Defuzzification Based on Feature Distance Minimization

Fuzzy Sets and Fuzzy Techniques
Lecture 13

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Outline

Introduction
- Fuzzy sets in image processing
- Defuzzification

Defuzzification by feature distance minimization
- Feature distance minimization
- Defuzzification at increased spatial resolution
- Scale space approach
- Extension to 3D

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Fuzzy sets in image processing

- In the digitization process, fuzzy representations preserve information about the original object better than crisp representations.
- At the same spatial resolution, many object **features** can be estimated with a higher precision from a fuzzy representation than from a crisp representation.

Among them are:
-area / volume
-perimeter / surface area
-geometric moments


crisp


fuzzy

Sometimes crisp is required

Reasons to use a crisp representation may be:
- to provide easier and less subjective interpretation of images, especially if the dimension of the image is higher than 2;
- in cases when fuzzy image analysis tools are not yet developed, while crisp analogues exist.

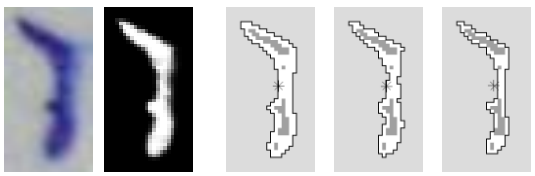
Defuzzification

to generate a crisp representation of a fuzzy set.

Defuzzification by α-cutting

Thresholding of the membership function.

An example:

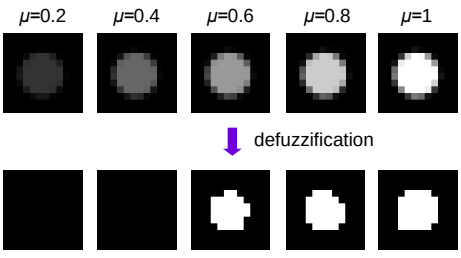


A part of a microscopy image of a bone implant. Original and a fuzzy segmented bone region.

α-cut at α=0.25 α-cut at α=0.5 α-cut at α=0.75

Which α to choose?

Defuzzification by minimizing point-wise difference



Stability?

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Defuzzification by feature distance minimization

Defuzzification formulated as an optimization problem

Definition: An optimal defuzzification of a fuzzy set A is a crisp set C (on the reference set X) such that the distance between A and C is minimal:

$$\mathcal{D}(A) \in \{C \in \mathcal{P}(X) \mid d(A,C) = \min_{B \in \mathcal{P}(X)} [d(A,B)]\} .$$

Find the crisp set closest to the given fuzzy set.
- Which distance to use?
- How to perform the optimization?

Feature distance

- Extract both local and global "features" of the sets.
- Make a vector representation of each set using the extracted features.
- Measure the distance between two sets in this high dimensional feature space.

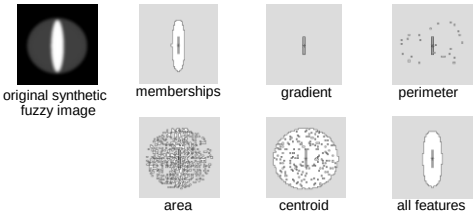
The **feature distance** between sets *A* and *B*, with feature representations Φ(*A*) and Φ(*B*), is

$$d^{\Phi}(A,B) = d(\Phi(A), \Phi(B)) .$$

We use the Minkowski distance, with *p=1*, as distance measure in the feature space.

$$d_p(\mathbf{x},\mathbf{y}) = \left(\sum_{i=1}^{n} |x_i - y_i|^p \right)^{1/p}$$

Examples of defuzzification based on different features



A combination of many features provides a good defuzzification result.

Minimizing the distance

- An analytic solution is, in general, not know.
- 2^n configurations – exhaustive search is not an option.
- Some heuristics must be applied.

Simulated annealing has shown to provide a good trade off between speed and performance.

The optimal *α*-cut provides a fast alternative (and a good starting point for the simulated annealing).

