Uppsala University
Department of Information technology

Examination in Scientific computing 3, 2018-10-22

Time: 08.00 – 13.00
Allowed resources: Calculator, Beta Mathematics Handbook

Start a new solution on a new sheet of paper and write your code on every sheet.

Each problem can give up to 5 credits. To get full credit for your solution good arguments for your method of solution and detailed calculations are required. The total credit sum determines the grade.

Grade limits: grade 3 at least 12 points, grade 4 at least 18 points, grade 5 at least 24 points.

1. For linear system of equations $Ax = b$ with

$$A = \begin{pmatrix} 10 & 0 & 1 \\ 0 & -8 & 1 \\ -1 & 0 & 6 \end{pmatrix}, \qquad b = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

   a) Do one iteration with the Gauss-Seidel method and the starting vector $x^{(0)} = [1, 1, 1]^T$.

   b) Is Gauss-Seidel convergent for the linear system above, motivate.

   c) Can we use the Conjugate-Gradient method to solve the problem above, motivate.

2. For the PDE

$$\begin{aligned} &u_t = \lambda u_{xx}, \quad \lambda > 0, \quad 0 < x < 1, \quad t > 0 \\ &u(0,t) = \alpha, \quad u(1,t) = \beta \\ &u(x,0) = f(x) \end{aligned}$$

   we want to use the following Finite Difference Method

$$\frac{u_j^{n+1} - u_j^{n-1}}{2\Delta t} = \lambda \left(\frac{u_{j+1}^n - 2u_j^n + u_{j-1}^n}{\Delta x^2} \right) \tag{1}$$

   This is an explicit multistep method that requires two time levels to compute the next level. Explain how we can start up the method and write a *Matlab-code* for implementing the method.

3. For the problem 2 above with the Finite Difference Method (1), derive the order of accuracy.

4. For the problem 2 above with the Finite Difference Method (1), use the Fourier method (von Neumann approach) to analyze conditions for stability. What are your conclusions about the method?

5. Formulate the FEM for the boundary value problem, with both Dirichlet and Neumann boundary conditions,

$$\begin{aligned} &u'' = f(x), \quad 0 \leq x \leq 1 \\ &u(0) = \alpha \\ &u'(1) = \beta \end{aligned}$$

using piecewise linear basis functions. Derive the resulting linear system of equations. You may assume equal step length $x_{i+1} - x_i = h$ between all nodes.

6. Consider the following PDE

$$\begin{aligned} &u_t + xu_x = 0, \quad 0 \leq x \leq 5, \quad t > 0 \\ &u(x, 0) = f(x) \end{aligned}$$

a) What boundary conditions should we set up for this problem?

b) Suggest a convergent explicit Finite Difference method for the problem.

c) Set up necessary conditions for stability.

Good Luck!