Algorithms & Data Structures 2 (1DL231) **Exam of 5 January 2018**

Instructions: This is a multiple-choice exam, in order to save you the time of tidying up the presentation of your answers. There is exactly ***one*** correct answer per question. You can keep the question sheets and should ***hand in only the answer sheet***: you are ***not*** expected to explain your answers. Unfortunately, the head teacher cannot attend this exam. ***Also read the instructions on the answer sheet before starting***.

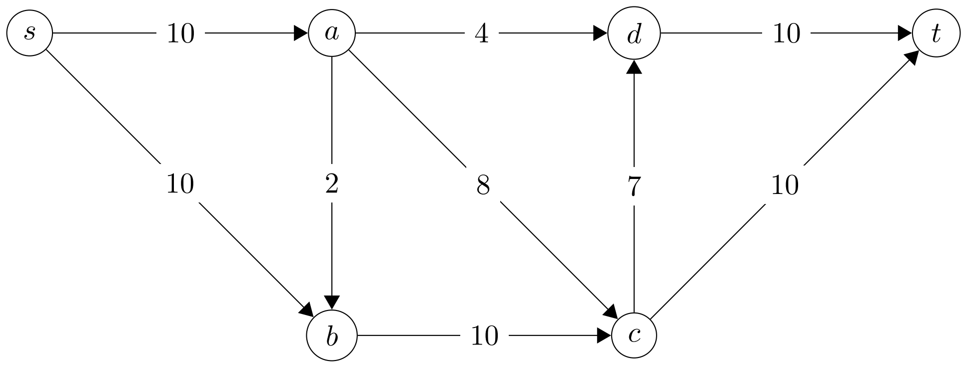
0 Warming Up

Question 0: When do I read ***all*** the instructions on this page ***and*** the answer sheet?

[A] at breakfast [B] if I get stuck [C] if I fail the exam [D] tonight [E] before starting

1 Maximum Flow

Consider the following flow network with source s and sink t:



Question 1: After augmenting along the path $s \to a \to c \to t$, along $s \to a \to b \to c \to t$, and finally along $s \to b \to c \to d \to t$, what is the augmenting path of highest capacity?

[A] $s \to b \to a \to d \to t$, capacity 2

[B] none, the reached flow value is optimal

[C] $s \to b \to a \to c \to t$, capacity 2

[D] $s \to b \to c \to d \to t$, capacity 1

[E] $s \to b \to c \to a \to d \to t$, capacity 1

Question 2: Are the flows across ***all*** cuts after the 3 augmentations of Question 1 equal?

[A] yes: 16 [B] yes: 17 [C] yes: 18 [D] yes: 19 [E] no

Question 3: What is the maximum flow value (after ***all*** possible augmentations)?

[A] 16 [B] 17 [C] 18 [D] 19 [E] 20

Question 4: Which is a source set S of a minimum-capacity (s,t)-cut (S,T)?

[A] $\{s\}$ [B] $\{s,a\}$ [C] $\{s,a,b\}$ [D] $\{s,a,c\}$ [E] $\{s,a,b,c\}$

Question 5: What is the capacity of a minimum-capacity (s,t)-cut?

[A] 16 [B] 17 [C] 18 [D] 19 [E] 20

2 Greedy Algorithms

Consider lectures $\ell_1, \ell_2, \ldots, \ell_n$ for which the same classroom is requested. Each lecture ℓ_i has a start time s_i and a finish time f_i, where $0 \leq s_i < f_i < \infty$. We wish to select a largest subset of lectures of which no two overlap in time. If selected, then ℓ_i happens during the half-open time interval $[s_i, f_i)$. For example, consider the following set of lectures:

i	1	2	3	4	5	6	7	8	9	10	11
s_i	1	3	0	5	3	5	6	8	8	2	12
f_i	4	5	6	7	9	9	10	11	12	14	16

The subset $\{\ell_3, \ell_9, \ell_{11}\}$ has non-overlapping lectures but $\{\ell_1, \ell_4, \ell_8, \ell_{11}\}$ is larger. The latter is a largest subset of non-overlapping lectures; another largest subset is $\{\ell_2, \ell_4, \ell_9, \ell_{11}\}$. Consider the following greedy algorithm template:

```
GREEDY-LECTURE-SELECTOR(n, [s_1, ..., s_n], [f_1, ..., f_n])
1  in-place sort the lectures by some criterion
2  L := {ℓ_1}        // L is the current subset of selected non-overlapping lectures
3  for i := 2 to n
4      if lecture ℓ_i does not overlap with any lecture already in L
5          L := L ∪ {ℓ_i}
6  return L
```

Question 6: Assume we in-place sort in line 1 the lectures by monotonically increasing finish time, giving $f_1 \leq f_2 \leq \cdots \leq f_n$: on which inputs (such as the example above) does the greedy algorithm above return a largest subset of non-overlapping lectures?

