```
datatype Colour = B | R
datatype Tree = E | T of Colour * int * Tree * Tree
val member = fn : int -> Tree -> bool
val balance = fn : Colour * int * Tree * Tree -> Tree
val insert = fn : int -> Tree -> Tree
val it = () : unit
- insert 4 E;
val it = T (B,4,E,E) : Tree
-
Why?
```

```
insert 4 E =
    let
    val ins = T(R,4,E,E)
    val T(_,y,a,b) = ins s
    in T(B,y,a,b)
= T(B,4,E,E)
```

Continued from previous slide:-

```
- insert 5 it;
val it = T (B,4,E,T (R,5,E,E)) : Tree
```

```
insert 5 T(B,4,E,E) =
 let
 val ins(s as T(B,4,E,E)) =
    balance(B,4,E,ins E)
 val T(_,y,a,b) = ins s
 in T(B,y,a,b)
```

So we have to find out what ins E is.

ins E = T(R,5,E,E)

This means the call to balance is :-

```
balance(B,4,E,T(R,5,E,E))
```

But since we don't have two reds in a row we simply get the tree returned. So we get the result.

Now things get more complicated :-

```
- insert 6 it;
val it = T (B,5,T (B,4,E,E),T (B,6,E,E)) : Tree
```

Notice the tree has been re-balanced in the ordinary search tree we would have got something different.

```
insert 6 T(B,4,E, T(R,5,E,E)) =
  let val ins(s as T(B,4,E, T(R,5,E,E)) =
    balance(B,4,E,ins T(R,5,E,E))
  val T(_,y,a,b) = ins s
    in T(B,y,a,b)
end
```

So we have to work out ins T(R,5,E,E)

```
ins T(R,5,E,E) =
```

```
balance(R,5,E,ins E) = balance(R,5,E,T(R,6,E,E)) =
T(R,5,E,T(R,6,E,E))
```

Again we don't match any of the clauses for balance since we don't have a black followed by two reds.

But we know now that :-

balance(B,4,E,ins T(R,5,E,E)) = balance(B,4,E, T(R,5,E,T(R,6,E,E))

Black followed by two reds.

# Red Black Trees Examples Matching balance clause. balance(B,4,E, T(R,5,E,T(R,6,E,E))Matches :balance(B,x,a,T(R,y,b,T(R,z,c,d)) So we get :-T(R,5,T(B,4,E,E),T(B,6,E,E)

Then by relabelling the top node black we get the answer.

# Red Black Trees Summary

- Insert a new node labelled red with the value in the correct place as you would with a binary search tree.
- Remove Black, Red, Red violations replacing them with a red node with two black children.

The net effect is that the tree remains balanced.

#### Implementation Approaches - Persistence versus updating

val a = [1,2,3] : int list
- val b = map (fn x=>x+1) a;
GC #0.0.0.1.3.45: (20 ms)
val b = [2,3,4] : int list

The list **a** stills stays around. ML works by creating a copy of the list **a**.

ML only copies the bits that you need to copy, sharing the bits that don't need to be copied.

Garbage collection gets rid of the bits you don't use.

## Implementation Approaches - Persistence versus updating

In C, the natural style is to create data-structures and modify them in place.

```
For example (not tested)
```

```
int add_one(int a[],int size) {
    int i;
    for(i=0;i<size;i++) {
        a[i]=a[i]+1
    }
}</pre>
```

In C it is often hard to make persistent data-structures. In ML it is more natural to make data-structures persistent and sometimes make the code easier to write. Compare the red-black tree in the book.