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Grey Weighted Polar Distance Transform for Outlining Circular and Approximately Circular Objects

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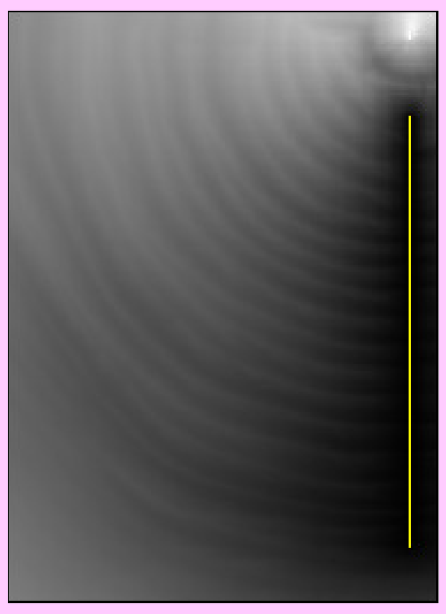
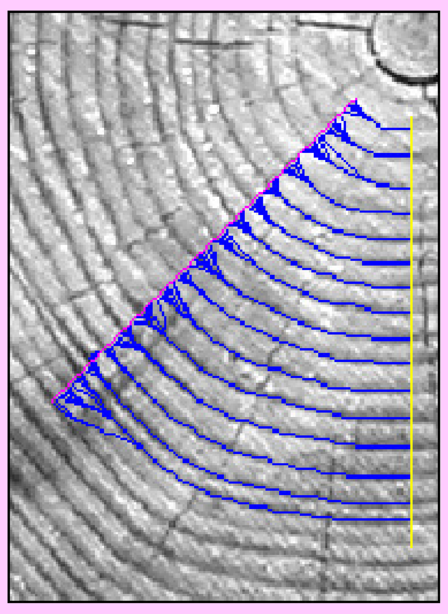
The **Polar Distance Transform** (PDT) is a weighted distance transform where local steps in the radial direction, with respect to a given origin, have a different (higher) weight than tangential steps. In this way, circular paths will be preferred over paths in the radial direction. The PDT can be combined with a cost function to create the **Grey Weighted Polar Distance Transform** (GWPDT). Here, the length of a local step is multiplied with the value of a cost function image, so that paths along low values are shorter than paths along high values.

The GWPDT is a useful tool for finding shortest paths corresponding to approximately circular shapes in images.

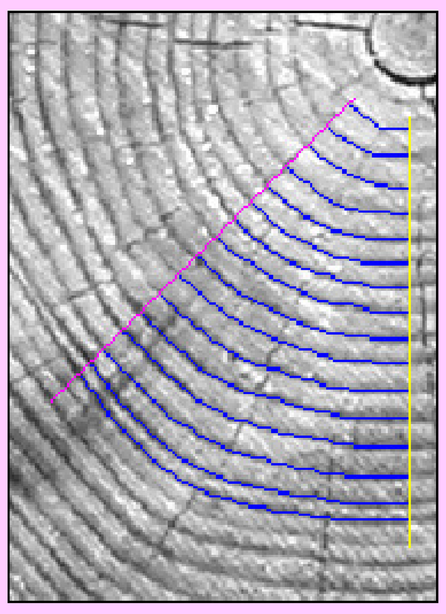
Identification of annual rings in log end face images



GWPDT with the object (yellow) superimposed. $\gamma(r) = 1/(5\sqrt{r})$



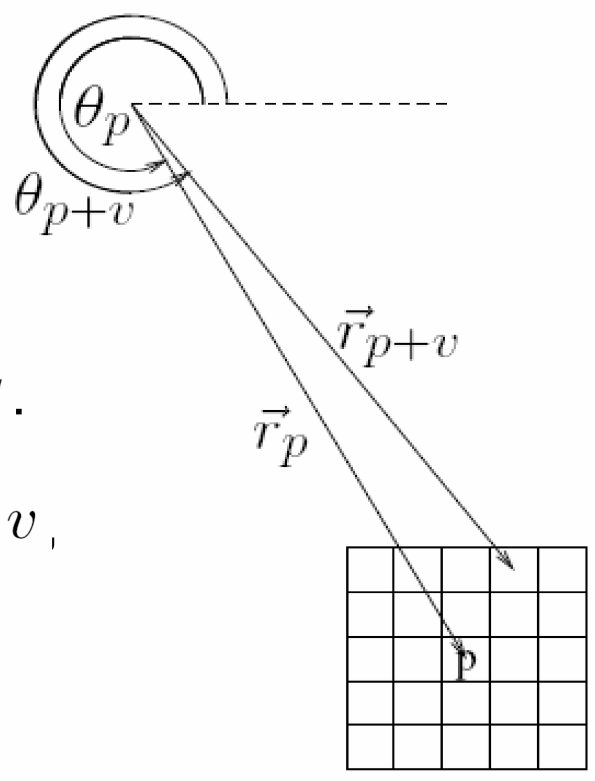
Shortest paths (blue) found by path growing from the red pixels to the object.



Shortest path from each object pixel found by back tracking.

Polar Distance Transform

Given
- Image with a circular pattern
- Origin of the circular pattern



The distance between two pixels, p and q, is equal to the length of the shortest path between p and q.