- [A] all
- [B] none
- [C] some
- [D] undecidable
- [E] NP-hard

Question 7: If lecture ℓ_j was most recently added to L, then how to test the Boolean condition in line 4 of the greedy algorithm in Question 6 in ***constant*** time and space?

- [A] $f_i < s_j$
- [B] $f_i \leq s_j$
- [C] $f_j < s_i$
- [D] $f_j \leq s_i$
- [E] impossible

Question 8: What is the ***tightest*** time complexity of the greedy algorithm in Question 6?

- [A] $\mathcal{O}(n)$
- [B] $\mathcal{O}(n \cdot \lg n)$
- [C] $\mathcal{O}(n^2)$
- [D] $\mathcal{O}(n^2 \cdot \lg n)$
- [E] $\mathcal{O}(n^3)$

Question 9: Assume we in-place sort in line 1 the lectures by monotonically increasing start time, giving $s_1 \leq s_2 \leq \cdots \leq s_n$: on which inputs does the greedy algorithm above return a largest subset of non-overlapping lectures?

- [A] all
- [B] none
- [C] some
- [D] undecidable
- [E] NP-hard

Question 10: Assume we in-place sort in line 1 the lectures by monotonically increasing duration, giving $f_1 - s_1 \leq f_2 - s_2 \leq \cdots \leq f_n - s_n$: on which inputs does the greedy algorithm above return a largest subset of non-overlapping lectures?

- [A] all
- [B] none
- [C] some
- [D] undecidable
- [E] NP-hard

3 Dynamic Programming

Consider the problem in Section 2 of selecting a largest subset of non-overlapping lectures. Assume the lectures are sorted by monotonically increasing finish time: $f_1 \leq f_2 \leq \cdots \leq f_n$. Let B_{ij} be the lectures that can happen between ℓ_i and ℓ_j; note that $B_{ij} \subseteq \{\ell_{i+1}, \ldots, \ell_{j-1}\}$. In the example of Section 2: $B_{2,9} = \{\ell_4\}$; $B_{9,2} = \varnothing = B_{3,8} = B_{8,3} = B_{11,4}$; $B_{4,11} = \{\ell_8, \ell_9\}$. Create two fictitious lectures ℓ_0 and ℓ_{n+1} with $f_0 = 0$ and $s_{n+1} = f_n$. Consider the following recurrence, parametrised by $\langle \alpha_1, \alpha_2, \alpha_3, \beta, \gamma \rangle$, on a numeric quantity $C[i,j]$:

$$C[i,j] = \begin{cases} 0 & \text{if and only if } \beta \\ \gamma\ \{C[\alpha_1, k] + \alpha_2 + C[k, \alpha_3] \mid \ell_k \in B_{ij}\} & \text{if and only if } \neg\beta \end{cases}$$

Question 11: If $C[0, n+1]$ is returned by a correct algorithm for computing the size of a largest subset of non-overlapping lectures, then what is $C[i,j]$, with $0 \leq i, j \leq n+1$?

- A the size of B_{ij}
- B the size of $\{\ell_i, \ell_{i+1}, \ldots, \ell_j\}$
- C the size of $\{\ell_i, \ell_{i+1}, \ldots, \ell_j\} \setminus \{\ell_0, \ell_{n+1}\}$
- D the size of a largest subset of non-overlapping lectures in B_{ij}
- E the size of a largest subset of non-overlapping lectures in $\{\ell_i, \ell_{i+1}, \ldots, \ell_j\}$

Question 12: For the example of Section 2, what is the sum $|B_{1,11}| + C[1, 11]$?

A 6 B 7 C 8 D 9 E other

Question 13: What is the Boolean condition β?

A $i = j$ B $i > j$ C $i \geq j$ D $B_{ij} = \varnothing$ E $B_{ij} \neq \varnothing$

Question 14: What is the index expression α_1?

A 1 B $i - 1$ C i D $j - 1$ E j

Question 15: What is the numeric expression α_2?

A -1 B $+1$ C size of B_{kk} D $C[k,k]$ E $f_k - s_k$

Question 16: What is the index expression α_3?

A $i - 1$ B i C $j - 1$ D j E n

Question 17: What is the single-argument set operator γ?

