Distributed Spectral Efficiency Maximization in Full-Duplex Cellular Networks

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Why full-duplex at the base station?

- Half-Duplex (HD) systems → Inefficient resource utilization
- Full-Duplex (FD) systems → $\sim 2 \times$ spectral efficiency
Outline

1. Introduction

2. System Model for Spectral Efficiency Maximization

3. Centralized Solution Based on Lagrangian Duality

4. Distributed Solution Based on Auction Theory

5. Numerical Results

6. Conclusions
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FD Characteristics in cellular networks

**Benefits**

- **Spectral efficiency:** $\sim 2 \times$
- **MAC layer:** hidden terminal, collision avoidance, reduced end-to-end delay...
FD Characteristics in cellular networks

Challenges

- **Severe** self-interference (SI)
- UE-to-UE interference
- User to frequency channels pairing and power allocation
### Research Gap in FD cellular networks

**Need of distributed schemes**
- Processing burden at the BS is high*
  - Dense deployment of user
  - New SI cancellation mechanisms
  - Radio Resource Management

**Lack of fair and efficient PHY procedures**
- How to mitigate UE-to-UE interference and assess fairness?
  - Pairing $\rightarrow$ UL and DL users to share the frequency resource
  - Power allocation $\rightarrow$ mitigate interference
  - Fairness $\rightarrow$ weighted sum spectral efficiency maximization

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Contributions

- Study sum spectral efficiency maximization and fairness problem
  - Joint pairing and power allocation $\rightarrow$ maximize weighted sum spectral efficiency
- Solve this MINLP problem
- Provide distributed mechanisms for FD cellular networks
- Show spectral efficiency gains over HD with distributed schemes
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- Provide distributed mechanisms for FD cellular networks
  - Distributed auction algorithm $\rightarrow$ resource assignment to UL and DL users
- Show spectral efficiency gains over HD with distributed schemes
  - Realistic system simulations $\rightarrow$ Yes, 89%!
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Definitions (1)

- Single-cell cellular system + only BS is FD-capable
- UL users $\rightarrow I$; DL users $\rightarrow J$; Frequency channels $\rightarrow F$
- Effective path gain values $\rightarrow G_{ib}, G_{bj}, G_{ij}$
- SI cancellation coefficient $\rightarrow \beta$
- Assignment matrix $\rightarrow \mathbf{X} \in \{0, 1\}^{I \times J}$

$$x_{ij} = \begin{cases} 1, & \text{if the UL UE}_i \text{ is paired with the DL UE}_j, \\ 0, & \text{otherwise.} \end{cases}$$
Definitions (2)

- Power vectors \( p^u = [P^u_1 \ldots P^u_I], \quad p^d = [P^d_1 \ldots P^d_J] \)
- SINR at the BS and at DL user
  \[
  \gamma^u_i = \frac{P^u_i G_{ib}}{\sigma^2 + \sum_{j=1}^{J} x_{ij} P^d_j \beta}, \quad \gamma^d_j = \frac{P^d_j G_{bj}}{\sigma^2 + \sum_{i=1}^{I} x_{ij} P^u_i G_{ij}}.
  \]
- Achievable spectral efficiency
  \[
  C^u_i = \log_2(1 + \gamma^u_i), \quad C^d_j = \log_2(1 + \gamma^d_j).
  \]
- Weights \( \alpha^u_i, \alpha^d_j \)
  \( \alpha^u_i = \alpha^d_j = 1, \forall i, j \rightarrow \) Sum spectral efficiency maximization
  \( \alpha^u_i = G_{ib}^{-1}, \quad \alpha^d_j = G_{bj}^{-1} \rightarrow \) Path loss compensation
Problem Formulation

- Weighted sum spectral efficiency maximization (P-OPT)

\[
\begin{align*}
\text{maximize} \quad & \sum_{i=1}^{I} \alpha_i^u C_i^u + \sum_{j=1}^{J} \alpha_j^d C_j^d \\
\text{subject to} \quad & \gamma_i^u \geq \gamma_{\text{th}}^u, \forall i, \\
& \gamma_j^d \geq \gamma_{\text{th}}^d, \forall j, \\
& P_i^u \leq P_{\text{max}}^u, \forall i, \\
& P_j^d \leq P_{\text{max}}^d, \forall j, \\
& \sum_{i=1}^{I} x_{ij} \leq 1, \forall j, \\
& \sum_{j=1}^{J} x_{ij} \leq 1, \forall i, \\
& x_{ij} \in \{0, 1\}, \forall i, j.
\end{align*}
\]
Problem solution approaches

