Timed Petri Nets

Signatures

1 5
6 3
2 0

1 5
2 6
3 0

ω

Time
Timed Petri Nets

Signatures

1

5

6

3

0

2

1

2

3

5

ω

6

0

ω

time
Timed Petri Nets
Signatures

1 5 6 3 0

! 1 5 6 3 0

time

! 1 5 6 3 0

time
Timed Petri Nets

Signatures
Timed Petri Nets

Signatures

\[\begin{array}{cccc}
1 & 2 & 6 & 3 \\
5 & \omega & 6 & 0 \\
\end{array}\]

\[\begin{array}{cccc}
4 & 1 & 2 & 6 \\
0 & 5 & \omega & 6 \\
\end{array}\]
Timed Petri Nets

Signatures

![Diagram of Timed Petri Nets]
Timed Petri Nets

Signatures
Timed Petri Nets

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Signatures
Timed Petri Nets

Signatures
$c_1 \equiv c_2 : \quad \text{sig}(c_1) = \text{sig}(c_2)$
Timed Petri Nets

Equivalence

\( c_1 \equiv c_2 : \)

\[ \operatorname{sig}(c_1) = \operatorname{sig}(c_2) \]
Timed Petri Nets
Equivalence

\[ c_1 \equiv c_2 : \quad \text{sig}(c_1) = \text{sig}(c_2) \]
$c_1 \trianglerighteq c_2$:

$\exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2)$
Timed Petri Nets

Ordering

$c_1 \sqsubseteq c_2 :$

$\exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2)$
Timed Petri Nets

Ordering

$c_1 \sqsubseteq c_2 :\exists c_3. \ (c_1 \equiv c_3) \land (c_3 \subseteq c_2)$
Timed Petri Nets

Ordering

$\exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2)$
Timed Petri Nets

Ordering

$c_1 \sqsubseteq c_2 :\exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2)$

$c_1$

\[
\begin{array}{cccccc}
5.0 & 1.7 & 8.2 & 4.7 & 3.2 & 6.5 & 1.0 \\
\end{array}
\]

$c_2$

\[
\begin{array}{cccccc}
5.0 & 3.2 & 2.5 & 4.8 & 3.1 & 6.6 & 1.0 \\
1.8 & 9.1 & 9.1 & 1.1 & 9.9 & 6.6 \\
\end{array}
\]

$c_3$

\[
\begin{array}{cccccc}
5.0 & 4.8 & 3.1 & 6.6 & 1.0 \\
1.8 & 9.1 & 9.1 & 1.1 & 9.9 & 6.6 \\
\end{array}
\]
Timed Petri Nets

Ordering

\[ c_1 \supseteq c_2 : \exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2) \]
Timed Petri Nets
Ordering

\[
\forall c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2)
\]
Timed Petri Nets

Ordering

$c_1 \sqsubseteq c_2 : \exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2)$
\[ c_1 \sqsubseteq c_2 : \exists c_3. (c_1 \equiv c_3) \land (c_3 \subseteq c_2) \]
$s_1 \subseteq s_2$: Derive $s_1$ from $s_2$ by:

- removing elements from multisets
- removing multisets
Derive $s_1$ from $s_2$ by:

- removing elements from multisets
- removing multisets
$s_1 \subseteq s_2$: Derive $s_1$ from $s_2$ by:
- removing elements from multisets
- removing multisets
Derive $s_1$ from $s_2$ by:

- removing elements from multisets
- removing multisets
Derive $s_1$ from $s_2$ by:

- removing elements from multisets
- removing multisets
$s_1 \subseteq s_2$: Derive $s_1$ from $s_2$ by:

- removing elements from multisets
- removing multisets

\begin{align*}
\text{s}_1 &:& \text{1} \ 5 \ \omega \ 4 \ 3 \ 6 \ 0 \ 1 \ 2 \\
\text{s}_2 &:& \text{1} \ 2 \ 4 \ 3 \ 6 \ 0 \ 1 \ 2
\end{align*}
Timed Petri Nets

Ordering

\[ c \models s \quad : \\
\exists c'. (c' \subseteq c) \land (\text{sig}(c') = s) \]
\[ c \models s : \exists c'. (c' \subseteq c) \land (\text{sig}(c') = s) \]
\( \exists c'. (c' \subseteq c) \land (\text{sig}(c') = s) \)

\[
\begin{bmatrix}
5.0 & 1.7 & 8.2 & 4.7 & 3.2 & 6.5 & 1.0 \\
1.8 & 9.1 & 9.1 & 1.1 & 9.9 & 6.6
\end{bmatrix}
\]
c' \models s : \exists c'. (c' \subseteq c) \land (\text{sig}(c') = s)

\text{c' \models s} : \exists c'. (c' \subseteq c) \land (\text{sig}(c') = s)
Timed Petri Nets

- Model
  - Configurations
  - Ordering
  - Monotonicity
  - Upward Closed Sets
  - Backward Reachability

- Transitions
  - Computing Predecessors
Timed Petri Nets

- Model
  - Configurations
  - Ordering
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  - Upward Closed Sets
  - Computing Predecessors
- Backward Reachability
Timed Petri Nets

Denotation

\[ [s] = \{ c \mid c \models s \} \]
Denotation

\[ [s] = \{ c | c \models s \} \]
\[ [s] = \{ c \mid c \models s \} \]
Timed Petri Nets

Denotation

\[ [s] = \{ c \mid c \models s \} \]
Timed Petri Nets

Denotation

\[ [s] = \{c \mid c \models s \} \]
Timed Petri Nets

Denotation

\[ [s] = \{ c \mid c \models s \} \]
$[s] = \{ c \mid c \models s \}$

Property:

