Background

Parameterized Systems

Petri Nets

Lossy Channel Systems

Timed Petri Nets
Lossy Channel Systems
Lossy Channel Systems

Model

Configurations

Ordering

Transitions

Monotonicity

Upward Closed Sets

Computing Predecessors

Backward Reachability
Lossy Channel Systems

Model

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Lossy Channel Systems

Model

- \( P_1 \): finite-state process
- \( P_2 \): finite-state process
- \( P_3 \): finite-state process
- Lossy unbounded FIFO buffer
- \( C_1 \)
- \( C_2 \)
- \( C_3 \)
Lossy Channel Systems

Model

P_1: finite-state process

P_2: lossy unbounded FIFO buffer

P_3: finite-state process

C_1: lossy unbounded FIFO buffer

C_2: lossy unbounded FIFO buffer

C_3: finite-state process
Lossy Channel Systems

Telecommunication Protocols

sender

lossy unbounded FIFO buffer

receiver

C

d

lossy unbounded FIFO buffer
Lossy Channel Systems

Telecommunication Protocols

sender

receiver

lossy unbounded FIFO buffer

c

d

lossy unbounded FIFO buffer
Lossy Channel Systems

Telecommunication Protocols
Lossy Channel Systems

Weak Memory Models

Shared Memory

P1 → lossy unbounded FIFO buffer

P2 → lossy unbounded FIFO buffer

P3 → lossy unbounded FIFO buffer

x = 2

y = 1

z = 3
Lossy Channel Systems

Weak Memory Models

P1

P2

P3

threads

lossy unbounded FIFO buffer

Shared Memory

intelmachine

\( x = 2 \)

\( y = 1 \)

\( z = 3 \)
Lossy Channel Systems

Weak Memory Models

- Lossy unbounded FIFO buffer
- Shared Memory
- intel x86

Threads:
- $P_1$
- $P_2$
- $P_3$

Values:
- $x=2$
- $y=1$
- $z=3$

FIFO Buffer:
- $y=2$
Lossy Channel Systems

Weak Memory Models

Thread

P1

y=2

Thread

P2

z=1

Thread

P3

lossy unbounded FIFO buffer

Shared Memory

x=2

y=1

z=3

lossy unbounded FIFO buffer
Lossy Channel Systems

Weak Memory Models

Thread

P_1

y=2

Thread

P_2

z=1

Thread

P_3

Lossy unbounded FIFO buffer

Shared Memory

x=2

y=1

z=3
Lossy Channel Systems

Weak Memory Models

-thread

P_1

y=2

thread

P_2

z=1

thread

P_3

lossy unbounded FIFO buffer

Shared Memory

x=2

y=1

z=3
Lossy Channel Systems

Weak Memory Models

Shared Memory

thread

P_1

lossy unbounded FIFO buffer

x=3

y=2

z=1

P_2

thread

P_3

lossy unbounded FIFO buffer

x=2

y=1

z=3

thread
Lossy Channel Systems

Model

Weak Memory Models

thread

P₁

lossy unbounded FIFO buffer

x=3  y=2

P₂

z=1

lossy unbounded FIFO buffer

P₃

Shared Memory

x=2

y=1

z=3
Lossy Channel Systems

- FIFO buffer

Weak Memory Models

- Shared Memory

- Lossy unbounded FIFO buffer

Threads:

- \( P_1 \)
  - \( x=4 \)
  - \( x=3 \)
  - \( y=2 \)

- \( P_2 \)
  - \( z=1 \)

- \( P_3 \)
Lossy Channel Systems

Weak Memory Models

thread

P₁

x=4   x=3

P₂

z=1   y=2   z=1

P₃

x=0   x=1   y=0

Shared Memory

x=2

y=2

z=3

lossy unbounded FIFO buffer

thread
Lossy Channel Systems

Weak Memory Models

thread

P₁

P₂

P₃

lossy unbounded FIFO buffer

Shared Memory

x=2
y=2
z=3

x=4 x=3

z=1 y=2 z=1

x=0 x=1 y=0
**Lossy Channel Systems**

**Weak Memory Models**

- Lossy unbounded FIFO buffer

**Threads**:
- \( P_1 \)
- \( P_2 \)
- \( P_3 \)

**Shared Memory**:
- \( x=2 \)
- \( y=2 \)
- \( z=3 \)

**States**:
- \( x=4 \) and \( x=3 \)
- \( z=1 \), \( y=2 \), and \( z=1 \)
- \( x=0 \) and \( y=0 \)
Lossy Channel Systems

