Element-by-element FEM techniques

One example: sparse approximate inverse

Recall: $A = \sum_{i=1}^{ne} R_i A_i R_i^T$

small size guys

Idea:

$$\tilde{A}^{-1} = \sum_{i=1}^{ne} R_i (A_i)^{-1} R_i^T$$

Regarding Schur complem.

Assume,

(a) $A_i = \begin{bmatrix} A_{11}^i & A_{12}^i \\ A_{21}^i & A_{22}^i \end{bmatrix}$

(b) A_{11}^i is invertible

Then $S_i = A^i_{22} - A^i_{21} (A^i_{11})^{-1} A^i_{12}$

$$\tilde{S} = \sum_{i=1}^{ns} \bar{R}_i S_i \bar{R}_i^T$$

sparse and cheap to construct

Q: is $\hat{S}$ a good approx. of S – (global Schur)

Yes in many cases.

Cases: Given 2 meshes which are embedded

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{array}{l} \} \text{ fine} \setminus \text{coarse} \\ \} \text{ coarse pt.} \end{array}$$

Spd matrices.

$$\alpha S \leq \tilde{S} \leq 1 . S \quad (*)$$

$0 < \alpha \leq 1$

This idea works for

* $-\Delta u = f$

  $$\frac{\partial}{\partial x}\left(a \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(a \frac{\partial u}{\partial y}\right)$$

  a - has jumps

* elasticity in Stokes form
* piezo-electricity
* Stokes
* convection-diffusion (nonsymmetric)

! used the so-called Generalized Locally Toeplitz theory

* visco-elasticity
* inner solver (AMG) started getting problems

Estimating the eigenvalues of FEM matrices

via the element matrices

Paper: A. Wathen (1987)?

$$a \le \lambda(A_i) \le b$$

$$\tilde{a} \le \lambda(\Sigma A_i) \le \tilde{b}$$

$$\tilde{b} \downarrow d\,b$$

d - related to the connectivity of the FE mesh

Linear FEM

$$c_1 h^2 \le \lambda(A) \le c_2 h^2$$

$$\varkappa(M) = O(1)$$

Same technique to estimate eigenvalues for FEM matrices

$$\tilde{c}_1 h^2 \le \lambda(A) \le \tilde{c}_2$$

stiffness

Rayleigh quotient:

$$\frac{\tilde{c}_1}{c_2} \le \lambda(M^{-1}A) \le \frac{\tilde{c}_2}{c_1} h^{-2}$$