PRELIMINARY VERSION !

COPERNICUS Project: CP 94-0820

High Performance Computing in Geosciences

Safety of constructions with respect to rock deformations and movements

HIPERGEOS

Final report

June 8, 1998

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Introduction

The Copernicus project CP 94 0820, *High Performance Computing in Geosciences; Safety of Constructions with respect to Rock Deformations and Movements (HIPERGEOS)* belongs to the EC program of cooperation with Central European states and Independent States of the former Soviet Union (COPERNICUS). It falls in the framework of the theme of High Performance Computing and Networking (HPCN) which aims in exploitation of the opportunities and expansion of the application potential of HPCN towards industry.

The project was carried out with the joint efforts of seven partner groups from Belgium, Bulgaria, The Czech Republic and The Netherlands, consisting of industrial partners dealing with projects concerned with safety of constructions in mining and civil engineering, experts in constructing proper mathematical models of the problems posed by the industrial sites, specialists in developing efficient numerical solution methods to solve the mathematical models numerically, and specialists in implementing the numerical methods on serial as well as on parallel computer platforms. Having in this way completed the path between posing the real-life problem and applying the software to solve it, we present the achieved results obtained after three and half years of extensive collaborative research and contacts, with the expectation that the project will in this way contribute to the above formulated major aim in HPCN. As additional benefit from the project, we underline the new close collaborative contacts established between the partners from the EC and CE countries, the stabilization of the academic potential in the CE states and the successful solution of engineering problems which are of direct interest for the industrial partners.

Chapter 1

General overview of the project

1.1 Project objectives

The major objectives of this project are as follows:

(1) to formulate and improve presently used mathematical models for certain problems in Geosciences;

(2) to develop efficient numerical solution methods which are robust with respect to the problem and method parameters, and which can also perform efficiently on parallel computers;

(3) to implement the solution methods and create software packages for these, and test them on benchmark problems, close to the real-life problems solved by the industrial partners.

Two classes of problems in Geosciences, namely, target practical problems originating from mine exploration and foundation engineering have been considered. The problems are formulated in mathematical form as two- and three-dimensional partial differential equations as used in elasticity and plasticity.

What concerns the models, the major aim is to consider sufficiently accurate 3D mathematical models. This brings requirements for use of various material models (elasticity, plasticity, rheology) as well as for modelling of contacts of bodies and/or thermo-mechanical influences in some cases. The accuracy and reliability may require also to perform more accurate local analysis (submodelling) or to consider a number of variants for determining the influence of some uncertain input parameters.

Once the mathematical models have been defined, the second major task becomes the development of efficient numerical solution methods. Emphasis is given both to the finite element (discretization) methodology and solution techniques of the arising linear and nonlinear algebraic systems. The discretization strategies include adaptive refinement, based of patched (overlaid) meshes, where the solution gradients are large. This captures the solution peculiarities more efficiently and accurately than if a uniformly sized mesh is used.

The numerical solution requires handling of linear and nonlinear algebraic systems of equations. Nonlinear equations solvers are studied in detail, namely, certain nonlinear equation solvers based on inexact Newton methods for nonlinear elliptic and plastic flow problems are analysed and implemented. The nonlinearity studied is due to material nonlinearity and can involve non-differentiable functions. A rate of convergence of the outer (nonlinear) iterations independent of meshsize and certain problem parameters is striven for. The treatment on nonlinear problems include two-grid methods and local refinement as well.

The solution of linear systems is an inseparable ingredient of the numerical solution process both because the linear systems describe completely some model problems of interest and also they have to be solved as a part of the nonlinear solution procedures. A very important issue, however, is the size of the systems, which easily reaches ranges of several hundreds of thousands to several millions of degrees of freedom for real-life problems, to which the project is oriented. For instance, the ability to make more reliable prediction of safety of constructions sensitive to rock deformations, requires much more accurate modelling, and, hence, more degrees of freedom. Therefore, it is necessary to solve these systems by iteration using methods which are highly efficient on certain parallel computer architectures, having a fast rate of convergence (independent of meshsize and certain problem parameters) and at the same time, requiring little communication overhead. The aim is to use the parallel computer facility up to its full capacity achieving high Megaflop rates *and* high parallel efficiency which will result in total execution times at least 100 fold smaller than with presently used methods and computers for the type of problems considered. This turns out to be possible by combining accurate modelling and fast solution times in order to permit performing many simulation runs to support the decision taking process.

