Pixel coverage segmentation for improved feature estimation

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Introduction

- The task of **Image Analysis** is to **extract relevant information from images**.
- **Numerical descriptors**, such as area, perimeter, and moments of the objects are often of interest, for the tasks of shape analysis, classification, etc.

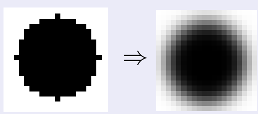
The standard image analysis task (and its solution)

1. Sample preparation and Imaging
2. Pre-processing (optional)
3. Segmentation
   - Usually crisp
4. Feature extraction
   - Discrete representation problematic
5. Classification

Instead of crisp

Going fuzzy

- Instead of representing an object using a crisp set of pixels (a binary mask), assign a graded (fuzzy) belongingness to the pixels.
- Estimate features directly from the fuzzy representation.

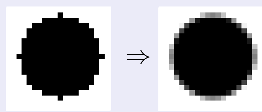


- Often a more robust approach. Good at handling noise etc.
- Membership can represent anything, a lot of freedom.
- Features of fuzzy sets are empirically good (precision, accuracy, stability), but what about theory?
- What do features values actually mean? How to interpret area and perimeter of a fuzzy set?

Too much freedom?

Coverage representation – constrained fuzziness

- Restrict to one specific meaning of memberships.
- Restrict to crisp imaged objects (no clouds or flames).



Pixel coverage digitization

Let the value of a pixel/voxel be equal to the part of it being covered by the object.

- Keep the good sides of fuzzy (robust, precise features, information rich representation).
- Enable strong theoretical results.
- Clear meaning of feature values – features describe the continuous crisp object.

Pixel coverage digitization

Instead of point sampling (Gauss centre point digitization), use area/volume sampling.

Definition

For a given continuous object $S \subset \mathbb{R}^2$, inscribed into an integer grid with pixels $p_{(i,j)}$, the *pixel coverage digitization* of S is

$$\mathcal{D}(S) = \quad (i,j), \frac{A(p_{(i,j)} \cap S)}{A(p_{(i,j)})} \quad (i,j) \in \mathbb{Z}^2 \quad ,$$

where $A(X)$ denotes the area of a set X.

- A useful and practical representation that stays close to the original image data.
- Only based on weak assumptions about the imaged objects.
- Utilizing the coverage information, **significant improvement in precision of extracted feature values** can be reached.

Some features that benefit from a pixel coverage representation.

Area and other geometric moments

- N. Sladoje and J. Lindblad. Estimation of Moments of Digitized Objects with Fuzzy Borders. ICIAP'05, LNCS-3617, pp. 188-195, Cagliari, Italy, Sept. 2005.

$$m_{p,q}(S) = \frac{1}{r^{p+q+2}}\,\tilde{m}(rS) + \mathcal{O}\left(\frac{1}{r\sqrt{n}}\right)$$

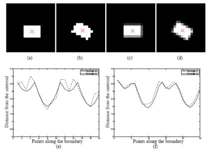
Perimeter and boundary length

- N. Sladoje and J. Lindblad. High Precision Boundary Length Estimation by Utilizing Gray-Level Information. IEEE Trans. on PAMI, Vol. 31, No. 2, pp. 357-363, 2009.

$$\gamma_n^{(0,q)} = \frac{2q}{q + \sqrt{(\sqrt{n^2+q^2} - n)^2 + q^2}}\,, \quad |\varepsilon_n| = \mathcal{O}(n^{-2})$$

Signature

- J. Chanussot, I. Nyström and N. Sladoje, Shape signatures of fuzzy star-shaped sets based on distance from the centroid, Pattern Recognition Letters, vol. 26(6), pp. 735-746, 2005.



Pixel coverage representations

We have nice theory ☺

Application = Real (noisy) data

How to go from image to pixel coverage representation?

Pixel coverage segmentation

Pixel coverage segmentation

Four segmentation methods that provide (approximate) pixel coverage images:

1. Direct assignment of pixel coverage values from a continuous segmentation model.
   - A. Tanács, C. Domokos, N. Sladoje, J. Lindblad, and Z. Kato. Recovering affine deformations of fuzzy shapes. SCIA, LNCS-5575, pp. 735–744, 2009.
2. A method based on mathematical morphology and a dual thresholding scheme.
   - N. Sladoje and J. Lindblad. High Precision Boundary Length Estimation by Utilizing Gray-Level Information. IEEE Trans. on PAMI, Vol. 31, No. 2, pp. 357–363, 2009.
3. A graph based approach for assigning partial belongingness.
   - F. Malmberg, J. Lindblad, and I. Nyström. Sub-pixel Segmentation with the Image Foresting Transform. IWCIA, LNCS-5852, 2009.
4. A method providing local classification, extending any existing crisp segmentation.
   - N. Sladoje and J. Lindblad. Pixel coverage segmentation for improved feature estimation. ICIAP, LNCS-5716, pp. 929–938, 2009.

