Type Checking

Outline

- General properties of type systems
- Types in programming languages
- Notation for type rules
  - Logical rules of inference
- Common type rules

Static Checking

- Refers to the compile-time checking of programs in order to ensure that the semantic conditions of the language are being followed

Examples of static checks include:

- Type checks
- Flow-of-control checks
- Uniqueness checks
- Name-related checks

Static Checking (Cont.)

Flow-of-control checks: statements that cause flow of control to leave a construct must have some place where control can be transferred;
e.g., **break** statements in C

Uniqueness checks: a language may dictate that in some contexts, an entity can be defined exactly once;
e.g., identifier declarations, labels, values in case expressions

Name-related checks: Sometimes the same name must appear two or more times;
e.g., in Ada a loop or block can have a name that must then appear both at the beginning and at the end

Types and Type Checking

- A *type* is a set of values together with a set of operations that can be performed on them
- The purpose of *type checking* is to verify that operations performed on a value are in fact permissible
- The type of an identifier is typically available from declarations, but we may have to keep track of the type of intermediate expressions

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Type Expressions and Type Constructors

A language usually provides a set of *base types* that it supports together with ways to construct other types using *type constructors*

Through *type expressions* we are able to represent types that are defined in a program

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Type Expressions

- A base type is a type expression
- A type name (e.g., a record name) is a type expression
- A type constructor applied to type expressions is a type expression. E.g.,
  - arrays: If T is a type expression and I is a range of integers, then array(I,T) is a type expression
  - records: If T1, …, Tn are type expressions and f1, …, fn are field names, then record((f1,T1),…,(fn,Tn)) is a type expression
  - pointers: If T is a type expression, then pointer(T) is a type expression
  - functions: If T1, …, Tn, and T are type expressions, then so is (T1,…,Tn) → T

Notions of Type Equivalence

Name equivalence: In many languages, e.g. Pascal, types can be given names. Name equivalence views each distinct name as a distinct type. So, two type expressions are name equivalent if and only if they are identical.

Structural equivalence: Two expressions are structurally equivalent if and only if they have the same structure; i.e., if they are formed by applying the same constructor to structurally equivalent type expressions.

Example of Type Equivalence

In the Pascal fragment

```
type nextptr = ^node;
     prevptr = ^node;
var  p : nextptr;
     q : prevptr;
```

`p` is not name equivalent to `q`,
but `p` and `q` are structurally equivalent.

Static Type Systems & their Expressiveness

- A static type system enables a compiler to detect many common programming errors
- The cost is that some correct programs are disallowed
  - Some argue for dynamic type checking instead
  - Others argue for more expressive static type checking
- But more expressive type systems are also more complex

Compile-time Representation of Types

- Need to represent type expressions in a way that is both easy to construct and easy to check

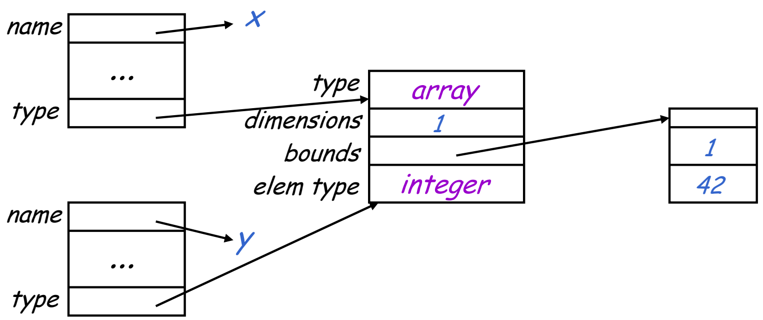
Approach 1: Type Graphs

- Basic types can have predefined "internal values", e.g., small integer values
- Named types can be represented using a pointer into a hash table
- Composite type expressions: the node for f(T1,…,Tn) contains a value representing the type constructor f, and pointers to the nodes for the expressions T1,…,Tn

Compile-time Representation of Types (Cont.)

Example:

```
var x, y : array[1..42] of integer;
```



Compile-Time Representation of Types

Approach 2: Type Encodings

Basic types use a predefined encoding of the low-order bits

BASIC TYPE	ENCODING
boolean	0000
char	0001
integer	0002

The encoding of a type expression op(T) is obtained by concatenating the bits encoding op to the left of the encoding of T. E.g.:

TYPE EXPRESSION	ENCODING
char	00 00 00 0001
array(char)	00 00 01 0001
ptr(array(char))	00 10 01 0001
ptr(ptr(array(char)))	10 10 01 0001

Compile-Time Representation of Types: Notes

- Type encodings are simple and efficient
- On the other hand, named types and type constructors that take more than one type expression as argument are hard to represent as encodings. Also, recursive types cannot be represented directly.

- Recursive types (e.g. lists, trees) are not a problem for type graphs: the graph simply contains a cycle

Types in an Example Programming Language

- Let’s assume that types are:
  - integers & floats (base types)
  - arrays of a base type
  - booleans (used in conditional expressions)

- The user declares types for all identifiers

- The compiler infers types for expressions
  - Infers a type for *every* expression

Type Checking and Type Inference

Type Checking is the process of verifying fully typed programs

Type Inference is the process of filling in missing type information

- The two are different, but are often used interchangeably

Rules of Inference

- We have seen two examples of formal notation specifying parts of a compiler
  - Regular expressions (for the lexer)
  - Context-free grammars (for the parser)
- The appropriate formalism for type checking is logical rules of inference

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Why Rules of Inference?