For each local step, from p to $p+v$, in the path, radial and tangential change is computed:

$$\Delta r(p,v) = |r_p - r_{p+v}|$$

$$\Delta\theta(p,v) = \min(|\theta_p - \theta_{p+v}|, 2\pi - |\theta_p - \theta_{p+v}|)$$

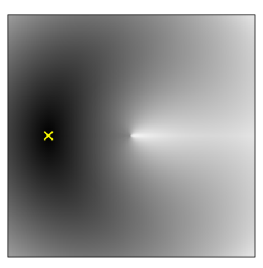
The total weight for a step is:

$$w(p,v) = \sqrt{(w_r \cdot \Delta r(p,v))^2 + (w_\theta \cdot \Delta\theta(p,v))^2}$$

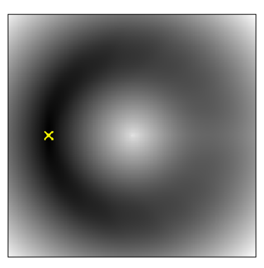
We let the relationship between the weights be a function of the distance, r, from the origin to the pixel.

$$w_r = \gamma(r) \cdot w_\theta$$

PDTs



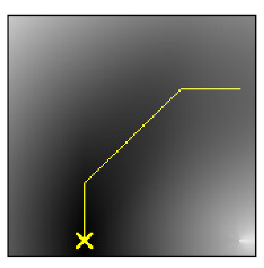
$\gamma = 0.02$



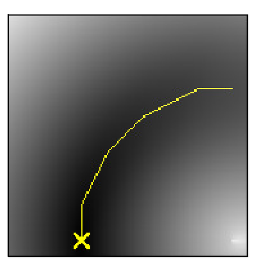
$\gamma = 0.10$

The choice of $\gamma(r)$ is application dependent. A large $\gamma(r)$ gives a high cost in the radial direction compared to the angular.

Neighbourhood sizes



3×3



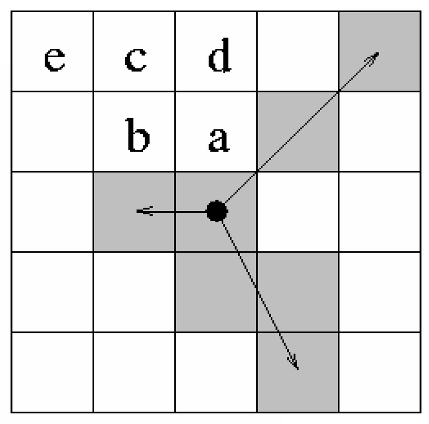
5×5

The neighbourhood size 5 x 5 pixels is used, due to the improved circular shape.

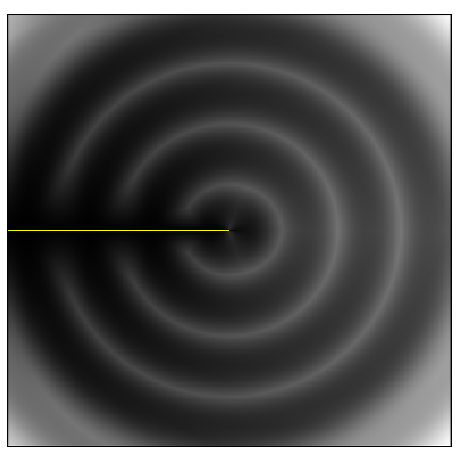
Grey Weighted Polar Distance Transform

An intensity image is used as a cost function, together with the 5 x 5 neighbourhood based PDT. All covered pixels in a local step must be taken into account in the grey weighting of the local distance. The local weights are multiplied with the mean value, μ, of the covered pixels. Five different types of steps are possible.

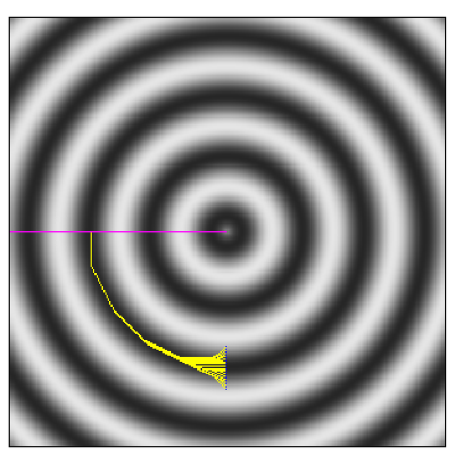
$$\begin{cases} \mu(p,0) = g(p) \\ \mu(p,a) = \frac{g(p)+g(p+a)}{2} \\ \mu(p,b) = \frac{g(p)+g(p+b)}{2} \\ \mu(p,c) = \frac{g(p)+g(p+a)+g(p+b)+g(p+c)}{4} \\ \mu(p,d) = \frac{g(p)+2\cdot g(p+a)+g(p+d)}{4} \\ \mu(p,e) = \frac{g(p)+2\cdot g(p+b)+g(p+e)}{4}, \end{cases}$$



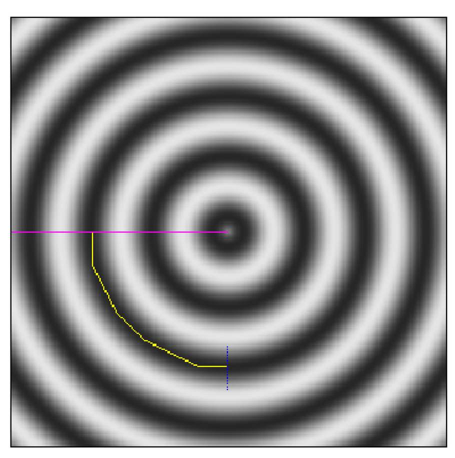
GWPDT computed on a sinusoidal pattern



GWPDT with the object pixels superimposed. $\gamma = 0.05$



Shortest paths (yellow) from 21 blue pixels to the object (pink).



Shortest path from the closest of the 21 pixels.

The GWPDT is useful for identifying approximately circular objects, e.g. annual rings in log end face images.

Poster presentation at
The International Conference on Image Analysis and Processing 2007