Defuzzification by feature distance minimization

Original | Fuzzy segmented | α-cut at α=0.5 | Best α-cut (α=0.46) | Simulated annealing

	Perimeter	Area	Centroid	Distance
Fuzzy object	91.46	182.78	(13.66,21.08)	0
α-cut at α=0.5	92.75	176.00	(13.57,21.18)	0.1271
α-cut at optimal α	93.30	182.00	(13.68,21.10)	0.1177
Sim. annealing	91.48	182.00	(13.65,21.08)	0.1053

Improved stability from global features

μ=0.2 μ=0.4 μ=0.6 μ=0.8 μ=1

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Membership based defuzzification

Stability?

Improved stability from global features

μ=0.2 μ=0.4 μ=0.6 μ=0.8 μ=1

Feature based defuzzification (simulated annealing)

Better stability

Additional constraints in the defuzzification

μ=0.2 μ=0.4 μ=0.6 μ=0.8 μ=1

Feature based defuzzification (simulated annealing)

With topological constraints

Defuzzification at increased spatial resolution

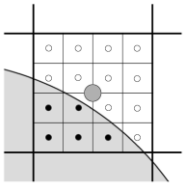
- Features of an original crisp continuous object can be estimated with a higher precision from a fuzzy representation, than from a crisp representation at the same spatial resolution.
- We use that information to find a crisp representative at an increased spatial resolution.

Distance between sets at different resolution

De-couple the feature representation from the image resolution.

- Global features
  - By proper rescaling of the feature values.
- Local features
  - By comparing individual pixels of the lower resolution image, with blocks of pixels of the higher resolution image.

Membership value – area of a block of pixels

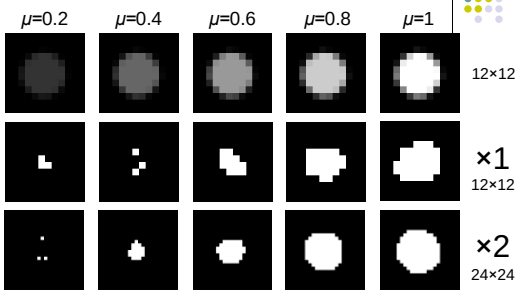


Fuzzy membership is (approximately) 5/16.

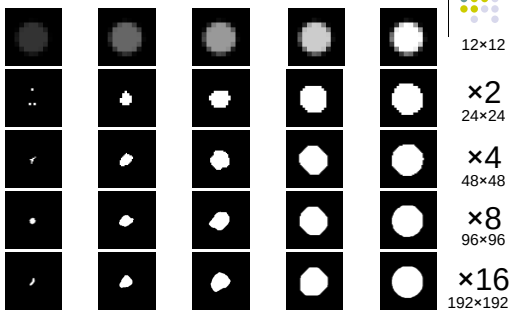
Area of the block of (crisp) subpixels is 5 × 1/16

The membership value of a fuzzy pixel corresponds to the area of a block of $r \times r$ crisp sub-pixels.

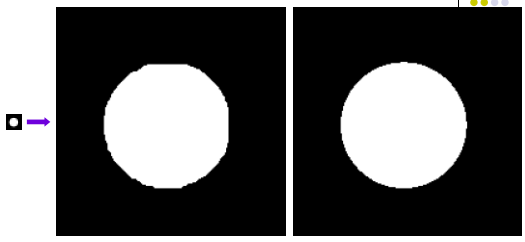
High resolution defuzzification



High resolution defuzzification



High resolution defuzzification



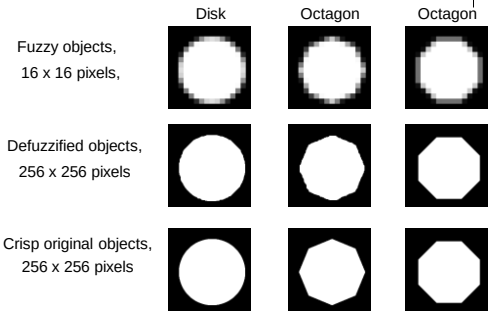
Fuzzy disk
12 ×12 pixels
8 bits per pixel
=1152 bits

Defuzzified set
192 ×192 pixels

32 times more information!