The computer implementation of the above numerical methods is based on some available codes for three-dimensional problems of elasticity and plasticity which are further enriched with possibilities to work with various finite element methods (including non-conforming ones) and various linear and nonlinear solution methods. The code is structured and open for future upgrading. It includes the new efficient solution techniques related to the local refinement and nonlinearity. Also, some linear equation solutions methods are implemented on various parallel computer platforms, namely, a parallel cluster of workstations, symmetric shared memory multiprocessors and a massively parallel distributed memory computer. These cover the commonly accessible computer architectures which can be expected to be available when solving the considered type of problems in the future.

A two-dimensional more research oriented code using mixed finite element methods for some model elasticity and plasticity problems has also been developed.

The developed software is tested on real sized problems and the obtained results have been subject to analysis from experts in the corresponding application field and also compared with in situ observations.

1.2 Applications

As already mentioned, an integral part of the project objectives consists in application of the developed numerical methods and high performance software to problems in geo-engineering and geo-sciences. These applications, which on the other hand have stimulated the research in high performance computing, originate from the following three areas.

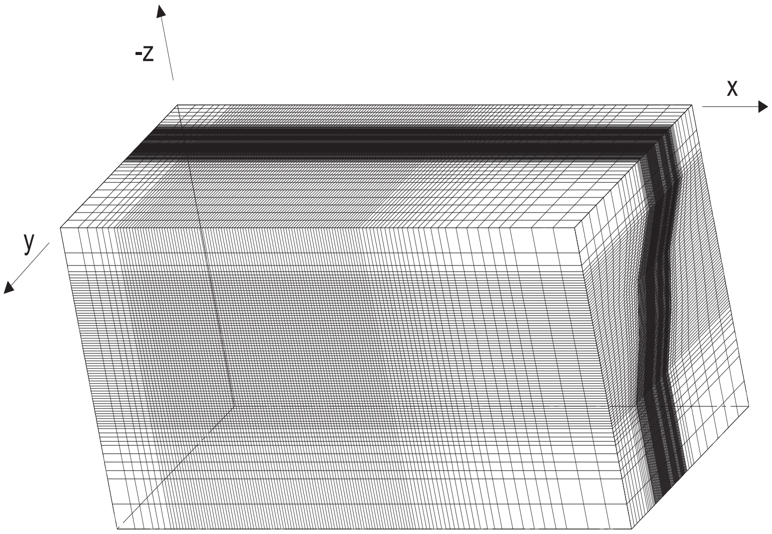


Figure 1.1: The target domain for Rožna mine

1.2.1 Mathematical models in mining

These applications represent the use of mathematical models for assessment of the stability and safety of mine openings. Such assessment is based on computation of the stress field in the investigated domains which often requires the solution of very large 3D elasticity problems with necessity of local refinement (submodelling).

More specifically, our effort was focused on the solution of a modelling problem formulated by our industrial partner GEAM D. Rožınka which operates its mining activities in the uranium ore deposit Rožna in the Czech Republic. At present, GEAM is forced to prepare new exploitation plan for the period of the years 1998 - 2002 which must reflect the nowadays economical conditions and must in the same time ensure stability of the mine openings, safety of the work and environmental protection. Careful assessment of all these conditions is required from the Czech Mining Authority.

Because this new exploitation plan deals with mining in greater depths (more than 1000 m below the surface) with complicated geological and geomechanical conditions, it was necessary to find suitable tools to model geomechanical aspects of the further advance of mining and to answering two basic questions: what is the risk of rock bursts in the deposit (such risk requires expensive measures to be taken) and should the former mining technology, which was the *overhand stoping*, be replaced by a new mining technology, the *underhand stoping*. Figure 1.1 illustrates the type of the exploited 3D grids which have to follow three ore veins in the considered domain.

