

# Propositional Satisfiability (SAT): Conflict-Driven Clause Learning (CDCL)

Tjark Weber

+ slides by Marijn Heule



# The CDCL Algorithm

The main difference between CDCL and DPLL is that CDCL's back jumping is non-chronological.

```
while not all variables are assigned :  
    1. select an unassigned variable and  
       assign True or False // decision  
    2. apply unit propagation  
    3. if there is a conflict :  
        3.1 analyze the conflict to learn a new clause  
        3.2 non-chronologically backtrack (back jump)  
            to an appropriate decision level  
            depending on the conflict clause  
            (return UNSAT if there is no such level)  
return SAT
```

# Mini-tutorial on conflict-driven clause learning solvers

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The University of Texas at Austin

20/01/2014 @ BIRS

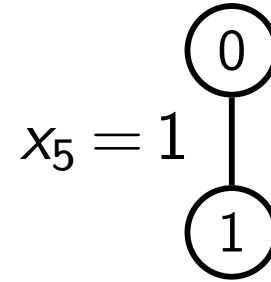
# Conflict-driven SAT solvers: Search and Analysis

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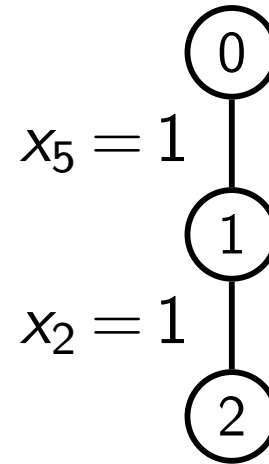
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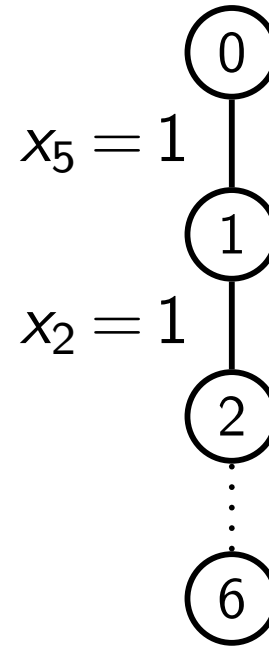
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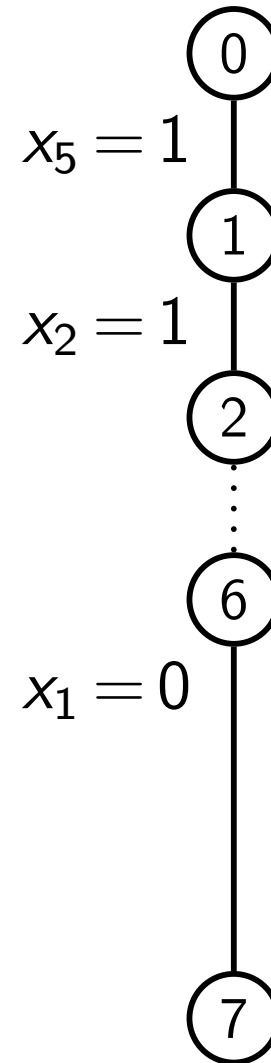