Model for a Telecommunication Protocol

sender

lossy unbounded FIFO buffer

c

receiver

d

lossy unbounded FIFO buffer
Lossy Channel Systems

Model

Telecommunication Protocol

sender

lossy unbounded FIFO buffer

receiver

Lossy unbounded FIFO buffer

c

d
Lossy Channel Systems

Model of Telecommunication Protocol

sender

lossy unbounded FIFO buffer

c

receiver

lossy unbounded FIFO buffer

d
Lossy Channel Systems

$\mathcal{P}_1$

$\mathcal{P}_2$

lossy unbounded FIFO buffer

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$d?1$ $c!0$

$d?1$ $d?0$

$d?0$ $c!1$

$P_2$

$c?1$ $d!1$

$c?1$ $c?0$

$c?0$ $d!0$

lossy unbounded FIFO buffer

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$P_2$

lossy unbounded FIFO buffer

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$P_2$

lossy unbounded FIFO buffer

lossy unbounded FIFO buffer
Lossy Channel Systems

Lossy unbounded FIFO buffer

$P_1$

$d!1$

d?1

d?0

d?0

c!1

c!0

$P_2$

c?1

d!1

c?0

c?0

d!0

c!1

d?1
Lossy Channel Systems

$P_1$

d?1  c!0

d?1  d?0

d?0  c!1

lossy unbounded FIFO buffer

$P_2$

c?1  d!1

c?1  c?0

c?0  d!0

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$P_2$

lossy unbounded FIFO buffer

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$d?1$ $c!0$

d$?1$ $d?0$

d$?0$ $c!1$

lossy unbounded FIFO buffer

$0$ $0$

$1$ $1$ $1$ $0$ $0$

$P_2$

c$?1$

c$?1$

c$?0$

c$?0$

d$!1$

d$!0$

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$d?1$  $c!0$

d?1  d?0

d?0  c!1

lossy unbounded FIFO buffer

$P_2$

c?1  d!1

c?1  c?0

c?0  d!0

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$P_2$

lossy unbounded FIFO buffer

$0 \ 0$

$0 \ 0$

$0 \ 0$
Lossy Channel Systems

$P_1$

$d?1$  $c!0$
$d?1$  $d?0$
$d?0$  $c!1$

Lossy unbounded FIFO buffer

$P_2$

$c?1$  $d!1$
$c?1$  $c?0$
$c?0$  $d!0$

Lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$P_2$

lossy unbounded FIFO buffer

lossy unbounded FIFO buffer
Lossy Channel Systems

$P_1$

$P_2$

(lossy unbounded FIFO buffer)
Lossy Channel Systems

Model

Configurations

Ordering

Transitions

Monotonicity

Upward Closed Sets

Computing Predecessors

Backward Reachability
Lossy Channel Systems

Con
gurations

State of \( P_1 \) State of \( P_2 \) Content of \( c \) Content of \( d \)

Configuration

\[ 00 \quad 011 \]
Lossy Channel Systems

Configurations

P₁

P₂

0 0

1 0 1

state of P₁
state of P₂
content of c
content of d

configuration
Lossy Channel Systems

Configurations

Initial configuration:
- State of $P_1$: 0
- State of $P_2$: 0
- Content of $c$: 00
- Content of $d$: 101

Initial state of $P_1$: $c = 0$
- Initial state of $P_2$: $c = c_0$

Initial state of $P_1$: $d = 0$
- Initial state of $P_2$: $d = d_1$
Lossy Channel Systems

- Model
- Configurations
- Transitions
- Ordering
- Monotonicity
- Upward Closed Sets
- Computing Predecessors
- Backward Reachability
Lossy Transitions

c \Rightarrow d

t_1 \Rightarrow t_2 \Rightarrow \cdots \Rightarrow t_{12} \Rightarrow c

d \Rightarrow c

\epsilon \Rightarrow \epsilon
Lossy Transitions
Lossy Transitions
Lossy Transitions