Some background

We are not the first ones to work with mixed/partially covered image elements.

- "Mixed pixels" - satellite imaging
- "Partial volume effects" - tomographic imaging
- Fuzzy segmentation techniques
- The presented pixel coverage model assumes **crisp** objects.
  - The membership of a pixel has a precisely defined meaning.

Pixel coverage segmentation for improved feature estimation
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Pixel coverage segmentation

Definition (pixel coverage segmentation)

A *pixel coverage segmentation* of an image I into m components $c_k, k \in \{1, 2, \ldots, m\}$ is

$$\mathcal{S}(I) = \left\{ \left((i,j), \alpha_{(i,j)} \right) \,\middle|\, (i,j) \in I_D \right\} ,$$

where

$$\alpha_{(i,j)} = (\alpha_1, \ldots, \alpha_m) , \quad \sum_{k=1}^{m} \alpha_k = 1 , \quad \alpha_k = \frac{A(p_{(i,j)} \cap S_k)}{A(p_{(i,j)})} ,$$

and where $S_k \in \mathbb{R}^2$ is the extent of the component c_k and $I_D \subseteq \mathbb{Z}^2$ is the image domain.

The continuous sets S_k are not known, and
the values α_k have to be estimated from the image.

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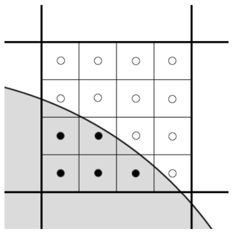
1. Use of a continuous segmentation model

From a continuous (crisp) representation it is, in general, straightforward to compute pixel coverage values, either analytically or numerically, e.g. based on supersampling.



Approximate coverages: $\alpha_1 = \frac{5}{16}, \alpha_2 = \frac{11}{16}$

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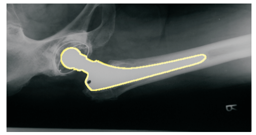
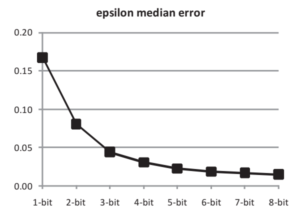
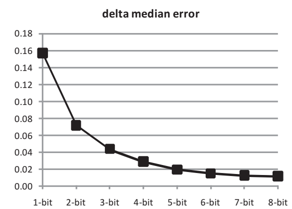
Application 1

Affine registration of digital X-ray images of hip-prosthesis implants for follow up examinations

Segmentation using active contours, modified to provide pixel coverage values utilized for improved moments' estimation in the registration process.







Evaluation of the registration results of 2000 synthetic images using different quantization levels of the fuzzy boundaries.

2. Image intensities + mathematical morphology

In many imaging situation, acquired pixel intensities correspond almost directly to pixel coverage values.

For example: Integration of photons over finite sized sensor elements, such as those of a digital camera.

- A reasonable model for low resolution images, where resolution is decided based on limited means for handling of the data, rather than the optical system. This is often the case for low-resolution video, but also for e.g. CT volumes.

However, noise may provide unreliable measurements. Appropriate pre-processing is recommended.

2. Image intensities + mathematical morphology

Properties of pixel coverage images

- The pixel coverage digitization leads to images where objects have grey edges which are never more than **one pixel thick** (if sampled at high enough resolution).
- The theoretical results of the perimeter estimation method relies on such thin grey boundaries.
- However, it is rarely the case that objects in real images exhibit such thin boundaries.

2. Image intensities + mathematical morphology

Grey-scale morphology and double thresholding.

Find foreground and a background thresholds, f and b such that the pixels with values $\in [b, f]$ form a one pixel thick separating band and the contrast between foreground and background, i.e., $f - b$ is maximal.

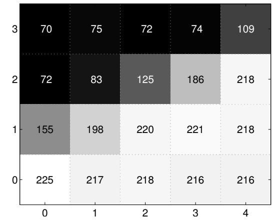
Input: A grey-scale image I.
Output: An approximates pixel coverage representation J with n positive grey-levels.