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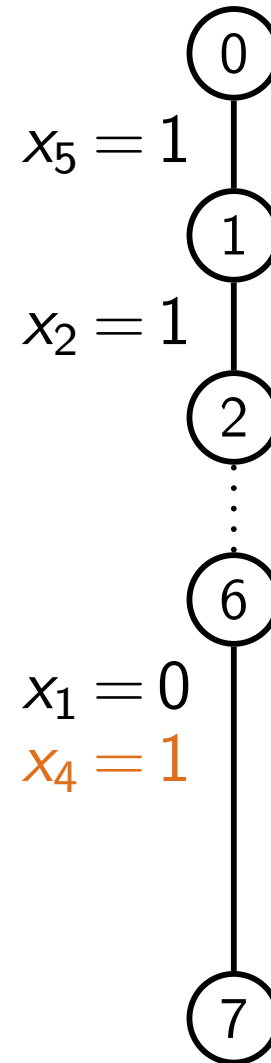
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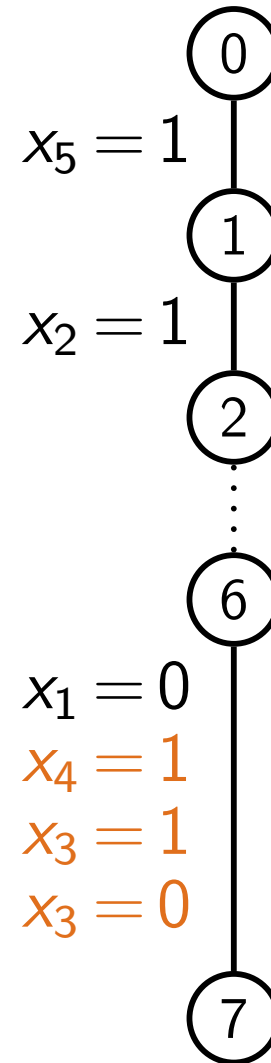
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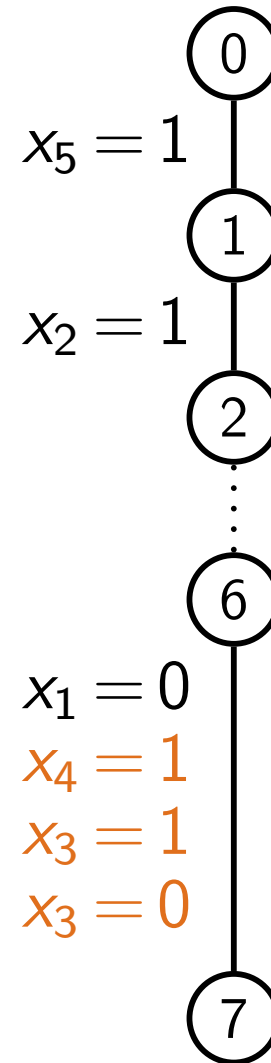
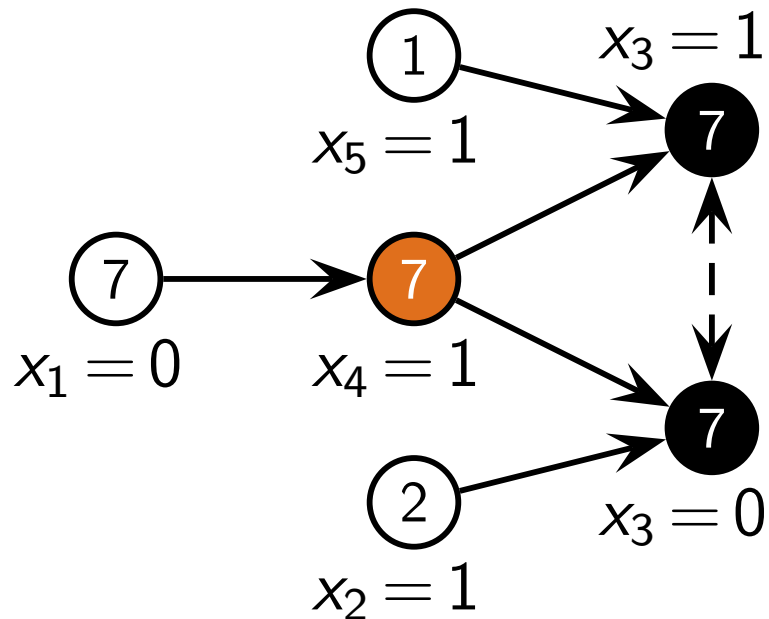
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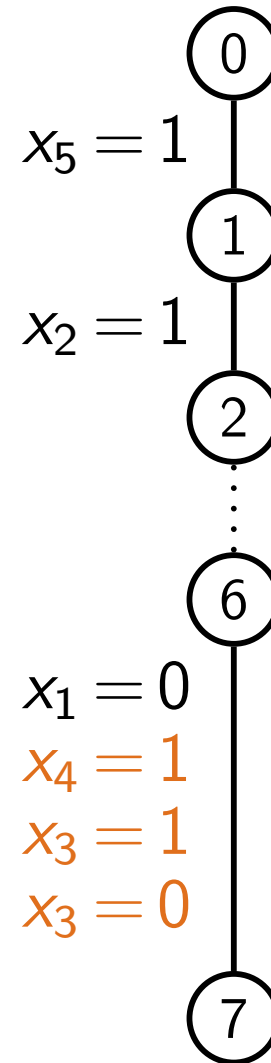