- Infinite
- Upward closed wrt. $\subseteq$
Timed Petri Nets

Denotation

\[ [s] = \{ c \mid c \models s \} \]

infinite

upward closed wrt. \( \subseteq \)

\[ s_1 \subseteq s_2 \]

implies

\[ [s_2] \subseteq [s_1] \]
Timed Petri Nets

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Timed Petri Nets

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Timed Petri Nets

Monotonicity

\[ 5.0 \ 1.7 \ 8.2 \ 4.7 \ 3.2 \ 6.5 \ 1.0 \]

\[ 5.0 \ 3.2 \ 2.5 \ 4.8 \ 3.1 \ 6.6 \ 1.0 \]

\[ 1.8 \ 9.1 \ 9.1 \ 1.1 \ 9.9 \ 6.6 \]
Timed Petri Nets

Monotonicity

time = 0.3

\[ [5.0, 1.7, 8.2, 4.7, 3.2, 6.5, 1.0] \rightarrow [5.3, 2.0, 8.5, 5.0, 3.5, 6.8, 1.3] \]
Timed Petri Nets

Monotonicity

time = 0.3

5.0 1.7 8.2 4.7 3.2 6.5 1.0

5.3 2.0 8.5 5.0 3.5 6.8 1.3

time = 0.2

5.0 3.2 2.5 4.8 3.1 6.6 1.0

1.8 9.1 9.1 1.1 9.9 6.6

5.2 3.4 2.7 5.0 3.3 6.8 1.2

2.0 9.3 9.3 1.3 10.1 6.8
Timed Petri Nets

Computing Predecessors

\[
\text{time} = 0.3
\]

\[
\begin{array}{cccccccc}
5.0 & 1.7 & 8.2 & 4.7 & 3.2 & 6.5 & 1.0 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
5.3 & 2.0 & 8.5 & 5.0 & 3.5 & 6.8 & 1.3 \\
\end{array}
\]

\[
\text{time} = 0.2
\]

\[
\begin{array}{cccccccc}
5.0 & 3.2 & 2.5 & 4.8 & 3.1 & 6.6 & 1.0 \\
1.8 & 9.1 & 9.1 & 1.1 & 9.9 & 6.6 \\
\end{array}
\]

\[
\begin{array}{cccccccc}
5.2 & 3.4 & 2.7 & 5.0 & 3.3 & 6.8 & 1.2 \\
2.0 & 9.3 & 9.3 & 1.3 & 10.1 & 6.8 \\
\end{array}
\]
Timed Petri Nets

Model

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Computing Predecessors

Backward Reachability
Timed Petri Nets

Model

Ordering

Transitions

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Computing Predecessors

Backward Reachability
Computing Predecessors

$P_{\text{time}}$
Timed Petri Nets

Computing Predecessors

\[ \text{Pre}_{\text{time}} \]
Timed Petri Nets

Computing Predecessors

\[ \text{\textit{Pre}}_{\text{time}} = \text{\textit{Pre}}_{\text{time}} \]
Timed Petri Nets

Computing Predecessors

Pre\textsubscript{time}

=
Timed Petri Nets

Computing Predecessors

\[ \text{Pre}_{\text{time}} = \]
Timed Petri Nets

Computing Predecessors

$\text{Pre}_{\text{time}} = 0$
Timed Petri Nets

Computing Predecessors

$\text{Pre}_{\text{time}} = \omega$

$\begin{array}{c}
1 & & 0 & & 4 \\
& & 4 & & \\
& & 1 & & 0 \\
6 & & 2 & & \omega
\end{array}$

$\begin{array}{c}
1 & & 0 & & 4 \\
& & 4 & & \\
& & 1 & & 0 \\
6 & & 2 & & \omega
\end{array}$
Computing Predecessors

$P_{\text{time}} = \omega$
Time Petri Nets
Computing Predecessors

\[ \mathbf{P}_{\text{time}} = \mathbf{0} \]
Timed Petri Nets

Computing Predecessors

$Pre_{time} = \omega$
Timed Petri Nets

Computing Predecessors

\[ \omega = 0 \]
Timed Petri Nets

Computing Predecessors

$Pre_{time}$

$= \omega$

$= \omega$
Timed Petri Nets

Computing Predecessors

Pre_{time}

Diagram showing the computing of predecessors in Timed Petri Nets.
Timed Petri Nets

Computing Predecessors

\( Pre_{time} \)

\[
\begin{array}{cccc}
2 & 6 & 1 & 0 \\
\omega & 4 & 4 & \omega \\
\end{array}
\]
Timed Petri Nets

Computing Predecessors
Timed Petri Nets

Computing Predecessors

\[ \text{Pre}_{\text{disc}} \]
Timed Petri Nets

Computing Predecessors

$\text{Pred}_{\text{disc}}$

$\begin{array}{cccc}
1 & 2 & 2 & 0 \\
5 & \omega & 6 & 4
\end{array}$

$\begin{array}{cccc}
1 & 2 & 6 & 1 \\
5 & \omega & 4 & 1
\end{array}$

$t_1[1..3)$

$[0..1)$

$[2..5)$
Timed Petri Nets

Computing Predecessors

$\text{Predisc}$
Timed Petri Nets

Model

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Model

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Backward Reachability
Timed Petri Nets

Backward Reachability
Timed Petri Nets
Backward Reachability

time
Timed Petri Nets
Backward Reachability
Timed Petri Nets
Backward Reachability
Timed Petri Nets - Backward Reachability

- Transition time: $t_1$, $t_2$

- Places and transitions connected by arcs.
Timed Petri Nets

Backward Reachability
Timed Petri Nets
Backward Reachability
symbolic representation = finite words over finite multisets

Termination: finite words over finite multisets well quasi-ordered