- Inference rules have the form

  If Hypothesis is true, then Conclusion is true

- Type checking computes via reasoning

  If E_1 and E_2 have certain types,
  then E_3 has a certain type

- Rules of inference are a compact notation for "If-Then" statements

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From English to an Inference Rule

- The notation is easy to read (with practice)
- Start with a simplified system and gradually add features
- Building blocks
  - Symbol $\wedge$ is "and"
  - Symbol $\Rightarrow$ is "if-then"
  - $x{:}T$ is "x has type T"

From English to an Inference Rule (2)

If e_1 has type int and e_2 has type int,
then $e_1 + e_2$ has type int

(e_1 has type int $\wedge$ e_2 has type int) $\Rightarrow$
$e_1 + e_2$ has type int

$(e_1\text{: int} \wedge e_2\text{: int}) \Rightarrow e_1 + e_2\text{: int}$

From English to an Inference Rule (3)

The statement

$$(e_1\text{: int} \wedge e_2\text{: int}) \Rightarrow e_1 + e_2\text{: int}$$

is a special case of

$$\text{Hypothesis}_1 \wedge \ldots \wedge \text{Hypothesis}_n \Rightarrow \text{Conclusion}$$

This is an inference rule

Notation for Inference Rules

- By tradition inference rules are written

$$\frac{\vdash \text{Hypothesis}_1 \quad \ldots \quad \vdash \text{Hypothesis}_n}{\vdash \text{Conclusion}}$$

- Type rules have hypotheses and conclusions of the form:

$$\vdash e : T$$

- $\vdash$ means “it is provable that . . .”

Two Rules

$$\frac{\text{i is an integer}}{\vdash \text{i : int}} \quad \text{[Int]}$$

$$\frac{\begin{array}{c}\vdash e_1 : \text{int} \\ \vdash e_2 : \text{int}\end{array}}{\vdash e_1 + e_2 : \text{int}} \quad \text{[Add]}$$

Two Rules (Cont.)

- These rules give templates describing how to type integers and + expressions
- By filling in the templates, we can produce complete typings for expressions

Example: 1 + 2

$$\frac{\dfrac{1 \text{ is an integer}}{\vdash 1 : \text{int}} \qquad \dfrac{2 \text{ is an integer}}{\vdash 2 : \text{int}}}{\vdash 1 + 2 : \text{int}}$$

Soundness

- A type system is *sound* if
  - Whenever $\vdash e : T$
  - Then e evaluates to a value of type T

- We only want sound rules
  - But some sound rules are better than others:

$$\frac{i \text{ is an integer}}{\vdash i : \text{number}}$$

Type Checking Proofs

- Type checking proves facts $e : T$
  - Proof is on the structure of the AST
  - Proof has the shape of the AST
  - One type rule is used for each kind of AST node
- In the type rule used for a node e:
  - Hypotheses are the proofs of types of e's subexpressions
  - Conclusion is the type of e
- Types are computed in a bottom-up pass over the AST

Rules for Constants

$$\frac{}{\vdash \text{false} : \text{bool}} \quad [\text{Bool}]$$

$$\frac{f \text{ is a floating point number}}{\vdash f : \text{float}} \quad [\text{Float}]$$

Two More Rules

$$\frac{\vdash e : \text{bool}}{\vdash \text{not } e : \text{bool}} \quad \text{[Not]}$$

$$\frac{\begin{array}{c}\vdash e_1 : \text{bool} \\ \vdash e_2 : T\end{array}}{\vdash \text{while } e_1 \text{ do } e_2 : T} \quad \text{[While]}$$

A Problem

- What is the type of a variable reference?

$$\frac{x \text{ is an identifier}}{\vdash x : ?} \quad \text{[Var]}$$

- The local, structural rule does not carry enough information to give x a type

A Solution

- Put more information in the rules!
- A *type environment* gives types for *free* variables
  - A type environment is a function from Identifiers to Types
  - A variable is free in an expression if it is not defined within the expression

Type Environments

Let E be a function from Identifiers to Types

The sentence $E \vdash e : T$

is read: Under the assumption that variables have the types given by E, it is provable that the expression e has the type T

Modified Rules

The type environment is added to the earlier rules:

$$\frac{i \text{ is an integer}}{E \vdash i : \text{int}} \text{ [Int]}$$

$$\frac{\begin{array}{c} E \vdash e_1 : \text{int} \\ E \vdash e_2 : \text{int} \end{array}}{E \vdash e_1 + e_2 : \text{int}} \text{ [Add]}$$

New Rules

And we can write new rules:

$$\frac{E(x) = T}{E \vdash x : T} \text{ [Var]}$$

Type Checking of Expressions

Production	***Semantic Rules***
E → id	{ if (declared(id.name)) then E.type := lookup(id.name).type else E.type := error(); }
E → int	{ E.type := integer; }
E → E1 + E2	{ if (E1.type == integer AND E2.type == integer) then E.type := integer; else E.type := error(); }

Type Checking of Expressions (Cont.)

May have automatic *type coercion*, e.g.

E1.type	E2.type	E.type
integer	integer	integer
integer	float	float
float	integer	float
float	float	float

Type Checking of Statements: Assignment

Semantic Rules:

S → Lval := Rval {check_types(Lval.type,Rval.type)}

Note that in general Lval can be a variable or it may be a more complicated expression, e.g., a dereferenced pointer, an array element, a record field, etc.

Type checking involves ensuring that:

- Lval is a type that can be assigned to, e.g. it is not a function or a procedure
- the types of Lval and Rval are “compatible”, i.e, that the language rules provide for coercion of the type of Rval to the type of Lval

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Type Checking of Statements: Loops, Conditionals

Semantic Rules:

Loop → while E do S {check_types(E.type,**bool**)}

Cond → if E then S1 else S2 {check_types(E.type,**bool**)}