d\ldots 1 \rightarrow t_3 \rightarrow t_4 \rightarrow c!0

d\ldots 1 \rightarrow t_1 \rightarrow t_2 \rightarrow d?0

d\ldots 0 \rightarrow t_5 \rightarrow t_6 \rightarrow c!1

d\ldots 0 \rightarrow t_7 \rightarrow t_8 \rightarrow c?0

c?1 \rightarrow t_9 \rightarrow t_{10} \rightarrow d!1

c?0 \rightarrow t_{11} \rightarrow t_{12} \rightarrow d!0

c!0 \rightarrow t_4 \rightarrow c!0

\epsilon \quad \epsilon

0 \quad \epsilon

\rightarrow t_4
Lossy Transitions

Diagram showing transitions with labels and arrows.
Lossy Transitions

d?1 t3 c!0
d?1 t1 t2 d?0
d?0 t5 t6 c!1

c?0 t7 t8 c?0
c?0 t11 t12 d?0

d?1 t4 c!0

c

d

t4

\[ P_2 \]

d?0 ?0

d?1 ?1
t1 t2 t3 t4 t5 t6 t7 t8 t9 t10 t11 t12

\[ t_4 \]
Lossy Channel Systems

Transitions

\[d?1(t_3 \leftrightarrow t_4) c!0\]

\[d?1(t_1 \leftrightarrow t_2) d?0\]

\[d?0(t_5 \leftrightarrow t_6) c!1\]

\[c?1(t_9 \leftrightarrow t_{10}) d!1\]

\[c?1(t_7 \leftrightarrow t_8) c?0\]

\[c?0(t_{11} \leftrightarrow t_{12}) d?0\]

\[t_4\]

\[t_4\]
Lossy Transitions

\[ P_2 \]

\[ t_4 \]

\[ t_4 \]

\[ t_4 \]

\[ t_4 \]
Lossy Transitions

\[
\begin{align*}
\text{Input Transition:} & \quad (d_1 t_3, t_4) c_0 \\
\text{Input Transition:} & \quad (d_1 t_1, t_2) d_0 \\
\text{Input Transition:} & \quad (d_0 t_5, t_6) c_1 \\
\text{Output Transition:} & \quad (c_1 t_7, t_8) c_0 \\
\text{Output Transition:} & \quad (c_0 t_11, t_{12}) d_0 \\
\text{Output Transition:} & \quad (t_{10}) d_1 \\
\end{align*}
\]
Lossy Transitions

\[ t_4 \]

\[ t_4 \]

\[ t_{10} \]
Lossy Transitions

- Transition $t_4$
- Transition $t_{10}$

Diagram shows states and transitions with labels $c, d, t_1, t_2, t_3, t_4, t_5, t_6, t_7, t_8, t_9, t_{10}, t_{11}, t_{12}$.
Lossy channel systems transitions
Lossy Transitions

```
<table>
<thead>
<tr>
<th></th>
<th>ε</th>
<th>ε</th>
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<tbody>
<tr>
<td>t4</td>
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<td>t10</td>
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<th>11</th>
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<tbody>
<tr>
<td>t10</td>
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</tbody>
</table>
```
Lossy Transitions

\[
\begin{align*}
&d?1(t_3 \rightarrow t_4 \rightarrow c!0) \\
&d?1(t_1 \rightarrow t_2 \rightarrow d?0) \\
&d?0(t_5 \rightarrow t_6 \rightarrow c!1) \\
&c?1(t_7 \rightarrow t_8 \rightarrow d!1) \\
&c?0(t_10 \rightarrow t_11 \rightarrow t_12 \rightarrow d?0) \\
\end{align*}
\]
Lossy Transitions

Diagram showing transitions and states in a lossy channel system.
Lossy Transitions

\[ \begin{align*}
\text{Transition } t_4 & \quad \text{from state } \epsilon \text{ to state } \epsilon \\
\text{Transition } t_4 & \quad \text{from state } 0 \text{ to state } \epsilon \\
\text{Transition } t_{10} & \quad \text{from state } 00 \text{ to state } \epsilon \\
\text{Transition } t_{10} & \quad \text{from state } 00 \text{ to state } 1 \\
\text{Transition } t_8 & \quad \text{from state } 00 \text{ to state } 11 \\
\text{Transition } t_{12} & \quad \text{from state } 00 \text{ to state } 11
\end{align*} \]
Lossy Channel Systems

