Algorithms & Data Structures 2 (1DL231) **Exam of 16 December 2016**

Instructions: This is a multiple-choice exam, to save you the time of tidying up the presentation of your answers. There is exactly ***one*** correct answer per question. You can keep the question sheets and should ***hand in only the answer sheet***: you are ***not*** expected to explain your answers. Unfortunately, the teacher cannot attend this exam. ***Also read the instructions on the answer sheet before starting***.

0 Warm-Up Questions: General Culture

Question −2: How is Edsger W. Dijkstra's surname pronounced in his native language?

A dəkstra B dəikstra C dikstra D dɜikstra E dɛikstra

Question −1: In which year was the Alan M. Turing Award ***not*** won by a scientist whose work was discussed in this course?

A 1972 B 1974 C 1976 D 1978 E 1982

1 String Matching

Question 1: On which of the following length-m patterns P does the naïve string matching algorithm reach its ***worst***-case runtime when looking for ***all*** occurrences of P in the text $T = 0^n$ (that is, a string of n occurrences of the character '0'), with $n \geq m \geq 3$?

A 1^m B $1^{m-1}0$ C $1(0^{m-1})$ D $0(1^{m-1})$ E $0^{m-1}1$

Question 2: For the Rabin-Karp string matching algorithm, let p denote the fingerprint of the length-m pattern P, and let t_s denote the fingerprint of the length-m substring T_s for shift s in text T (of length at least m). On which assumption does the algorithm rely?

A $p = t_s \Rightarrow P = T_s$

B $p = t_s \Leftrightarrow P = T_s$

C $p = t_s \Leftarrow P = T_s$

D $p \neq t_s \Leftarrow \exists k \in 1 \ldots m : P[k] \neq T_s[k]$

E $p \neq t_s \Rightarrow \forall k \in 1 \ldots m : P[k] \neq T_s[k]$

Question 3: How many spurious hits does the Rabin-Karp string matching algorithm encounter in the text T = "31415126538497 92" when looking for ***all*** occurrences of the pattern P = "26", working modulo $q = 11$ and over the alphabet $\Sigma = \{0, 1, 2, \ldots, 9\}$?

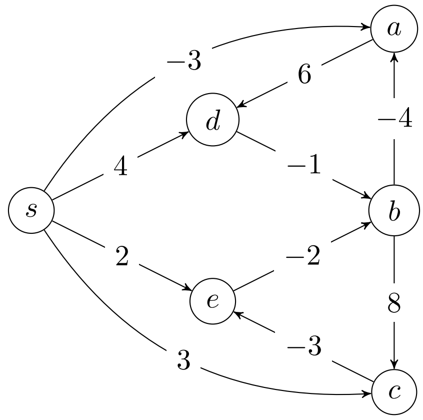
A 0 B 1 C 2 D 3 E 4

Question 4: On which of the following patterns P does the Rabin-Karp string matching algorithm reach its ***best***-case runtime when looking for ***all*** occurrences of P in the text $T = 0^n$ (that is, a string of n occurrences of the character '0'), with $n \geq 3$, working modulo $q = 3$ and over the alphabet $\Sigma = \{0, 1, 2, \ldots, 9\}$?

A "660" B "300" C "099" D "007" E "000"

2 Greedy Algorithms

Consider a weighted directed graph G with a set V of n vertices, a set E of m edges, and an edge-weight function $w\colon E \to \mathbb{R}$, such as in the figure to the right. In this example, some edges have ***negative*** weights: Dijkstra's single-source shortest paths algorithm is ***not*** applicable for finding shortest paths (that is, minimum-weight paths) from vertex s to all vertices; for instance, a shortest path from s to d is s-c-e-b-a-d and has weight $3 + (-3) + (-2) + (-4) + 6 = 0$.



Question 5: Let G have some negative weights: which condition below is ***sufficient*** for the existence of a shortest path from a source vertex s to a vertex reachable from s?