- Primal Problem (P-OPT)
- Dual Problem
- Centralized Solution
  - Opt power + Hungarian Alg. (C-HUN)
- Distributed Solution
  - Opt power + Auction theory (D-AUC)
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Lagrangian function

- Formulate partial Lagrangian function

\[
L(\lambda^u, \lambda^d, X, p^u, p^d) \triangleq - \sum_{i=1}^{I} \alpha^u_i C_i^u - \sum_{j=1}^{J} \alpha^d_j C_j^d + \\
+ \sum_{i=1}^{I} \lambda^u_i (\gamma^u_{th} - \gamma^u_i) + \sum_{j=1}^{J} \lambda^d_j (\gamma^d_{th} - \gamma^d_j)
\]

- The dual function is

\[
g(\lambda^u, \lambda^d) = \inf_{X \in \mathcal{X}, p^u, p^d \in \mathcal{P}} L(\lambda^u, \lambda^d, X, p^u, p^d)
\]
Dual problem and closed-form solution for assignment

- Rewrite the dual as

\[
g(\lambda^u, \lambda^d) = \inf_{X \in \mathcal{X}, p^u, p^d \in \mathcal{P}} \sum_{n=1}^{N} \left( q^u_{in}(X, p^u, p^d) + q^d_{jn}(X, p^u, p^d) \right),
\]

with

\[
q^u_{in}(X, p^u, p^d) \triangleq \lambda^u_{in} \left( \gamma^u_{th} - \gamma^u_{in} \right) - \alpha^u_{i} C^u_{in},
\]

\[
q^d_{jn}(X, p^u, p^d) \triangleq \lambda^d_{jn} \left( \gamma^d_{th} - \gamma^d_{jn} \right) - \alpha^d_{j} C^d_{jn}.
\]

- Closed-form expression for the assignment

\[
x^*_ij = \begin{cases} 
1, & \text{if } (i,j) = \arg \max_{i,j} \left( q^u_{in}^{\text{max}} + q^d_{jn}^{\text{max}} \right) \\
0, & \text{otherwise}
\end{cases}
\]
Dual problem and closed-form solution for assignment

- Rewrite the dual as

\[
g(\lambda^u, \lambda^d) = \inf_{X \in \mathcal{X}} \sum_{n=1}^{N} \left( q_{i_n}^u(X, p^u, p^d) + q_{j_n}^d(X, p^u, p^d) \right),
\]

with

\[
q_{i_n}^u(X, p^u, p^d) \triangleq \lambda_{i_n}^u \left( \gamma_{th}^u - \gamma_{i_n}^u \right) - \alpha_{i_n}^u C_{i_n}^u,
\]

\[
q_{j_n}^d(X, p^u, p^d) \triangleq \lambda_{j_n}^d \left( \gamma_{th}^d - \gamma_{j_n}^d \right) - \alpha_{j_n}^d C_{j_n}^d.
\]

- Closed-form expression for the assignment

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0, & \text{otherwise}
\end{cases}
\]
Dual problem and optimal power allocation

- Analyse the dual problem

\[
\begin{align*}
\text{maximize} & \quad \lambda^u, \lambda^d \\
\text{subject to} & \quad \lambda_i^u, \lambda_j^d \geq 0, \forall i, j,
\end{align*}
\]

- Turn our attention to the power allocation problem

\[
\begin{align*}
\text{minimize} & \quad p^u, p^d - \sum_{i=1}^J \alpha_i^u C_i^u - \sum_{j=1}^J \alpha_j^d C_j^d \\
\text{subject to} & \quad p^u, p^d \in \mathcal{P}. 
\end{align*}
\] (1a)

Optimal solution for (1) available:

\[^\dagger\text{D. Feng, L. Liu, Y. Yuan-Wu, G. Y. Li, Q. Feng and S. Li, “Device-to-Device Communications Underlaying Cellular Networks,” IEEE TC, vol. 61, no. 8, pp. 3541-3551, August 2013.}\]
Dual problem and optimal power allocation