Transitions

\[ t_4 \]
\[ t_4 \]
\[ t_{10} \]
\[ t_{10} \]
\[ t_8 \]
Lossy Channel Systems

Transitions

\[
\begin{align*}
& d?1(t_3, t_4) c!0 \\
& d?1(t_1, t_2) d?0
\end{align*}
\]

\[
\begin{align*}
& c?1(t_9, t_{10}) d!1 \\
& c?1(t_7, t_8) c?0
\end{align*}
\]

\[
\begin{align*}
& t_{12} d!0
\end{align*}
\]

\[
\begin{align*}
& 00 \quad \epsilon
\end{align*}
\]

\[
\begin{align*}
& 00 \quad 0 \quad \epsilon
\end{align*}
\]

\[
\begin{align*}
& 00 \quad 1
\end{align*}
\]

\[
\begin{align*}
& 00 \quad 11
\end{align*}
\]

\[
\begin{align*}
& t_4 \quad t_4
\end{align*}
\]

\[
\begin{align*}
& t_{10} \quad t_{10}
\end{align*}
\]

\[
\begin{align*}
& t_8 \quad t_{12}
\end{align*}
\]
Lossy Transitions

- Transition $t_4$
- Transition $t_{10}$
- Transition $t_8$
- Transition $t_{12}$
Lossy Channel Systems

Transitions

- Transition 1: $(c?1, t9, t10, d?1, c?0) \rightarrow (c?1, t7, t8, c?0)$
- Transition 2: $(d?1, t3, t4, c!0) \rightarrow (d?0, t5, t6, c!1)$

- Transition 3: $(c?1, t9, t10, d?1, c?0) \rightarrow (c?0, t11, t12, d?0)$
- Transition 4: $(d?1, t3, t4, c!0) \rightarrow (d?0, t5, t6, c!1)$

- Transition 5: $(00, c?0) \rightarrow (00, 1)$
- Transition 6: $(00, c?1) \rightarrow (00, 11)$
- Transition 7: $(00, 1) \rightarrow (00, 11)$
- Transition 8: $(00, 11) \rightarrow (00, 011)$
- Transition 9: $(00, 011) \rightarrow$ (loss)

- Transition 10: $(00, c?0) \rightarrow (00, 1)$
- Transition 11: $(00, c?1) \rightarrow (00, 11)$
- Transition 12: $(00, 1) \rightarrow (00, 11)$
- Transition 13: $(00, 11) \rightarrow (00, 011)$
- Transition 14: $(00, 011) \rightarrow$ (loss)
Lossy Channel Systems

Transitions

\[ c \rightarrow \epsilon \rightarrow \epsilon \]

\[ d \rightarrow 0 \rightarrow \epsilon \]

\[ t_4 \]

\[ t_{10} \]

\[ t_{10} \]

\[ t_8 \]

\[ t_{12} \]

loss
Lossy Channel Systems

Transitions

\[ \begin{align*}
  & P \quad P^2 \\
  & t_4 \\
  & t_4 \\
  & t_{10} \\
  & t_{10} \\
  & t_8 \\
  & t_{12} \\
  & \text{loss} \\
  & \text{loss}
\end{align*} \]
Lossy Channel Systems

Transitions

\[ t_4 \]
\[ t_{10} \]
\[ t_8 \]
\[ t_{12} \]
loss
loss

[Diagram showing transitions and states]
Loss, Transitions

- Transition $t_4$
- Transition $t_{10}$
- Transition $t_8$
- Transition $t_{12}$
- Loss
- Loss
Lossy Channel Systems

Transitions
Lossy Channel Systems

Transitions

- $t_2 \rightarrow d?0$
- $c?1 \rightarrow t_9 \rightarrow t_10 \rightarrow d?1$
- $c?0 \rightarrow t_11 \rightarrow t_12 \rightarrow d?0$
- $d?1 \rightarrow t_3 \rightarrow t_4 \rightarrow c?0$
- $d?1 \rightarrow t_1 \rightarrow t_2 \rightarrow d?0$
- $d?0 \rightarrow t_5 \rightarrow t_6 \rightarrow c?1$