1.2.2 Modelling in foundation engineering

The design of proper foundation systems for bridges and other big constructions requires also large scale numerical simulations to clear up the properties of the designed foundation in its interaction with the soil medium. This simulation leads to the solution of large scale 3D elasticity or plasticity problems depending on the properties of the surrounding media (clay soil in the considered case). While for the models in mining one of the most difficult factors

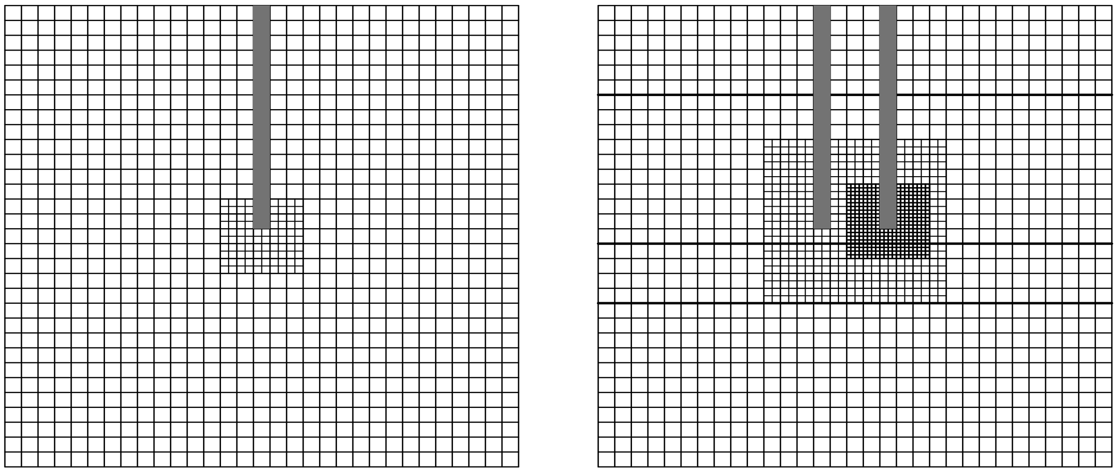


Figure 1.2: 3-D simulation of pile systems with patched local refinement

is the complicated geometry of the target domain, in this case the major difficulties are due to the strong coefficient jumps between the different soil layers, and especially between the foundation and the soil, where the jump is of order several thousands.

The main effort was to develop an appropriate software and to apply it for solving realistic model problems provided by the second industrial partner MOST XXI from Sofia which is involved in civil engineering constructions. The model problems concerns multiple pile foundation system, e.g. a part of the foundation system of the "Asparukhov Most" in Varna, one of the biggest bridges in Bulgaria. The soil media is inhomogeneous and its behaviour is in general nonlinear, described by tables of experimentally determined stress - strain data. The model assumes that the construction technology guarantees ideal contact between the piles and the surrounding soil layers. Figure 1.2 is a 2D cross-section of the 3D benchmark problem and also illustrates the subregions where local refinement is required.

1.2.3 Complex models with contact, rheology and thermal field influence

The above described models exploit the solution of large scale 3D elasticity and plasticity problems in their classical formulation. However, when modelling many other engineering problems, also other important phenomena have to be taken into consideration. For instance, one can consider the contact conditions between geological bodies without or with friction present in the contact. Another phenomena is the rheology which is a property of many geomaterials under specific conditions. Such complicated models are also investigated in the project. Among the rheology models, we consider the Bingham visco-plastic rheology model. The last phenomena studied in the project in relation with the elasticity and plasticity, is the influence of the thermal fields to structural mechanics models which results in thermo-elasticity and thermo-plasticity.

The latter complex models were applied to problems of collision and subduction of lithospheric plates in several regions: the Bohemian massif, the Himalayas and the Central Aleutian arc. These applications brought into consideration also another phenomena, namely, free boundaries between the solid and melted rocks (the so-called Stephan problem).

These complex models are directly applicable to some engineering industrial problems of the nearest future, for instance, to the assessment of designs of the radioactive waste repository which should be constructed in the Czech Republic. These models can be used for mathematical simulations of
- geodynamical processes in the upper parts of the Earth,
- mutable slopes and conditions in rock massive with deep fault,
- stress - strain measurements in deep boreholes and post-processing of the measured data.

1.3 Theoretical results

Successful large scale mathematical modelling requires the use of new achievements in Numerical Analysis. Within the HIPERGEOS project, a lot of work has been done also in this respect, namely, development, analysis and use of highly efficient numerical procedures. We mention here some of these results, the other are described in more detail in the connection with applications in a later Section.