- Analyse the dual problem

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\begin{align*}
\text{maximize} \quad & \lambda^u, \lambda^u \\
\text{subject to} \quad & g(\lambda^u, \lambda^d) \\
& \lambda^u_i, \lambda^d_j, \geq 0, \forall i, j,
\end{align*}
\]

- Turn our attention to the power allocation problem

\[
\begin{align*}
\text{minimize} \quad & p^u, p^d \\
\text{subject to} \quad & p^u, p^d \in \mathcal{P}.
\end{align*}
\]

Optimal solution for (1) available\(^*\)


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\footnote{D. Feng, L. Lu, Y. Yuan-Wu, G. Y. Li, G. Feng and S. Li, "Device-to-Device Communications Underlaying Cellular Networks," IEEE TC, vol. 61, no. 8, pp. 3541-3551, August 2013}
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Problem Reformulation

Reformulate closed-form assignment with optimal power allocation as

\[
\text{maximize } \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} x_{ij}
\]

subject to

\[
\sum_{i=1}^{I} x_{ij} = 1, \forall j,
\]

\[
\sum_{j=1}^{J} x_{ij} = 1, \forall i,
\]

\[
x_{ij} \in \{0, 1\}, \forall i, j.
\]

- Centralized solution → Hungarian Algorithm
- Distributed solution → Auction Theory
Problem Reformulation

- Reformulate closed-form assignment with optimal power allocation as

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\begin{align*}
\text{maximize} & \quad \sum_{i=1}^{I} \sum_{j=1}^{J} c_{ij} x_{ij} \\
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& \quad x_{ij} \in \{0, 1\}, \forall i, j.
\end{align*}
\]

- Centralized solution \(\rightarrow\) Hungarian Algorithm
- Distributed solution \(\rightarrow\) Auction Theory
## Distributed Auction

- **Input:** $c_{ij}$, and tolerance $\epsilon$

### Bidding Phase
- UL bids for a DL user that maximizes $c_{ij} - \hat{p}_j$
- Wait for acknowledgement on assignment or update price

### Assignment Phase
- BS is responsible for DL users
- BS selects the highest bid and update the prices $\hat{p}_j$
- Send updates and wait until the assignment matrix $X$ is feasible

- Messages exchanged using control channels, e.g., PUCCH or PDCCH
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Simulation Parameters

- $F = I = J$ and $I + J = 8, \ldots, 50$
- Path-loss compensation $\rightarrow \alpha_i^u = G_{ib}^{-1}$, $\alpha_j^d = G_{bj}^{-1}$
- SI cancellation $\beta = [-70, -100]$ dB
- Proposed algorithm
  - **D-AUC**: Dual solution with distributed Auction compared to
    - **P-OPT**: Primal optimal from brute-force solution
    - **C-HUN**: Centralized solution based on Duality and Hungarian algorithm
    - **R-EPA**: Random assignment + equal power allocation
    - **HD**: Traditional Half-Duplex scheme
Optimality Gap Comparison with $\beta = -100\text{dB}$

Negligible difference between P-OPT, C-HUN and D-AUC

Possible to use distributed solutions without losing too much
Optimality Gap Comparison with $\beta = -100\,\text{dB}$

- Negligible difference between P-OPT, C-HUN and D-AUC
- Possible to use distributed solutions without losing too much
Sum Spectral Efficiency Comparison for different $\beta$

\[
\begin{align*}
\beta &= -110 \text{dB} \rightarrow \text{UE-to-UE interference is the limiting factor} \\
\beta &= -70 \text{dB} \rightarrow \text{residual SI is the limiting factor}
\end{align*}
\]
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Take home message

- Distributed algorithms to FD cellular networks
  - Perform close to centralized schemes
    - Fair and efficient
    - Almost double spectral efficiency (89% gain)
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- Trade-off: residual SI × UE-to-UE interference
  - UE-to-UE interference is the limiting factor for low $\beta$
  - Residual SI is the limiting factor for high $\beta$
Distributed algorithms to FD cellular networks
- Perform close to centralized schemes
- Fair and efficient
- Almost double spectral efficiency (89% gain)

Trade-off: residual SI $\times$ UE-to-UE interference
- UE-to-UE interference is the limiting factor for low $\beta$
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