A if G is acyclic B if G is connected C if real weights D if rational weights E if integer weights

Question 6: What weight does Dijkstra's algorithm find for a shortest path from s to d?

A it loops forever B 0 C 3 D 4 E it crashes

Question 7: If G has some negative weights, violating the precondition of Dijkstra's algorithm, then let us first modify G by adding $-\ell$ to all weights, where $\ell < 0$ is the smallest weight, giving us a new graph G'. When does Dijkstra's algorithm find shortest paths in G' that correspond (but with exaggerated weights) 1-to-1 to shortest paths in G?

A always B sometimes C never D undecidable E we do not know

Question 8: Dijkstra's greedy algorithm for the single-source shortest paths problem performs n iterations: does it stay correct if we drop the inner loop of relaxation checks at the nth iteration of the outer loop (extraction of a minimum from a min-priority queue)?

A always B sometimes C never D undecidable E we do not know

Question 9: Let G possibly have some negative weights: what is the maximum number of edges of a shortest path that is simple (that is, it does not re-visit vertices)?

A $-\infty$ B $n-1$ C n D $n+1$ E $+\infty$

3 Dynamic Programming

Consider a weighted directed graph G with a set V of n vertices, a set E of m edges, an edge-weight function $w\colon E \to \mathbb{R}$ with no negative-weight cycles, and a source vertex s. Consider the following recurrence, parameterised by $\langle \alpha, \beta_1, \ldots, \beta_4, \gamma \rangle$, for a quantity $T[i, v]$:

$$T[i,v] = \begin{cases} 0 & \text{if } \alpha \wedge (v = s) \\ \beta_1 & \text{if } \alpha \wedge (v \neq s) \\ \gamma\left(T[i-1, \beta_2],\quad \min\limits_{(x,v)\in E}(T[i-1,\beta_3] + \beta_4)\right) & \text{if } \neg\alpha \end{cases}$$

Question 10: If $T[n-1, v]$ is returned by a correct algorithm for computing the weight of a shortest path in G from s to v, then $T[i, v]$, with $0 \leq i < n$ and $v \in V$, denotes the weight of a shortest path from s to v with how many edges?

[A] $< i$ [B] $\leq i$ [C] $= i$ [D] $\geq i$ [E] $> i$

Question 11: For the graph in the figure of Section 2, what is the sum $T[3, a] + T[3, b]$?

[A] -8 [B] -6 [C] -5 [D] -4 [E] -3

Question 12: What is the logical condition α?

[A] $i = 0$ [B] $i = n - 1$ [C] $i = m - 1$ [D] $i = m$ [E] $i \cdot v = 0$

Question 13: What is the numeric expression β_1?

[A] $-\infty$ [B] -1 [C] 0 [D] $+1$ [E] $+\infty$

Question 14: What is the index expression β_2?

[A] s [B] $v - 1$ [C] v [D] $v + 1$ [E] x

Question 15: What is the index expression β_3?

[A] s [B] $v - 1$ [C] v [D] $v + 1$ [E] x

Question 16: What is the numeric expression β_4?

[A] $w(s, v)$ [B] $w(s, x)$ [C] $w(v, x)$ [D] $w(x, v)$ [E] 1

Question 17: What is the two-argument operator γ (written in prefix form above)?

[A] $+$ [B] Σ [C] Π [D] w [E] min

Question 18: What is an ordering of the indices i and v under which the elements $T[i, v]$ of the table T can be filled without performing any redundant computations?

[A] by lexicographically increasing $\langle v, i \rangle$

[B] any order

[C] by lexicographically increasing $\langle -i, v \rangle$

[D] by lexicographically decreasing $\langle v, -i \rangle$

[E] by lexicographically decreasing $\langle -i, v \rangle$

We say that $\langle a, b \rangle$ is *lexicographically smaller* than $\langle c, d \rangle$ if either $a < c$ or $a = c \wedge b < d$.