- *Linear solution methods*
  As already stated, the successful solution of large scale numerical models depends to a great extent on the availability of suitable *high performance linear solvers*. In this respect, as a main result obtained within the project we point out the development and the analysis of various types of preconditioning techniques, used in the framework of iterative solution procedures, such as the standard conjugate gradient (CG) method or some variants of the generalized conjugate gradient (GCG) method. Namely, the new results concern various types of block incomplete (ILU) factorizations, block-size reduction (BSR) method, block-circulant (CBF), fast adaptive composite grid (FAC) and BEPS[1] preconditioners, algebraic multilevel iteration (AMLI) methods of additive and multiplicative type and domain decomposition (DD) techniques. This broad class of preconditioning techniques ranges from methods which exploit fully the structure of the matrices arising from the finite element method (FEM) discretization models and, thus, are very fast but tightly connected to the problem type and the discretization technique (such as the block-ILU preconditioners), via methods which are not optimal with respect to computational work but converge in a very general setting of the FEM problem (such as BSR) or others which possess high degree of parallelism (such as the block-circulant preconditioner), to general algebraically constructed methods (such as AMLI) which are of optimal order of computational complexity and are also parallelizable. A comparison of the performance of these methods was done, based on the solution times needed to solve some of the benchmark problems.

- *Parallelization of the linear solution methods*
  Special attention was paid to *parallelization* of the linear iterative solution methods. Here, various methods have been implemented for parallel computer environment (DD, CBF, AMLI) utilizing various types of parallelism - coarse-grained, suited for computers systems with few (two to four) processors and fine-grained, utilizing the capabilities of computer systems with many processors (up to 64 tested). The major result is that the developed parallel software shows very good parallel efficiency and scalability, and is fully portable, based on the Message Passing Interface (`MPI`) standard.

[1]BEPS stands for Bramble, Ewing, Pasciak and Schats, the authors of the article introducing the method.

- *Local refinement* Real-life industrial problems require very often some local grid refinements and/or submodelling in order to capture some details which are not "visible" on the global grid. Note, that due to the large target domains (sometimes of several tens of kilometers) the "fine" discretization grid may turn out to be too coarse to represent some effects of particular interest, which take place within a meter, for instance. For these cases a lot of results have been obtained using iterative procedures on local problems defined via *additional local grids* (patches). These results enable the analysis of the stress fields in localized critical zones where the interaction processes have to be followed on a micro-scale, compared to the whole problem.

- *Nonconforming FE*
  Our investigations in development of efficient solution methods for nonconforming FEM systems (Crouzeix-Raviart linear FE, rotated bilinear FE, and rotated trilinear FE) are motivated by their application to construction of *locking-free* approximation of parameter dependent problems. New optimal iterative solution algorithms for the arising linear algebraic systems were proposed and analysed. This study is more theoretically oriented following the framework of the present project. The use of nonconforming FE is considered as an alternative to the commonly used conforming FE as well as to the more recently developed robust algorithms based on mixed FEM approximations.

- *Nonlinear solution methods*
  Special attention was devoted to the solution of nonlinear problems. Strong theoretical results were obtained about convergence of inexact Newton type methods for problems of nonlinear elasticity and plasticity and two-level and multi-level nested Newton methods also with an use of the first order least squares FE formulation. Several methods including conjugate gradients with projection and duality based methods have been studied for the solution of problems with contact of the bodies.

- *Modelling of contact problems*
  Significant progress was obtained also in numerical methods for solving Bingham visco-plasticity, thermo-elasticity and Stephan problems. These are contributions to theoretical geodynamics, technology and geomechanics (treatment of problems of geophysics), with the application to different parts of the Earth. During the project, a monography, entitled *Numerical Modelling in Geodynamics with Applications* was written by prof. J. Nedoma (ICSAS) and published by John Wiley and Sons, Ltd, where several chapters contain the results and experience gained during the HIPERGEOS project.

1.4 Practical results and applications

1.4.1 Benchmark problems

For the development of high performance solvers for the solution of large scale problems arising in geo-sciences and geo-engineering, it was essential to specify appropriate benchmark problems and to have software which will enable the work with the benchmarks for all research partners.

Based on the requirements for mathematical modelling from both industrial partners, such benchmarks were formulated by IGAS Ostrava and CICT Sofia groups. These benchmarks were developed with increasing complexity to involve gradually all important aspects of the real-life engineering problems, namely 3D modelling and high resolution of the models resulting in large number of degrees of freedom of the finite element (FE) discretizations, local refinement to capture particular details in the models, nonlinear behaviour of some materials (soil), and elasto-plastic transition phases.

