



# Validation and generation of geographical data using a domain theory

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# An industrial experience

- This talk will illustrate how domain theories are used in Industrilogik's work in formal specification and verification of railway systems.
- Specification, simulation, modelling and verification (refinement proof) is supported by the GTO toolset.
- The GTO language is predicate logic with finite domains and limited temporalities (previous-moment operator). Independent SAT solvers are used for proving.

# Geographical data

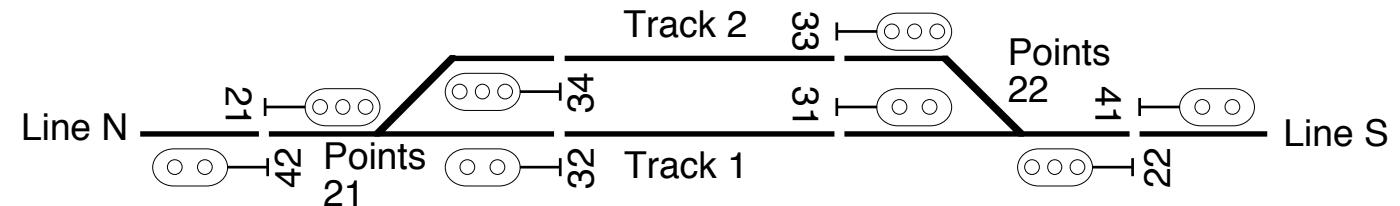
- Railway interlockings are systems governed by general rules. On an abstract level different installations perform the same functions.
- Formal req's specs can be written in a generic manner and instantiated using *geographical data*. Example: Industrilogik's specifications of Swedish and Norwegian signalling.
- Interlocking software can be written as generic modules which are combined and parameterised using geographical data. Examples: Ebilock (Bombardier), Alister (Vossloh/Banverket)



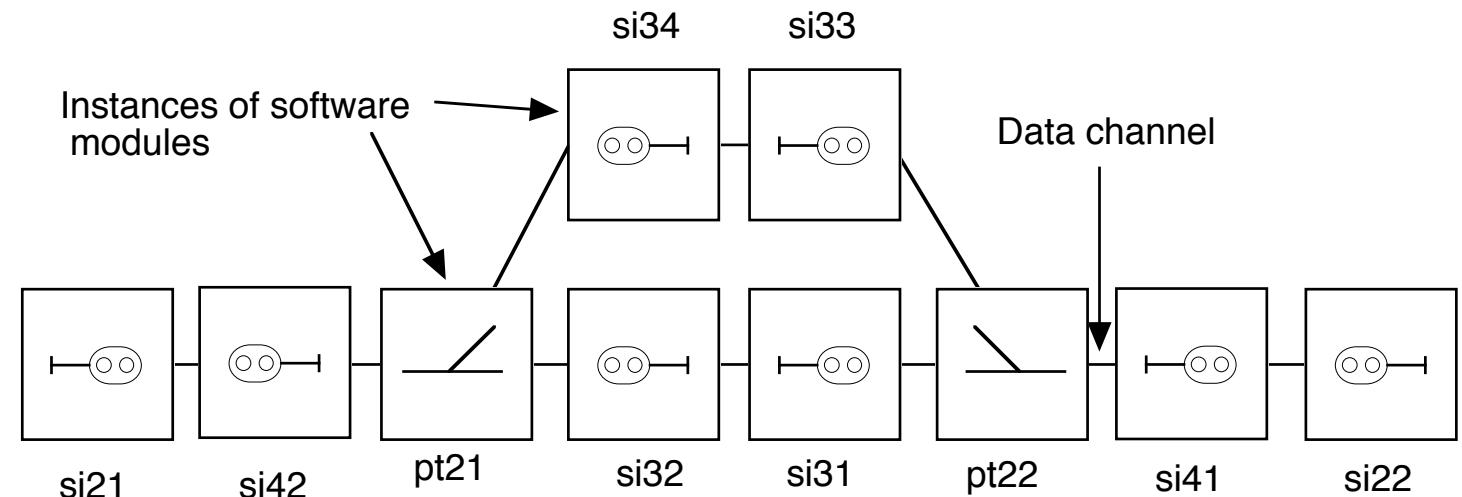
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# Ebilock software structure

## (Bombardier transportation)

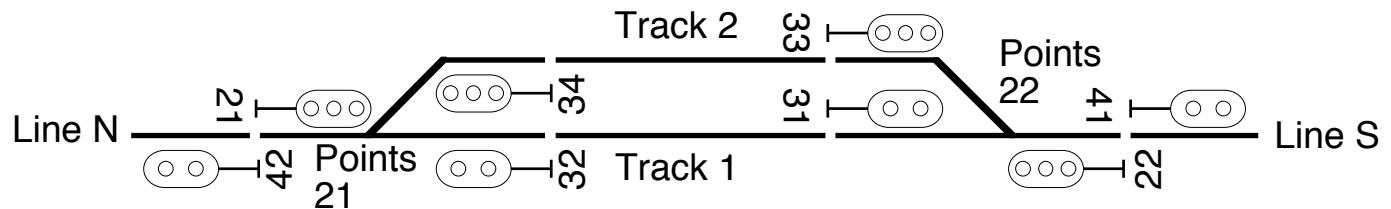


- Software modules:



# Primary geographical data

- Primary geographical data relates directly to concrete properties of the installation.



