Heterogeneous Multiscale Models for the Landau-Lifshitz equation with highly oscillatory coefficient

Lena Leitenmaier and Olof Runborg

KTH Royal Institute of Technology, Dept. of Mathematics, Lindstedtsvägen 25, Stockholm, Sweden

e-mail: `lenalei@kth.se`

We use a simplified version of the Landau-Lifshitz equation to study a composite ferromagnetic object that consists of magnetic materials with different interaction behavior. In the model this is represented by a rapidly varying material coefficient a^ϵ, $\varepsilon \ll 1$. More precisely, we study

$$\partial_t \mathbf{m} = -\mathbf{m} \times \mathbf{H} - \alpha \mathbf{m} \times \mathbf{m} \times \mathbf{H}\,, \tag{1}$$

where

$$\mathbf{H} = \boldsymbol{\nabla} \cdot (a^\varepsilon \boldsymbol{\nabla} \mathbf{m})\,.$$

Direct numerical simulation of the problem is expensive as the small ε-scale must be resolved; however, the small scale cannot be ignored either, since it has a significant influence on the magnetization behavior on the coarse scale. We aim to design an efficient numerical method for this case using the framework of heterogeneous multiscale methods (HMM). Here the idea is to combine a coarse macro scale model with a micro problem which is solved exactly but only on a small domain in time and space. The thus obtained micro solution is then averaged to obtain the information that is needed to update the macro model.

In order to understand which behavior to expect from the solution and to obtain a good reference solution, we derive a homogenized equation where $\mathbf{H}$ in (1) is replaced by $\bar{\mathbf{H}} = \boldsymbol{\nabla} \cdot (A \boldsymbol{\nabla} \mathbf{m})$ with a constant matrix A. We also derive equations for higher order correction terms to the homogenized solution $\mathbf{m_0}$. Investigating the correction terms, we find that there is a rapidly oscillating contribution that dominates the short-term behavior but decays exponentially with time t/ε^2. This is especially important in the context of HMM where we average over short time intervals in order to approximate the effective long time behaviour.

It is then possible to obtain estimates for both the difference $\mathbf{m_0} - \mathbf{m}^\varepsilon$ and the error introduced in the averaging process in terms of ε. We use these results to analyze an HMM method and also show numerical results that confirm the accuracy of the method.

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