Another set of benchmarks was developed by ICSAS Prague group. These benchmarks, mostly 2D, enable testing of the above mentioned complex models with contact of bodies, rheology, free boundaries and thermo-mechanical influence.

1.4.2 Analysis of the results and benefits for the industrial partners

Mining technology

The modelling performed for the assessment of the further advance of mining at the Rožna deposit with the HIPERGEOS software includes the solution of several boundary value problems corresponding to various stages of mining and two limit cases of the pre-mining stress field measured in the investigated domain. The solution of each boundary value problem means dealing with a discrete FE model with nearly 4 million degrees of freedom. Due to the complex pre-mining stress, also some additional analyses had to be performed, which lead to the solution of large singular systems. To compare the mining technologies of underhand and overhand stoping, the submodelling with iterative local refinement techniques was done.

The results of modelling were compared with the existing experience in GEAM. Several numerical approaches were tested to reduce the computational demands and to fulfil all aims of the modelling. The performed modelling then resulted in several practical recommendations. Two of them, directly corresponding to the described aims of the modelling are the following.

- A remarkable difference between the considered mining technologies was confirmed. Overhand stoping method should not be recommended for greater depths due to extremely high stress concentrations in ore pillar above the advancing front. On the other hand, the underhand stoping method can be recommended for continuous mining through next 3 - 4 levels.

- Mining in depths greater than 1000m below the surface induces changes of the stress field in far field zone which are comparatively small which indicates no reasons for increased rock burst hazard.

The modelling enables also quantification the influence of other factors, e.g. the pre-mining stress existing in the deposit. Figures 1.3 and 1.4 illustrate the computed stress fields for the case of anisotropic pre-mining stress.

Foundation engineering

An engineering expertise of the numerical results obtained from the solution of selected pile foundation problem from bridge engineering has been given by the industrial partner from MOST XXI. Let us note once again that the considered benchmarks correspond to a real upper bridge structure, that transfers the external loads to the foundation bed soil, through a pile system. The next concluding remarks summarize in a brief form the results of the engineering analysis.

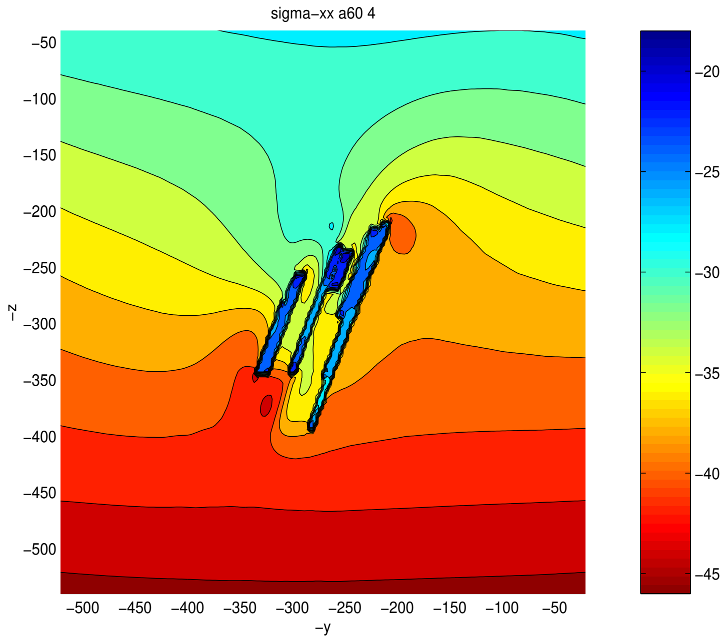


Figure 1.3: Horizontal stress σ_{xx} in the cross section I=60 shows how the initial stress field is influenced by the mining.