- **Units:** ln, pt21, pt22, t1, t2, pt22, ls
- **Signals:** si21, si22, si31, si32, si33, si34, si41, si42
- **Relational data:**

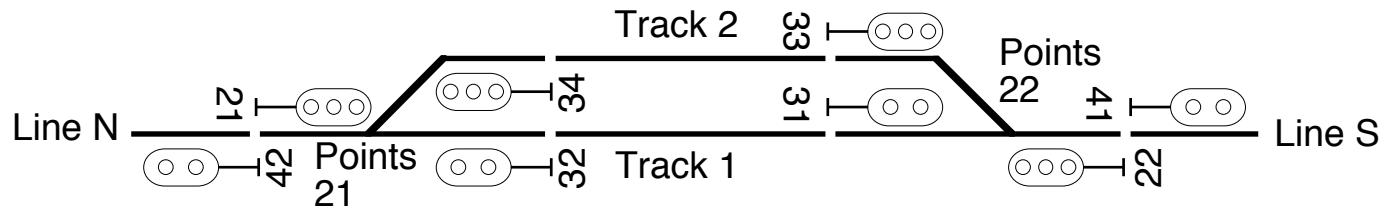
connectsTo(pt21,ln),	connectsTo(ln, pt21),
connectsTo(pt21,t1),	connectsTo(t1,pt21),
connectsTo(pt21,t2),	connectsTo(t2,pt21),
leftBranch(pt21,t2),	rightBranch(pt21,t1),...

ahead(si21,SLln),	inRear(si21,pt21),...
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# Secondary geographical data

- Secondary geographical data describe abstract properties wholly or in part determined by primary geographical data.



- Routes:** rt2131, rt2133, rt2232, rt2234, rt3141, rt3242, rt3341, rt3442
- Relational data:** conflict(rt2131,rt3242),.....  
entry(si21,rt2131), exit(si31,rt2131),  
before(rt2131,ln), first(rt2131,pt21),  
partOf(rt2131,pt21), partOf(rt2131,t1),.....

(Routes are properly sets of units, but the GTO tool is first order...)

## Formal req's specification (part)

- $\forall pt \in \text{UNIT} \ (\exists rt \in \text{ROUTE} \ (\text{routelocked}(rt) \wedge \text{partOf}(pt, rt)) \wedge \text{points}(pt) \rightarrow \text{pointslocked}(pt))$
- $\forall rt \in \text{ROUTE} \ (\text{ready}(rt) \rightarrow \neg \exists rt_1 \in \text{ROUTE} \ (\text{conflict}(rt) \wedge \text{routelocked}(rt)))$
- $\forall si \in \text{SIGNAL} \ (\text{proceed}(si) \rightarrow \exists rt \in \text{ROUTE} \ (\text{entry}(si, rt) \wedge \text{ready}(rt)))$
- $\text{routelocked}(rt)$ :  $rt$  is a locked route
- $\text{pointslocked}(pt)$ :  $pt$  are locked for reversals
- $\text{ready}(rt)$ : Trains may enter  $rt$
- $\text{proceed}(si)$ :  $si$  displays a proceed aspect

# Validation of geographical data

- Correct function of specification/interlocking depends on correctness of geographical data.
- Primary geographical data can be checked for internal consistency.
- Secondary geographical data can be verified against the primary geographical data.
- Verification uses a domain theory for railways.
- The geo. data determines a finite interpretation of the logical language so verification amounts to computing truth values.