4 Complexity

Consider a weighted directed graph G with a set V of n vertices, a set E of m edges, and an edge-weight function $w\colon E \to \mathbb{R}$. Some of the following questions refer to the recurrence on $T[i, v]$ of Section 3, but they can be answered without knowing the correct answers to the questions of that section.

Question 19: What is the ***space*** complexity of a dynamic program obtained directly (that is, without improvements) from the recurrence on $T[i, v]$?

A $\Theta(n \cdot m)$ B $\Theta(n \cdot m^2)$ C $\Theta(n^2)$ D $\Theta(n^2 \cdot m)$ E $\Theta(n^3)$

Question 20: What is the ***tightest*** *time* complexity of a dynamic program for the recurrence on $T[i, v]$?

A $\mathcal{O}(n \cdot m)$ B $\mathcal{O}(n \cdot m^2)$ C $\mathcal{O}(n^2)$ D $\mathcal{O}(n^2 \cdot m)$ E $\mathcal{O}(n^3)$

Question 21: A ***useful*** decision version of the shortest path problem, which is an optimisation problem, asks whether from a given source vertex to a given target vertex there exists a path whose weight, say ω, is in what relationship to a given weight W?

A $\omega = n - 1$ B $\omega = n$ C $\omega \leq W$ D $\omega = W$ E $\omega \geq W$

Question 22: What is the ***tightest*** *time* complexity class, as ***currently*** known, of a decision version of the single-source shortest paths problem, considering the existence of a dynamic program for the recurrence on $T[i, v]$?

A P B pseudo-polynomial C NP-complete D NP-hard E we do not know

Question 23: What is the ***tightest*** *time* complexity class, as ***currently*** known, of a decision version of the optimisation problem of finding ***longest*** paths from a given source vertex to all other vertices of a weighted directed graph?

A P B pseudo-polynomial C NP-complete D NP-hard E we do not know

Answer Sheet — AD2 Exam (1DL231) of 16 December 2016

Instructions: Do ***not*** alter the drawing above. Using a ***very dark*** colour, ***fill in*** entirely ***at most one*** answer box (A to E) per question: we will use an optical character recognition (OCR) system that ignores circles, crosses, ticks, etc. Transfer your answers from the question sheets to this answer sheet ***just before handing in***; if an answer becomes ambiguous to an OCR system, then please request another answer sheet. Every correct answer gives 2 points. Every multiple answer or incorrect answer gives 0 points. Partial credit of 1 point may be given in exceptional circumstances. If you think a question is unclear or wrong, then mark its number with a $\star$ on ***this sheet***, and explain ***on the backside of this sheet*** what your difficulty with the question is ***and*** what additional assumption underlies the candidate answer that you have chosen or the new answer that you indicate.

Grading: Your grade is as follows, when your mark is e points:

Grade	Condition
5	$38 \leq e \leq 46$
4	$30 \leq e \leq 37$
3	$23 \leq e \leq 29$
U	$00 \leq e \leq 22$

1 String Matching

Question 1: A B C D ■

Question 2: A B ■ D E

Question 3: A B ■ D E

Question 4: A B C ■ E

2 Greedy Algorithms

Question 5: ■ B C D E

Question 6: A B ■ D E

Question 7: A ■ C D E

Question 8: ■ B C D E

Question 9: A ■ C D E

3 Dynamic Programming

Question 10: A ■ C D E

Question 11: A ■ C D E

Question 12: ■ B C D E

Question 13: A B C D ■

Question 14: A B ■ D E

Question 15: A B C D ■

Question 16: A B C ■ E

Question 17: A B C D ■

Question 18: A B C D ■

4 Complexity

Question 19: A B ■ D E

Question 20: ■ B C D E

Question 21: A B ■ D E

Question 22: ■ B C D E

Question 23: A B ■ D E

Again: Please use a ***very dark*** colour to ***fill in*** your chosen boxes ***entirely***!

Your anonymous exam code:						