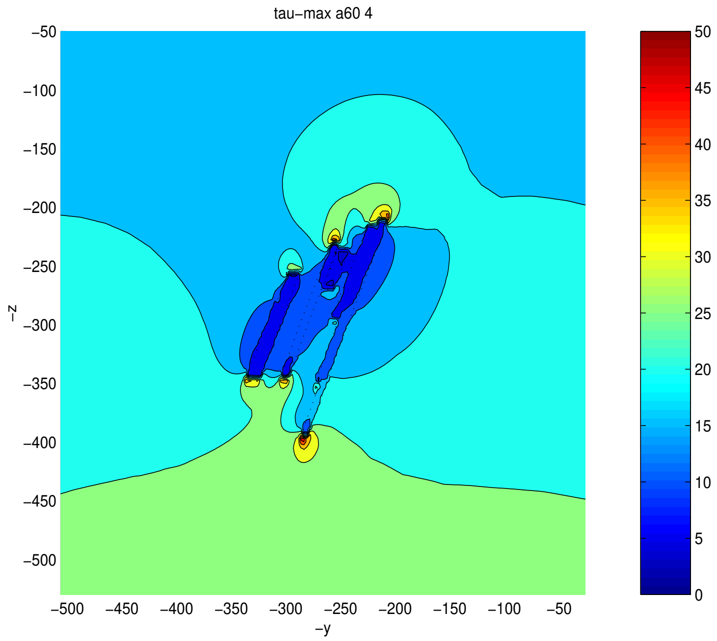


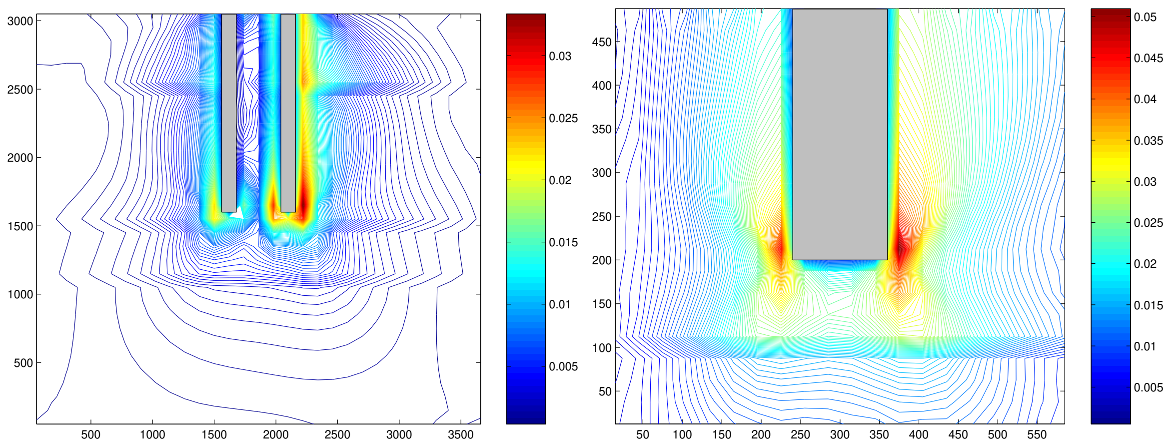
Figure 1.4: The maximal shear stress in the vicinity of the mine openings (in the cross section I=60) shows the zones where a failure of rocks may be expected.

- The developed FEM code produces numerical solution of the problem taking into account all important phenomena (especially nonlinear interactions in multi-layer soil media) as described in the work packages of the project.

- One of the most important advantages of the 3D FEM solution is that the end-user obtains complete information about the spatial distribution of the parameters of the studied engineering problem.

- From engineering point of view, the developed local refinement FEM code provides new powerful tools for detailed analysis of the zones of active interaction between the foundation elements and the surrounding soil media, which is not possible to be performed by other conventional means.

Figures 1.5(a) and 1.5(b) present results of the patched local refinement FEM code for nonlinear multi-layer interaction (MOST XXI - Benchmark 2). Figure 1.5(a) corresponds to the numerical solution on the initial regular FEM mesh ($n_x = n_y = n_z = 32$), while Figure 1.5(b) presents results in the patch of the second level of local refinement. It is important to note the achieved high level of numerical resolution in the zone of most active interactions under the top of the right pile. The nonlinear behavior of the solutions is also well expressed. The different levels of strain intensity correspond to different values of the deformation module $E_{soil} = E_{soil}(u)$. The influence of the multi-layer structure of the soil media is clearly visible.



(a) (b)

Figure 1.5: 3-D simulation of a double pile system in a nonlinear multi-layer media. Isolines of the strain intensity ϵ_0. MOST XXI-Benchmark 2. Cross section $y = 18.6\ m$.

