Math308, Quiz 6, 03/17/14

First Name: .... **Last Name:** ....

Grade: ....

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Problem 1. 100%. Solve the following non-homogeneous differential equation:

$$y'' + 2y' + y = 2\cos t. \tag{1}$$

Solutions

Problem 1. a) First we find the general solution of corresponding homogeneous differential equation:

$$y'' + 2y' + y = 0. \tag{2}$$

The characteristic equation is

$$r^2 + 2r + 1 = 0, \tag{3}$$

which has two repeated real roots: $r_{1,2} = -1$. Therefore, one solution of the homogeneous equation is $y_1(t) = e^{-t}$ and the second solution is $y_2 = te^{-t}$. The general solution of Equation (**??**) is then written as

$$y_{\text{hom}}(t) = C_1 e^{-t} + C_2 t e^{-t}. \tag{4}$$

b) Let us now search for a particular solution of the non-homogeneous equation (1). The right hand side of (1) is a trigonometric function, thus we assume that $Y(t) = A \sin t + B \cos t$, where A and B are constants to be determined. By inserting $Y(t)$ to Equation (1) we obtain:

$$\begin{aligned}
&Y(t)'' + 2Y(t)' + Y(t) \\
&\qquad = (A \sin t + B \cos t)'' + 2(A \sin t + B \cos t)' + (A \sin t + B \cos t) \\
&\qquad = -A \sin t - B \cos t + 2A \cos t - 2B \sin t + A \sin t + B \cos t \\
&\qquad = (-A - 2B + A) \sin t + (-B + 2A + B) \cos t \\
&\qquad = -2B \sin t + 2A \cos t \\
&\qquad = 2 \cos t.
\end{aligned}$$

Where by matching the coefficient we find that $A = 1$ and $B = 0$. Therefore,

$$Y(t) = \sin t,$$

and the general solution of the non-homogeneous equation is

$$y_{\text{nonhom}}(t) = C_1 e^{-t} + C_2 t e^{-t} + \sin t. \tag{5}$$