A argmax B average C max D median E set-size

Question 18: Assuming the desired quantity $C[0, n+1]$ is in the upper-right corner of the table C, what is an ordering of filling C without referring to yet non-computed elements?

- A columns left-to-right, bottom-up in the columns
- B columns left-to-right, top-down in the columns
- C rows top-down, left-to-right in rows
- D rows top-down, right-to-left in rows
- E rows bottom-up, right-to-left in rows

4 Complexity

Question 19: Assuming $d = \max\ \{f_i - s_i \mid 1 \leq i \leq n\}$ is the duration of a longest lecture among the n lectures in the problem of Sections 2 and 3, what is the ***tightest*** time complexity of a dynamic program for the recurrence on $C[i, j]$ of Section 3? (This question can be answered ***without*** knowing the correct answer to any question of Section 3!)

- A $\mathcal{O}(n^2)$
- B $\mathcal{O}(n^2 \cdot \lg n)$
- C $\mathcal{O}(n^2 \cdot d)$
- D $\mathcal{O}(n^3)$
- E $\mathcal{O}(n^3 \cdot d)$

Question 20: If the best known algorithm for solving a decision problem D takes $\mathcal{O}(k^n)$ time on an instance of size n, for a constant $k > 1$, then what is the ***tightest*** complexity class of D, according to current knowledge?

- A P
- B NP
- C NP-complete
- D NP-hard
- E we cannot conclude

Question 21: If the best known solution checker for a decision problem D takes $\mathcal{O}(n^k)$ time on an instance of size n, for a constant $k > 0$, then what is the ***tightest*** complexity class of D, according to current knowledge?

- A P
- B NP
- C NP-complete
- D NP-hard
- E we do not know

Question 22: The classical algorithm for computing naïvely (without knowledge of arithmetic progressions) the sum $1 + 2 + \cdots + n$ for a given natural number n takes $\Theta(n)$ time: what is the most accurate description of this time complexity?

- A logarithmic
- B linear
- C pseudo-polynomial
- D super-exponential
- E we do not know

Question 23: In order to prove that a decision problem D is NP-complete, one has to:

- A prove that D reduces to (denoted by $\leq_P$) some known problem in P
- B prove that D reduces to some known NP-complete problem
- C prove that D reduces to some known NP-complete problem and that D is in NP
- D prove that some known NP-complete problem reduces to D
- E prove that some known NP-complete problem reduces to D and that D is in NP

Answer Sheet — AD2 Exam (1DL231) of 5 January 2018

Instructions: Do ***not*** alter the drawing above. Using a ***very dark*** colour, ***fill in*** entirely ***at most one*** answer box (A to E) per question: we will use an optical character recognition (OCR) system that ignores circles, crosses, ticks, etc. Transfer your answers from the question sheets to this answer sheet ***just before handing in***; if an answer becomes ambiguous to an OCR system, then please request another answer sheet. Every correct answer gives 2 points. Every multiple answer or incorrect answer gives 0 points. Partial credit of 1 point may be given in exceptional circumstances. If you think a question is unclear or wrong, then mark its number with a ⋆ on ***this sheet***, and explain ***on the backside of this sheet*** what your difficulty with the question is ***and*** what additional assumption underlies the candidate answer that you have chosen or the new answer that you indicate.

Grading: Your grade is as follows, when your mark is e points:

Grade	Condition
5	$38 \leq e \leq 46$
4	$30 \leq e \leq 37$
3	$23 \leq e \leq 29$
U	$00 \leq e \leq 22$

1 Maximum Flow

Question 1: ■ B C D E

Question 2: A ■ C D E

Question 3: A B C D ■

Question 4: ■ B C D E

Question 5: A B C D ■

2 Greedy Algorithms

Question 6: ■ B C D E

Question 7: A B C ■ E

Question 8: A ■ C D E

Question 9: A B ■ D E

Question 10: A B ■ D E

3 Dynamic Programming

Question 11: A B C ■ E

Question 12: A ■ C D E

Question 13: A B C ■ E

Question 14: A B ■ D E

Question 15: A ■ C D E

Question 16: A B C ■ E

Question 17: A B ■ D E

Question 18: ■ B C D E

4 Complexity

Question 19: A B C ■ E

Question 20: A B C D ■

Question 21: A ■ C D E

Question 22: A B ■ D E

Question 23: A B C D ■

Again: Please use a ***very dark*** colour to ***fill in*** your chosen boxes ***entirely***!

Your anonymous exam code:						