraph which is connected

Example: Two blocks



WQO

Proof

$\preceq$ WQO:

- $g_1 \rightsquigarrow g_2$ implies $deg(g_1) \geq deg(g_2)$
- In back reachability scheme:
    - generated graphs have bounded degree
    - contain finitely many types of blocks (modulo contraction)
    - each graph can be encoded by a vector of multisets of vectors of natural numbers !
    - $\preceq$ WQO by Higman's lemma.

Experiments

Prog.	Prop.	Time	$\#C^{ons.}$	#Iter.	Prog.	Prop.	Time	$\#C^{ons}$
Concat	Deref	0.4 s	7	3	Delete	Deref	0.4 s	8
Fumble	Deref	0.3 s	3	2	Reverse	Deref	0.3 s	2
Walk	Deref	0.4 s	9	3	Zip	Deref	1.9 s	206
Fumble	Garbage	0.7 s	38	14	Reverse	Garbage	0.8 s	55
Reverse	Well-form.	1.7 s	48	20				