- $t_4 \rightarrow 
  \begin{array}{ll}
    \epsilon & \epsilon \\
  \end{array}$
- $t_4 \rightarrow 
  \begin{array}{ll}
    0 & \epsilon \\
  \end{array}$
- $t_{10} \rightarrow 
  \begin{array}{ll}
    00 & \epsilon \\
  \end{array}$
- $t_{10} \rightarrow 
  \begin{array}{ll}
    00 & 1 \\
  \end{array}$
- $t_8 \rightarrow 
  \begin{array}{ll}
    00 & 11 \\
  \end{array}$
- $t_{12} \rightarrow 
  \begin{array}{ll}
    0 & 11 \\
  \end{array}$
- $loss \rightarrow 
  \begin{array}{ll}
    00 & 011 \\
  \end{array}$
- $loss \rightarrow 
  \begin{array}{ll}
    00 & 01 \\
  \end{array}$
- $t_2 \rightarrow 
  \begin{array}{ll}
    0 & \epsilon \\
  \end{array}$
Lossy Channel Systems

Model

Transitions

Ordering

Monotonicity

Computing Predecessors

Upward Closed Sets

Backward Reachability

Configurations
Lossy Channel Systems

Model

Configurations

Transitions

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Lossy Channel Systems

- Model
- Configurations
- Transitions

- Ordering
- Monotonicity
- Upward Closed Sets
- Computing Predecessors

- Backward Reachability
Lossy Ordering
Subword Relation

\[ ab \sqsubseteq xaybz \]
Lossy

Ordering

Subword Relation

\[ ab \sqsubseteq xaybz \]
Lossy Ordering

Subword Relation

ab \sqsubseteq xaybzM

Ordering

Monotonicity
Lossy Channel Systems

- Model
- Ordering
- Transitions
- Monotonicity
- Computing Predecessors
- Upward Closed Sets
- Backward Reachability
Lossy Channel Systems

- Model
- Configurations
- Transitions
- Ordering
- Monotonicity
- Upward Closed Sets
- Computing Predecessors
- Backward Reachability
Lossy Upward-Closed Sets
Lossy Channel Systems

Model

Configurations

Ordering

Transitions

Monotonicity

Upward Closed Sets

Computing Predecessors

Backward Reachability
Lossy Channel Systems

- Model
- Configurations
- Ordering
- Transitions
- Monotonicity
- Upward Closed Sets
- Computing Predecessors
- Backward Reachability
Lossy Predecessors
Lossy Predecessors

$Pre(U)$

$U$

$m_1$

$m_3$
Predecessors

Lossy Channel Systems

\[ Pre(U) \rightarrow U \]

\[ m_2 \rightarrow m_1 \]

\[ m_3 \]
Loss\textsubscript{y}, Predecessors

\[ \text{Pre}(U) \]

\[ U \]
Monotonicity: UC persevered by $Pre$
Monotonicity: UC persevered by $Pre$

$m_1 \rightarrow m_2$

$m_3 \rightarrow m_4$

$Pre(U)$  

upward closed?
Monotonicity: UC persevered by $Pre$

Lossy Channel Systems

Predecessors

$m_1 \rightarrow m_2$

$m_3 \rightarrow m_4$

$Pre(U)$

$upward\ closed$?
Monotonicity: UC persevered by $Pre$.

$Pre(U)$ upward closed?
Monotonicity: UC persevered by $Pre$

$m_1 \rightarrow m_2$

$m_3 \rightarrow m_4$

$Pre(U)$

$U$

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Monotonicity: UC persevered by $Pre$

$Pre(U)$

upward closed

$U$
Monotonicity: UC persevered by \( Pre \)

Choose \( m_4 = m_2 \)

Lossy Channel Systems

Predecessors

\[ m_1 \rightarrow m_2 \]
\[ m_3 \rightarrow m_4 \]

upward closed

\( Pre(U) \)
Monotonicity: UC persevered by $Pre$

Choose $m_4 = m_2$

Predecessors

Lossy Channel Systems

$Pre(U)$, upward closed?
Lossy Computing Predecessors
Loss, Computing Predecessors

\[ \text{Pre}_{t_2} \]

\[ d?0 \rightarrow d!1 \]

\[ c?1 \rightarrow c!0 \]

\[ \epsilon \rightarrow 01 \]

\[ t_2 \rightarrow d?0 \]
Lossy Computing Predecessors

\[ \text{Pre}_{t_2} \]

\[ = \]

\[ \text{Pre}_{t_2} \]

\[ t_2 d?0 \]
Lossy Computing Predecessors

$\text{Pre}_{t_2}$

$\text{Pre}_{t_{10}}$
Loss, Computing Predecessors

\[ \text{Pre}_{t_2} \]