## Domain theory (part)

- $\forall u_1, u_2 \in \text{UNITS} (\text{connectTo}(u_1, u_2) \rightarrow \text{connectsTo}(u_2, u_1))$
- $\forall u \in \text{UNITS} \neg \text{connectTo}(u, u)$
- $\forall w, u \in \text{UNITS} (\text{points}(w) \wedge \text{rightBranch}(u, w) \rightarrow \text{connectsTo}(u, w))$
- $\forall w \in \text{UNITS} (\text{points}(w) \rightarrow \exists u_1, u_2, u_3 \in \text{UNITS} (\text{connectsTo}(w, u_1) \wedge \text{connectsTo}(w, u_2) \wedge \text{connectsTo}(w, u_3) \wedge u_1 \neq u_2 \wedge u_1 \neq u_3 \wedge u_2 \neq u_3 \wedge \forall u_4 \in \text{UNITS} (\text{connectsTo}(w, u_4) \rightarrow u_1 = u_4 \vee u_2 = u_4 \vee u_3 = u_4)))$
- $\forall s \in \text{SIGNALS} \exists u \in \text{UNITS} (\text{ahead}(s, u) \wedge \forall u_1 \in \text{UNITS} (\text{ahead}(s, u_1) \rightarrow u = u_1))$

# Consistency validation

- Suppose `connectsTo(pt21, t1)` is included in the data but `connectsTo(t1, pt21)` is not.
- Sample GTO session:

```
Welcome to the GTO Formal Tool 0.5.26
> load sampleGeodata1
> listinv domain_1
ALL u1:UNIT ALL u2:UNIT
(connectsTo(u1,u2)->connectsTo(u2,u1))
> evf domain_1
FALSE
> why
Formula is FALSE because
~connectsTo(t1,pt21)
```

## Domain theory (more)

- $\forall r \in \text{ROUTES} \exists u \in \text{UNITS} (\text{before}(r, u) \wedge \forall u_1 \in \text{UNITS} (\text{before}(r, u_1) \rightarrow u = u_1))$
- $\forall r \in \text{ROUTES} \forall u \in \text{UNITS} (\text{before}(r, u) \rightarrow \neg \text{partOf}(r, u) \wedge \exists u_1 \in \text{UNITS} (\text{partOf}(r, u_1) \wedge \text{connectsTo}(u, u_1)))$
- $\text{first}(r, u) \equiv \text{partOf}(r, u) \wedge \forall u_1 \in \text{UNITS} (\text{before}(r, u_1) \rightarrow \text{connectsTo}(u, u_1))$
- $\forall r \in \text{ROUTES} \exists s \in \text{SIGNALS} (\forall u \in \text{UNITS} (\text{ahead}(s, u) \rightarrow \text{before}(r, u)) \wedge \forall u \in \text{UNITS} (\text{inRear}(s, u) \rightarrow \text{first}(r, u)))$
- $\forall r_1, r_2 \in \text{ROUTES} (\text{conflict}(r_1, r_2) \leftrightarrow r_1 \neq r_2 \wedge \exists u \in \text{UNITS} (\text{partOf}(u, r_1) \wedge \text{partOf}(u, r_2)))$



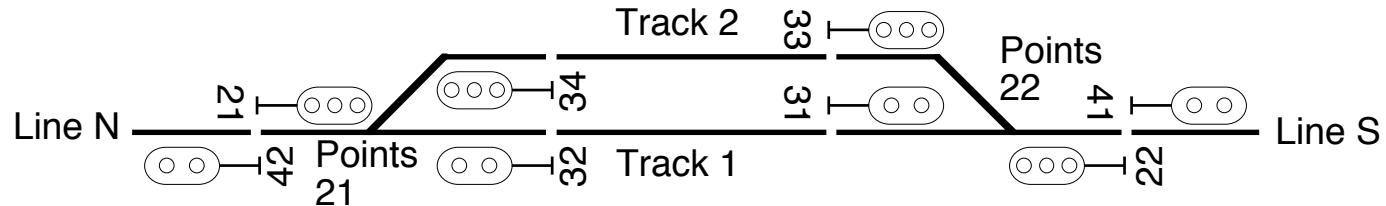
# Generating secondary geo. data

- Some axioms can be made into definitions and computed

$$\text{conflict}(r_1, r_2) \equiv \\ r_1 \neq r_2 \wedge \exists u \in \text{UNITS} \\ (\text{partOf}(u, r_1) \wedge \text{partOf}(u, r_2))$$

- A SAT solver can be used to find assignments to predicates (given assignment to primary preds).
- In the particular case of train routes where the number of routes is not known in advance, a single route identifier can be used and successive SAT solutions will generate all routes.