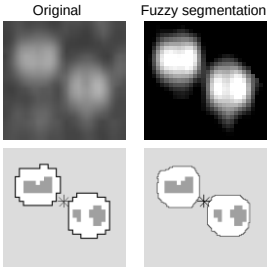
Crisp disk
192 ×192 pixels
1bit per pixel
=36 864bits

High resolution reconstruction disk vs. octagon



Realistic example: Blood vessels

A fuzzy segmentation, and defuzzifications, of a slice in a 3D MRA of a human aorta, where it separates into the two iliac arteries.

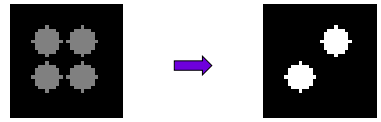


Defuzzification at the same spatial resolution

Defuzzification at 4 times increased spatial resolution

Motivation for a scale space approach

- Example with four discrete fuzzy disks



Synthetic fuzzy disks with membership values 0.5

Optimal defuzzification based on (local and global) feature distance minimization

Perimeter, area and centroid are all perfectly matched!

The method can "transport" area from one part of the image to another.

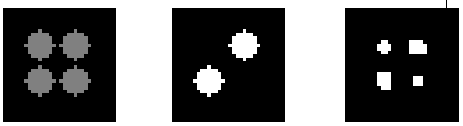
Meso-scale features

- Meso-scale is scale in-between the local (pixel-size) and the global (the whole object) scale.
- Characteristics of a fuzzy set often have different importance at different scales.
- Our first choice of feature to observe at meso-scale is area (volume in 3D case).

Feature vector representation for scale space defuzzification

- Features included in the vector representation of a 2D fuzzy set are
  - Area, at a number of scales (including pixel-size, and global)
  - Perimeter
  - Centroid
- Resolution pyramid representation of sets is utilized for storing areas of blocks of pixels.
- Comparison of meso-scale areas reduces to a comparison of the corresponding pixels in the sets at corresponding levels of the resolution pyramid.
- All measures are appropriately rescaled.

Defuzzification of 4 circles



Four fuzzy disks

A. Defuzzification based on local and global feaures

B. Defuzzification using scale space approach

	Perimeter	Area	Centroid	Membership	Meso-scale	Dist 1	Dist 2
A.	0.0000	0.0000	0.0000	0.0957	0.3828	0.0957	0.4785
B.	0.0015	0.0381	0.0000	0.0957	0.1758	0.1353	0.3111

Contribution of different features to the distance measure.
Dist1 without meso-scale, Dist2 with meso-scale features.

Scale space defuzzification of 3D fuzzy sets

- Defuzzification method for 2D fuzzy sets is straightforwardly generalized to the 3D case.
- The features included in the 3D defuzzification are local, meso-scale, and global volumes, obtained by iterative grouping of blocks of 2 x 2 x 2 voxels, together with surface area and centroid.
- Larger search space puts additional demands on the efficiency of the optimization method.
  - Simulated annealing with re-annealing is a reasonable option, if started from an optimal α-cut.

3D Example - image of a bone implant

Slice through the 3D image

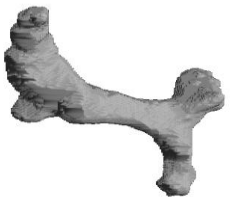
Slice through the 3D fuzzy segmentation

Slice through the 3D defuzzification

Rendering of the optimal α-cut defuzzification

Rendering of defuzzification (simulated annealing) at four times the original resolution

3D rendering of the optimal defuzzification

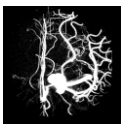


Instead of a conclusion…

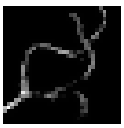
A better solution to a crisp problem is found by looking first in a larger space of fuzzy sets

which has different (usually less) constraints and therefore allows the algorithms more freedom and reduces errors caused by forced crisp answers at intermediate steps.

3D Example – X-ray scan of the arteries of the right half of a human head

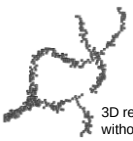


MIP through the image volume



MIP through a selected part of the image

3D rendering of defuzzification using under-estimated surface area.



3D rendering of defuzzification without meso-scale volume

3D rendering of defuzzification with meso-scale volume

3D rendering of a defuzzification using only volume at all scales