\[ \text{Pre}_{t_{10}} \]

\[ c_{10} \]

\[ d_{10} \]

\[ = \]

\[ c_{10} \]

\[ d_{10} \]
Lossy Computing Predecessors

\[\text{Pre}_{t_{10}}\]

\[d!1 \quad t_{10} = \epsilon 0\]
Lossy Backward Reachability

symbolic representation = finite words
Lossy Backward Reachability

Termination: words well quasi-ordered

symbolic representation = finite words
Lossy Channel Systems

- Model
- Configurations
- Ordering
- Transitions
- Monotoncity
- Upward Closed Sets
- Computing Predecessors
- Backward Reachability
Lossy Channel Systems

- Model
- Configurations
- Ordering
- Transitions
- Monotoncity
- Upward Closed Sets
- Computing Predecessors
- Backward Reachability
Lossy Channel Systems

- Model
- Configurations
- Ordering
- Transitions
- Monotoncity
- Upward Closed Sets
- Computing Predecessors

Backward Reachability

*a more systematic algorithm*
Lossy Backward Reachability
Lossy Backward Reachability

- Initial state of $P_1$
- Initial state of $P_2$
- Content of $c$
- Content of $d$

Initial configuration
Lossy Backward Reachability

Initial state of $P_1$
Initial state of $P_2$
Content of $c$
Content of $d$
Initial configuration

Target configurations
Loss-Backward Reachability

**Target configurations**

- **processes**:
  - red and black states
- **channel** `c`:
  - any content
- **channel** `d`:
  - at least a "1"

### Initial configuration

- **Initial state of** $P_1$
- **Initial state of** $P_2$
- **Content of** `c`
- **Content of** `d`
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

visited
Lossy Channel Systems
Backward Reachability

waiting

visited

\[ d_1, t_1, t_2, d_0, c_0 \]

\[ c_1, t_3, t_4, d_0 \]

\[ d_0, t_5, t_6, c_1 \]

\[ c_0, t_7, t_8, c_0 \]

\[ c_0, t_9, t_{10}, d_0 \]

\[ d_0, t_11, t_{12}, d_0 \]
Lossy Backward Reachability

Waiting

Visited
Lossy Backward Reachability

waiting

visited

visited

visited
Lossy Backward Reachability

Waiting

visited

Lossy Backward Reachability

Waiting

visited
Lossy Backward Reachability

waiting

visited

t_8 c?0
Lossy Backward Reachability
Lossy Backward Reachability

waiting

visited
Lossy, Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

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Lossy Backward Reachability
Lossy Backward Reachability

waiting

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Lossy Backward Reachability

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Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

visited

Lossy Channel Systems
Lossy Backward Reachability

waiting

visited

\[ t_1 \xrightarrow{c} t_2 \xrightarrow{d} t_3 \xrightarrow{e} t_4 \xrightarrow{f} t_5 \]

\[ t_6 \xrightarrow{g} t_7 \xrightarrow{h} t_8 \xrightarrow{i} t_9 \xrightarrow{j} t_{10} \xrightarrow{k} t_{11} \xrightarrow{l} t_{12} \]
Lossy Backward Reachability
Lossy Backward Reachability

waiting

visited

t10

d1!

c0

t1

t2
d0

t3
d1

visited

0 1

ε

0 10

epsilon

0 1

ε

0 10

visited
Lossy Backward Reachability

waiting

visited
Lossy Channel Systems

Backward Reachability

waiting

visited
Lossy Backward Reachability

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Lossy Backward Reachability
Lossy Backward Reachability

waiting

visited
Lossy, Backward Reachability

Waiting

Visited
Lossy Backward Reachability
Lossy Backward Reachability
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability
Lossy Backward Reachability

waiting

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Lossy, Backward Reachability
Lossy Channel Systems

Backward Reachability

waiting

visited
Lossy Backward Reachability
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waiting

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Lossy Channel Systems
Lossy Backward Reachability
Lossy Backward Reachability

Waiting

Visited
Lossy Backward Reachability

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Lossy Backward Reachability

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Lossy Channel Systems

Backward Reachability

waiting

visited
Lossy Backward Reachability

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Lossy Backward Reachability
Lossy Backward Reachability

