Division of Scientific Computing
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NGSSC: Numerical Methods in Scientific Computing
January 25, 2012

Assignment: Partial Differential Equations

Requirements: At the end of the lab session, please deliver a Matlab-generated report on the runs you performed. Give your comments and observations (included in the generated report or added by hand). Print the report on `pr1515`.

Where to run Matlab: After logging in on some of the Unix hosts, please issue the command `linuxlogin`. For some more details, see `http://www.it.uu.se/datordrift/maskinpark/linux`.

Exercise 1 (Parabolic equation)

1. Use the forward Euler scheme to solve the one-dimensional heat problem

$$\begin{array}{ll} u_t = \lambda u_{xx}, & 0 < x < 1, t > 0, \\ u(x,0) = f(x), & 0 < x < 1, \\ u(0,t) = u(1,t) = 0, & t > 0, \end{array} \tag{1}$$

   where

$$\begin{array}{l} f(x) = sin\pi x + 0.3 sin 2\pi x - 0.1 sin 3\pi x \\ \lambda = 0.1, h = 0.1, k = 0.05 \end{array}$$

   Compute an plot the values U_j^n, $n = 0, 10, 20, 30, 40, 50$.

2. The same problem as in (1), but with $k = 0.1$. Comment on the results.

3. Same problem again but with $\lambda = 0.05$. Choose k as large as possible but keep $k/h^2 \leq 1/2$. Plot the results an compare them with the results from (1). How does the reduction in the value of the parameter λ affect the results? Experiment with diferent values of λ.

4. Solve the same problem with the backward Euler method, this gives a system of $N - 1$ equations to solve at each time step. The solution of such a tridiagonal system can be computed using the direct solver in `MATLAB`, '\'. Verify the unconditional stability of the scheme.

5. Solve the same problem using the Crank-Nicolson method.

<u>Exercise</u> 2 (Hyperbolic Equations)

Introduction

Solve the equation $u_t + u_x = 0$ given a simple initial condition

$$u(x,0) = \begin{cases} 1 - |x|, & |x| \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

using various standard finite difference schemes in `diff_test.m`. Compare the outputs, no method is perfect, what are the good features and what are the bad features of the numerical solution in each case?

<u>Exercise</u> 3 (Flow in a channel)

The simulation of (inviscid) flow in a channel uses the Euler equations:

$$\frac{\partial}{\partial t}U + \frac{\partial}{\partial x}F(U) = 0$$

where

$$U = \begin{pmatrix} U_1 \\ U_2 \\ U_3 \end{pmatrix} = \begin{pmatrix} u \\ \rho u \\ \rho E \end{pmatrix}, F(U) = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ (\rho E + p)u \end{pmatrix}$$

and where ρ is the density, u is the velocity, p is the pressure and E is the total energy per unit mass. The following holds

$$E = \frac{1}{2}u^2 + e, \quad p = (\gamma - 1)\rho e$$

where e is the internal energy per unit mass and γ is a material constant ($\gamma = 1.4$ for air). Assume that it is a 1-dimensional channel, 20 units long (all quantities are dimensionless). The channel is initially ($t \leq 0$) at rest, with a partition in the middle, so that

$$\begin{aligned} \rho_{left} &= 1.0 \\ p_{left} &= 1.0 \\ \rho_{right} &= 0.125 \\ p_{right} &= 0.1 \end{aligned}$$

At $t = 0$ the partition is removed. Simulate the flow for $0 \leq t \leq 2$ by solving the Euler equations.

1. Download `Euler.m` and `F.m`, then run `Euler`. Use $M = 200$ and $\Delta t = 0.02$.
2. Plot your results.
3. Next, succesively increase Δt. What happens to the solution?