Math308, Quiz 1, 01/23/14

First Name: .... **Last Name:** ....

Grade: ....

SHOW ALL WORK!

Problem 1. Given the differential equation:

$$y'' + 2y' - 3y = 0. \tag{1}$$

(a) (20%) Determine **1. the order**, **2. the kind** (i.e. PDE or ODE), **3. linear or nonlinearity** of (1).

(b) (30%) Varify that $y_1(t) = e^{-3t}$ is a solution of (1).

Problem 2. (50%) Solve the following equation:

$$\frac{dy}{dx} = \frac{x - e^{-x}}{y + e^{y}}.$$

Solutions

Problem 1. (a) (20%)

1. 2nd order, since the highest derivative is $\frac{d^2y}{dt^2}$;
2. ODE, since only one ordinary derivative is used;
3. linear, since all terms depending on y are linear.

(b) (30%) Insert $y(t) = e^{-3t}$ into (1) and get:

$$(e^{-3t})'' + 2(e^{-3t})' - 3e^{-3t} = -3(-3)e^{-3t} + 2(-3)e^{-3t} - 3e^{-3t} = 9e^{-3t} - 9e^{-3t} = 0.$$

Problem 2. (50%)

First of all note that $y + e^y \neq 0$. Then, by multipling the equation by $\dfrac{dx}{y + e^y}$ we obtain that

$$(y + e^y)dy = (x - e^{-x})dx.$$

This equation is separable, so by method of direct integration we get

$$\frac{y^2}{2} + e^y = \frac{x^2}{2} + e^{-x} + C,$$

where C is some arbitrary constant.