Some of the performed numerical tests related to the MOST XXI Benchmarks have been presented to Dipl.-ing. Thomas Freund (a similar size as MOST XXI consulting office in Wien, Austria) to get an additional, qualified, and independent engineering evaluation about the possibilities for practical application of the developed during the HIPERGEOS project new methods, algorithms and codes. The responding letter (the original is signed by the chief of the office) expresses the opinion for abilities of the new developed codes to serve in the future as a tool for more detailed examination of bridge pile foundations. We find the next citation from this letter as especially important from potential future applications point of view: *"I believe that many chances exist for this technology process in particular with regard to difficult engineering solutions. I feel that, because the exact calculations, solutions can be found, that were considered impossible until now."*

1.4.3 Achievements in high performance computations

Here we address two issues.

The first one is the gain in total computing time for the serial implementations, compared, for instance, to the performance of existing commercially available finite element packages. As a rule, most commercial FE packages, available on the market nowadays, aim at being universal and able to solve any problem, discretized by some finite element method. This restricts the set of solution methods available in these packages to either *direct solvers*, based on some variant of Gauss elimination, where huge memory requirements occur as soon as a large problem has to be solved, or *iterative solvers* which can use very simple, such as *element-by-element preconditioning*, which exhibit in general very slow convergence rates. The requirement for absolute applicability sacrifices, therefore, the efficiency and makes such packages practically unusable for large 3D real-life problems.

The philosophy adopted within this project is to consider a class of problems and to develop solution methods for this particular class, which is still broad enough but makes possible to use advanced numerical techniques and ensure high efficiency of the solution methods. Here, high efficiency includes robustness with respect to both problem parameters (Poisson's ratio and Young's modulus for the elasticity problems) and discretization parameters (mesh-sizes), optimal (or almost optimal) computational complexity, measured as number of arithmetic operations per degree of freedom, and optimal (or almost optimal) rate of convergence, independent of the problem and discretization parameters. During the project, a series of solution techniques has been studied and implemented and a comparison of their performance on the benchmark problems is available in one of the project deliverables.

The second issue is the gain in total computing time for the parallel implementations, compared to the serial implementations of the solution methods developed during the project.

Here, two types of parallel implementations have been enhanced: a coarse-grained parallelism utilizing few workstations, and a medium- to fine-grained parallelism, suited also for massively parallel computer platforms.

The coarse-grained parallelism is adopted in the parallel implementation of the mining exploration models. The serial tests to model the mining activities for GEAM D. Rožınka lead to huge computational demands, in the beginning it was more than 90 hours (140 hours with the submodelling) when the codes were run on the workstation IBM RS/6000 3CT with 133 MFlops performance in the LINPACK test. After several improvements of the solution methods and program implementation, this time was reduced to a little more than 6 hours, i.e., a speedup of the serial code of about 14 was reached. This speed up was obtained by using of the Dirichlet modelling sequence (the most of the solved boundary value problems use pure Dirichlet boundary conditions) and by parallelizing the computations for a cluster of three workstations. For a fixed preconditioner based on displacement decomposition and incomplete factorization this parallelizing gives speed up of about 2.2. But the use of variable preconditioner (inner - outer iterations) based on the same displacement decomposition and incomplete factorization, which is in the sequential version as efficient as the fixed preconditioner, brings here very high superlinear speed up of about 10. For this type of algorithm, the communication and synchronization overhead is minimal, and most of the time on the individual workstations is spent without interruption on solving substantially smaller subproblems. We remark here, that for many industrial or research institutions it is relatively easy to buy or use similar computer configurations. More tests performed on benchmarks at KUN Nijmegen and ULB Brussels showed that a further speedup can be reached when using more efficient iterative methods (e.g. block ILU or AMLI preconditioners) and somewhat faster computer (SUN Ultra Sparc). The total speed up is then estimated to be in the range from 100 to 200.

For fixed preconditioner based on displacement decomposition and incomplete factorization this parallelizing gives speed up of about 2.2. But the use of variable preconditioner (inner - outer iterations) based on the same displacement decomposition and incomplete factorization, which is in the sequential version as efficient as the fixed preconditioner, brings here an incredible speed up of about 10. For this type of algorithm, the communication and synchronization overhead is minimal, and most of the time on the individual workstations is spent without interruption on solving substantially smaller subproblem. We remark here, that for many industrial or research institutions if it relatively easy to buy similar computer configurations. More tests performed on benchmarks at KUN Nijmegen and ULB Brussels showed that a further speedup 100 - 200 can be reached when using more efficient iterative methods (e.g. block ILU or AMLI preconditioners) and somewhat faster computer (SUN Ultra Sparc).

