Spatially-Variant Morphological Operations on Binary Images based on the Polar Distance Transform

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Application

Spatially-variant mathematical morphology by the polar distance transform can be useful when handling circular segments.

Dilation applied on the set of detected annual ring pixels (black) connects pixels that belong to the same ring, while keeping pixels from different rings separated. The result is shown in white, superimposed on the original image.

Morphological operations with a distance transform

Binary mathematical morphology can be performed by thresholding a distance transform (DT), provided that the DT is a metric.

Erosion an dilation of a set *X*, by thresholding a distance transform *D*, with the threshold *T* is given by:

$$\varepsilon_D^T(X) = \{x \in X \mid D(x) > T\}$$

and

$$\delta_D^T(X) = (\varepsilon(X^C))^C$$

respectively, where X^C is the complement of *X*.

The structuring element (SE) is equal to a disc defined by the DT, with the radius given by the threshold.

Spatially-Variant Mathematical Morphology

The shape of the SE changes with the spatial location in the image.

We use the polar distance transform (PDT), where the distance between pixels depends on the polar coordinates to define morphological operations.

Theorem: The PDT is a metric, if $\bar{\omega}_r(p,v) > 0$ and $\bar{\omega}_\theta(p,v) > 0,\ \forall p, v.$

The PDT defines distance in angular direction different from distance in the radial direction, giving “discs” that are elongated. Discs elongated in the angular direction are applicable when circular objects are considered.

The Polar Distance Transform

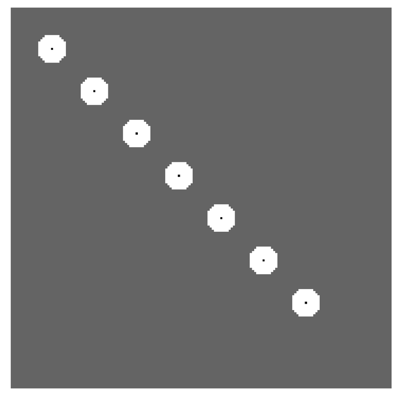
The distance between two pixels *p* and *p+v*, where *p+v* is in the 5x5 neighbourhood of *p* is given by:

$$d(p, p+v) = \sqrt{(\bar{\omega}_r(p,v)\Delta r(p,v))^2 + (\bar{\omega}_\theta(p,v)\Delta\theta(p,v))^2},$$

where $\Delta r(p,v)$ and $\Delta\theta(p,v)$ are the change in radius and angle, respectively, and $\bar{\omega}_r(p,v)$ and $\bar{\omega}_\theta(p,v)$ are weights for the step from pixel *p* to pixel *p+v*, that depends on the polar coordinates of the pixel.

Different structuring elements

By varying the weights and the threshold, the shape of the SEs can be varied. The larger the radial weight compared to the angular, the more elongated are the SEs in the angular direction. With a threshold that depends on the radius we can create SEs that cover equal angle on all radii.
Here we use $\omega_\theta = r$ for all examples.



$\omega_r = 1, T = 7$

$\omega_r = 10, T = 70$

$\omega_r = 10, T = 0.7r$

The original set is shown as black pixels.

Opening of a set

Opening is performed by erosion followed by dilation. With SEs elongated in the angular direction, disturbances connecting elements on different radii can be removed.
The parameters $\omega_\theta = r$, $\omega_r = 40$ and $T = 160$ were used.

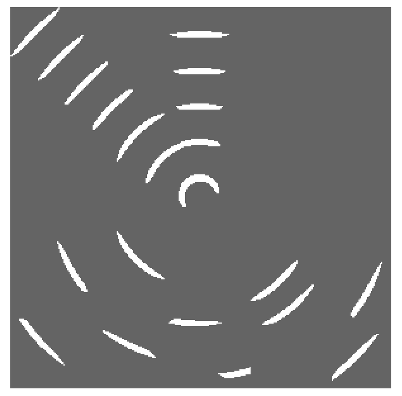
Original set

Eroded set

Opened set

Closing of a set

Closing is performed by dilation followed by erosion. Closing using the PDT connects elements along the same radius while other elements are preserved separated.
We used $\omega_\theta = r$, $\omega_r = 40$ and $T = 200$.



Original set

Dilated set

Closed set

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