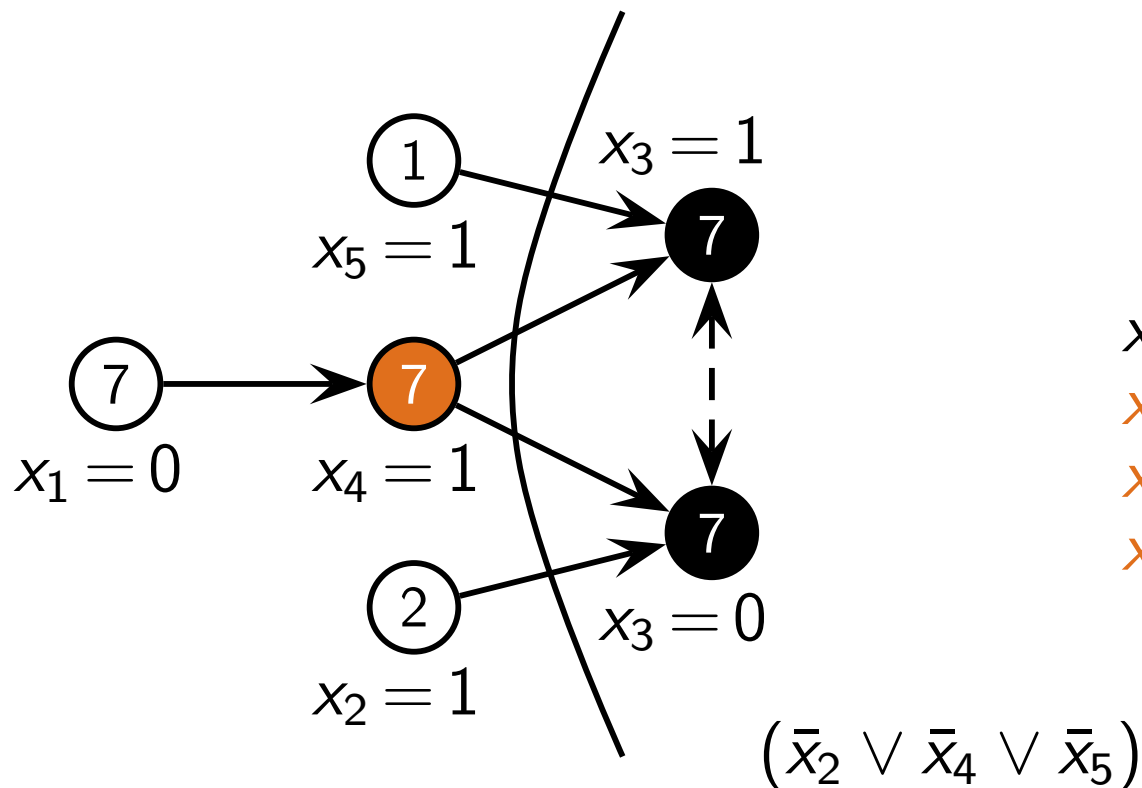
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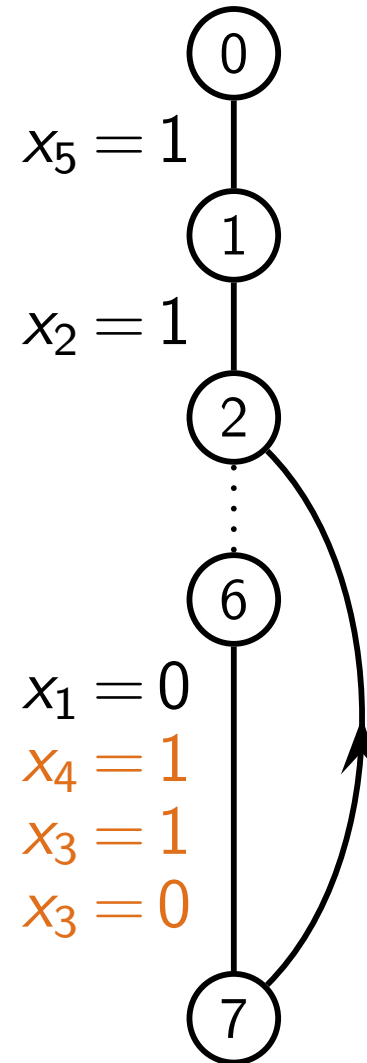
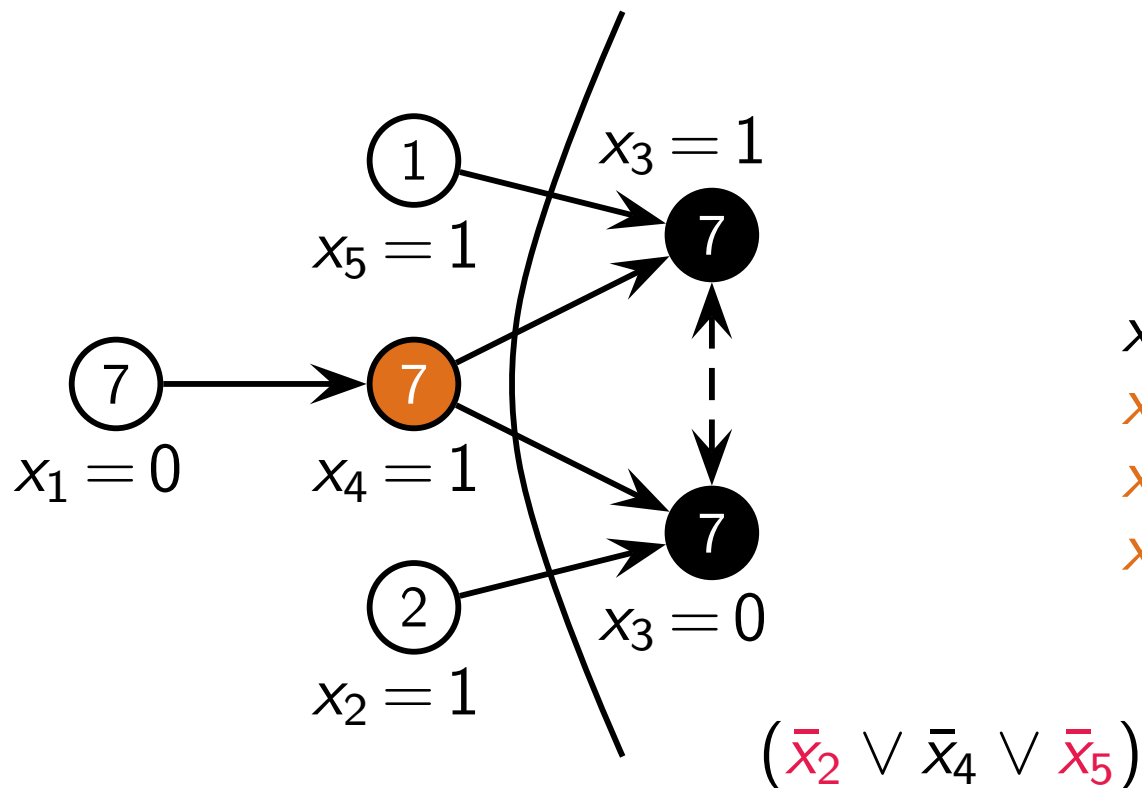
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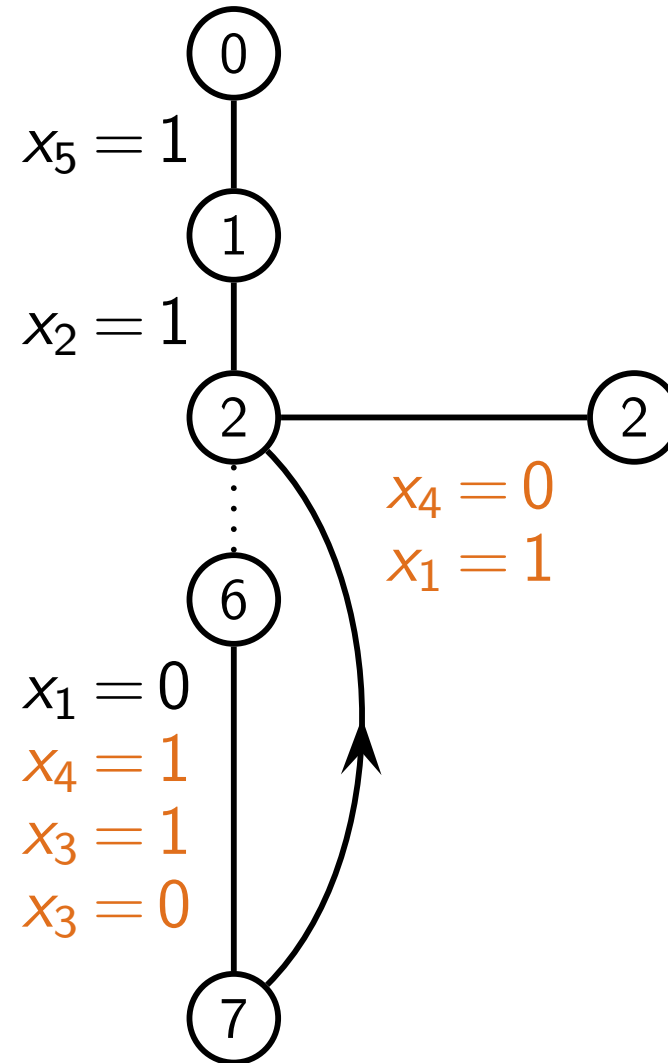
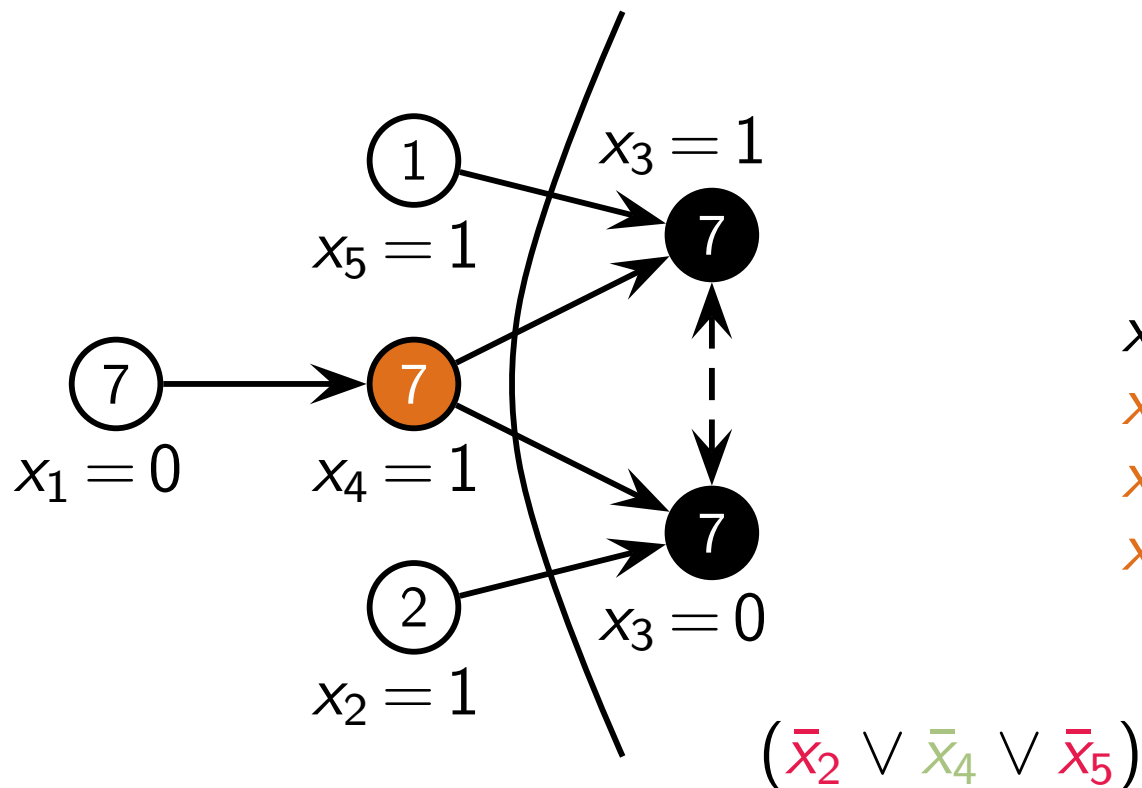
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