To the class of coarse-grained parallel implementations we attach also the DD-based 3D linear elasticity solver, which utilizes as black-box subdomain solvers some of the robust serial methods (BSR, for instance).

The implementations of CBF and AMLI methods are illustrations of medium- to fine-grained parallelism. They both show very good parallel efficiency and scalability. Due to the portability, the I-AMLI code was successfully executed both on SUN Enterprise 3000 with 8 processors and a Cray T3E massively parallel computer with 64 nodes, confirming the theoretically derived computational complexity estimates and scalability on both configurations.

1.5 Fields of applicability of the results, potential users and commercialization of the results

The successful applications performed within the HIPERGEOS project demonstrate applicability of the developed methods and software to many areas in geo-engineering and geosciences for problems ranging from tens of meters to hundreds of kilometers. The application areas involve:

- assessment of stability and safety of mine openings;
- numerical investigation of the influence of a new construction or new mining on the existing surrounding buildings and structures;
- numerical simulation of construction damages; re-estimation of the load capacity of existing bridges and other civil engineering structures;
- step-by-step simulation of complicated foundation systems taking into account the technology build-up sequence; development of new foundation technologies;

- assessment of constructions on unstable slopes;
- problems with collision and subduction of lithospheric plates.

There is a strong mutual interest in continuing the collaboration with our industrial partners, which we assume will go on. Also it will be striven to find common interests with other industrial subjects. We present some more possibilities for future applications of the HIPERGEOS project results.

(i) IGAS Ostrava was asked from GEAM D. Rožınka for an additional assistance in preparation of project materials for approving the new exploitation plan for the deposit D. Rožınka by the Czech Mining Authority. GEAM ordered also a continuation of the modelling process concerning the assessment of a modified exploitation plan which reflects some new economical circumstances. GEAM would also like to continue with the modelling in the phase of preparation of the next exploitation plan.

(ii) In case it is decided to construct a radioactive waste repository for the Czech Power Plants (ČEZ) in the locality Skalka near D. Rožınka, we are ready to take part in mathematical modelling of this locality in collaboration with GEAM D. Rožınka.

(iii) We are looking for another mining companies interested in similar mathematical modelling in Central or West Europe.

(iv) We suppose also a further application of the HIPERGEOS results in linear and nonlinear soil mechanics (foundation engineering) in collaboration with MOST XXI or other similar industrial subject in Central or West Europe.

(v) Many perspectives are opened in regard of applications of the complex thermo-elasto-plastic and contact models. Cooperation between applied scientific institutes and industrial companies are expected in connection with the preparation and future construction of a high level radioactive waste repository, during the first phase at least in countries of Central Europe and India, as well as in connection with construction of water dams in the Himalayas (India). The so-developed models are with a flexible structure and offer the possibility to be used in several domains of application, such as high level radioactive waste repositories, tunnels, deep mines, non-stable slopes, bridges on non-stable slopes, open pit mines, highways. At present the developed methods are tested on geodynamical problems of plate tectonics in several regions of the world - the Himalayas, the Andaman-Nicobar Island Arc System, the Central Aleutians, the Carpathians in Middle Europe, and data for the Andes and Auflan region are prepared for investigations. Moreover, the work in the Himalaya region as well as on the Andaman - Nicobar Island Arc System and the plate tectonics of the Middle Europe (Carpathians and Alps) will continue to serve the forthcoming construction of high level radioactive waste repositories by year 2015. Preparations to handle data for some big water dams in Lesser Himalayas in India are on the way, in cooperation with the Central Building Research Institute, Roorkee and similarly for the open mines in cooperation with the Central Mining Research Institute, Dhanbad. It is expected that the contact problem approach will be also applied for bridges on unstable slopes in cooperation with MOST and CICT Sofia.

Appendix

List of workshops

- Blagoevgrad, Bulgaria: June 15–16, 1995.
- Nijmegen, The Netherlands: June 16–18, 1996
- Rožnov, Czech Republic: September 7–9, 1996
- Bruxelles, Belgium: May 30-31,1997
- Prague, Czech Republic: January 15-17, 1998
- Prague, Czech Republic: July 6-7, 1998