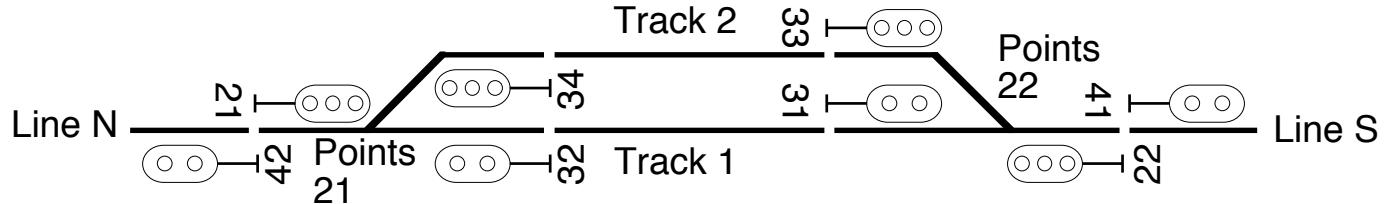
# Data generation



Welcome to the GTO Formal Tool 0.5.26

```
> load sampleGeodata2
> listdef conflict
conflict(r1,r2) == r1<>r2&SOME u:UNIT
(partOf(u,r1)&partOf(u,r2))
> list conflict
conflict(rt2131,rt2133)
conflict(rt2131,rt2232)
conflict(rt2131,rt2234)
conflict(rt2131,rt3242)
conflict(rt2131,rt3442)
conflict(rt2232,rt2234).....
```

# Train route generation



- A single train route identifier (route) is used. No data for secondary predicates.

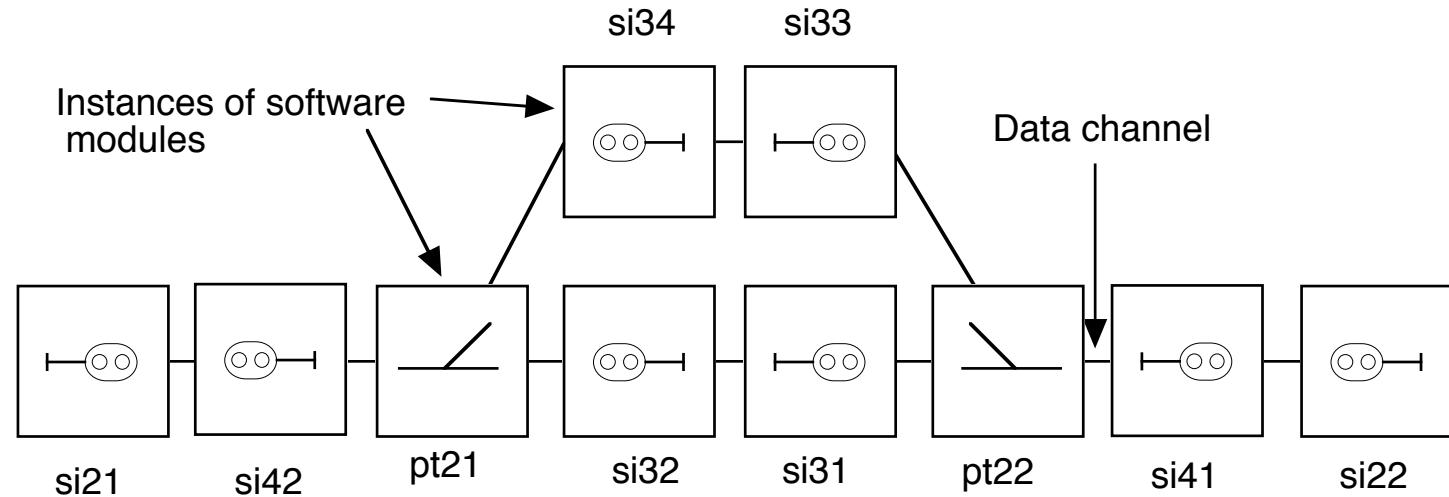
Welcome to the GTO Formal Tool 0.5.26  
> `load sampleGeodata3`  
> `satisfy TRUE`  
The formula is satisfiable. Time: 0.26  
> `list partOf entry exit`  
`partOf(route,pt21)`  
`partOf(route,t1)`  
`entry(si21,route)`  
`exit(si31,route)`

# Geographical data at Bombardier

- Formal verification of interlocking software, written in the proprietary language STERNOL.
- Ebilock interlocking systems use data files with geographical data for configuring the software.
- The tools SST (STERNOL specification tool) and SVT (STERNOL verification tool) perform formal verification of STERNOL programs, providing access to geographical data.
- SST+SVT can generate all train routes using domain axioms and exhaustive SAT solving.



# Using geographical data with SST



- Specialised specification language includes constructs to access this data:

```
legs(SI42) = 2,  
leg(SI42,1) = PT21  
leg(SI42,2) = SI21.....
```



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# Dynamic behaviour

(A comment on Dines' talk)

- Railway signalling uses protections to prevent (within reason) accidents when trains do not respect signals. E.g. overlaps, flank protection.
- Protections are typically not defined or determined in a very well-founded manner.
- A domain theory including dynamics of train movements and (probabilistic) behaviour of drivers and equipment could help design protections in a well-founded manner.