Waiting

Visited

\[ \text{Waiting} \]

\[ \text{Visited} \]
Lossy Backward Reachability

waiting

visited

$\epsilon$

$01$

$\epsilon$

$01$

$1$

$\epsilon$

$\epsilon$

$0$

$\epsilon$

$0$

$\epsilon$
Lossy Backward Reachability

Waiting

Visited
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability
Lossy Backward Reachability
Lossy, Backward Reachability

Waiting

Visited

\[ t_1 \]

\[ d_1 \]
Lossy Channel Systems
Backward Reachability
Lossy Backward Reachability

Waiting

Visited
Lossy Backward Reachability

waiting

t?1 t2 t3 t4 d?0
d?0 t5 t6 c?1
c?0 t7 t8 d!1

c?1 t9 t10 t11 d!1

c?0 t12 d!0

t8 c?0
Lossy Backward Reachability
Lossy Backward Reachability

waiting

visited

Lossy Channel Systems
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

c

d

c

d

c

d

c

c

c

c

c

c

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c

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c

c

visited
Lossy Backward Reachability
Lossy Backward Reachability
Lossy Backward Reachability
Lossy Backward Reachability

waiting

t2

d?0

t2

d?0

visited

0 0 1 ε

1 0 ε

ε 1

ε 10

ε 01

ε 0

ε 0
Lossy Backward Reachability

Waiting

Visited

$t_2$ $d?0$
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

d?1 t1 t2 t3 c?0
d?0 t5 t6 t7 c?1
c?1 t4 t6 t8 d?1
c?0 t11 t12 d?0

t8 c?0
Lossy Backward Reachability
Lossy Channel Systems
Backward Reachability

waiting

visited
Lossy Backward Reachability
Lossy Backward Reachability
Lossy Backward Reachability

- Waiting
- Visited
Lossy Backward Reachability

waiting

visited
Lossy Backward Reachability

waiting

c?1 t7
d?1 t3 t1 t2 d?0
d?0 t5 t0 c?0 c?1

c?0 t8 t6 c?0 c?1
c?1 t6 t8 d?1

t1 t2 t3 t4 t5 t6 t7 t8
c0 c1

visited
Lossy Backward Reachability
Lossy Backward Reachability
Lossy Backward Reachability

visited

waiting
Lossy Backward Reachability

waiting
Lossy Backward Reachability

waiting

T : target configurations
Lossy Backward Reachability

Initial state of $P_1$

Initial state of $P_2$

Content of $c$

Content of $d$

Initial configuration

$T$: target configurations
Lossy Backward Reachability

initial state of $P_1$
initial state of $P_2$
content of $c$
content of $d$

initial configuration

$T$: target configurations

$Pre^*(T)$
Lossy Backward Reachability

waiting

initial state of $P_1$
initial state of $P_2$
content of $c$
content of $d$

initial configuration

$Pre^*(T) \cap Init = \emptyset$

$T : \text{target configurations}$

$Pre^*(T)$
Lossy Backward Reachability

Waiting

\[ \text{Pre}^*(T) \cap \text{Init} = \emptyset \]

- \( T \): target configurations
- \( \text{Pre}^*(T) \)
- \( \emptyset \)
Well Quasi-Ordering

\[ w_0, w_1, w_2, \ldots, w_i, \ldots, w_j, \ldots \]

\[ \exists i < j : w_i \sqsubseteq w_j \]
Well Quasi-Ordering

\[ w_0, w_1, w_2, \ldots, w_i, \ldots, w_j, \ldots \]

\[ \exists i < j : w_i \sqsubseteq w_j \]
Well Quasi-Ordering

Infinite sequence of words

\[ w_0, w_1, w_2, \ldots, w_i, \ldots, w_j, \ldots \]

\[ \exists i < j : w_i \sqsubseteq w_j \]

Well Quasi-Ordering

Infinite sequence of configurations

\[ c_0, c_1, c_2, \ldots, c_i, \ldots, c_j, \ldots \]

\[ \exists i < j : c_i \sqsubseteq c_j \]
Well Quasi-Ordering

w_0, w_1, w_2, \ldots, w_i, \ldots, w_j, \ldots

\exists i < j : w_i \sqsubseteq w_j

Well Quasi-Ordering

c_0, c_1, c_2, \ldots, c_i, \ldots, c_j, \ldots

\exists i < j : c_i \sqsubseteq c_j
Ordering:
  • monotonicity
  • computing predecessors
  • well quasi-ordering