$b = 0; f = 0$
for each grey-level b'
  $F' = \{p \mid [\varepsilon I](p) > b'\}$ /* Foreground */
  if $F' \neq \emptyset$
    $f' = \min_{p \in F'} [\varepsilon \delta I](p)$
    if $f' - b' > f - b$ /* Better than previous */
      $f = f'; b = b'$
    endif
  endif
endfor

$n = f - b$

$$J(p) = \begin{cases} 0 & , \quad [\delta \varepsilon I](p) \le b, \\ 1 & , \quad [\varepsilon \delta I](p) \ge f, \\ \frac{I(p) - b}{n} & , \quad \text{otherwise.} \end{cases}$$

Evaluation

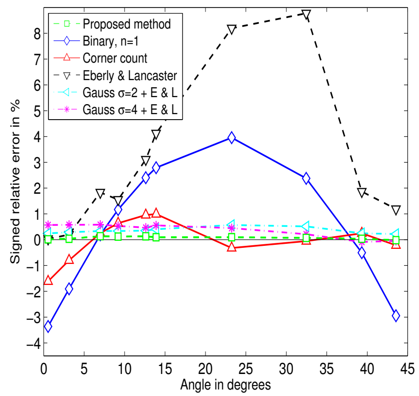


Photos of the straight edge of a white paper on a black background at a number of angles.

Observed noise is between 20 and 50 grey-levels, out of 255.

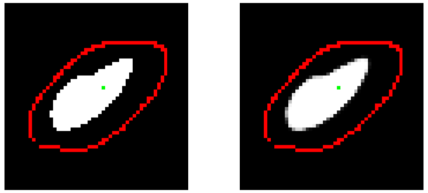
Observed maximal errors for boundary length estimation:

- Proposed method **0.14%**;
- Binary 3.95%;
- Corner count 1.61%;
- Eberly & Lancaster 8.78%;
- Gauss $\sigma = 2$ + E & L 0.57%;
- Gauss $\sigma = 4$ + E & L 0.58%.



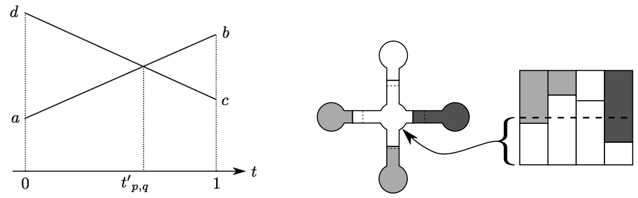
3. Sub-pixel Segmentation with the Image Foresting Transform

- The Image Foresting Transform (IFT) is a framework for supervised and often interactive segmentation.
- Given a set of seed-points with user-defined labels, the IFT completes the labelling by computing minimal cost paths from all image elements to the seed-points.
- Each image element is given the (crisp) label of its closest seed-point.
- We propose a modified version of the IFT that allow mixed labels at region boundaries.



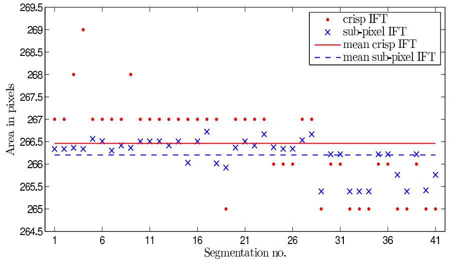
3. Sub-pixel Segmentation with the Image Foresting Transform

1. Compute the standard (crisp) IFT.
2. For each edge connecting two nodes (voxels) of different label, compute the intersection point.
3. Assign mixed labels to the nodes by integrating the labels of its associated edges.



3. Sub-pixel Segmentation with the Image Foresting Transform

The proposed sub-pixel segmentation provides **much more stable feature estimates** than the original IFT.



4. Un-mixing based on local classification

Instead of inventing a pixel coverage version of every existing segmentation method, we wanted a more generic approach.

Given an existing crisp segmentation, augment it to a pixel coverage segmentation.

4. Un-mixing based on local classification

Assumption

Partial pixel coverage exist only at the object boundaries of the existing crisp segmentation.

Approach

Re-assign class belongingness to the boundary pixels based on a local classification using the surrounding non-boundary pixels.

To obtain a pixel coverage segmentation, we propose a method composed of the following four steps:

1. Application of a crisp segmentation method, appropriately chosen for the particular task
2. Selection of pixels to be assigned partial coverage
3. Application of a liner mixture model for “de-mixing” of partially covered pixels and assignment of pixel coverage values
4. Ordered thinning of the set of partly covered pixel to provide one pixel thin 4-connected regions of mixed pixels

Steps 1 and 2.

1. **Any** crisp segmentation model.
   - For the example to come, we used Linear Discriminant Analysis in combination with Iterative Relative Fuzzy Connectedness[1]

2. **Selection of pixels to re-evaluate**
   - All pixel which are 4-connected to a pixel with a different label.

[1] J. Lindblad, N. Sladoje, V. Ćurić, H. Sarve, C.B. Johansson, and G. Borgefors. Improved quantification of bone remodelling by utilizing fuzzy based segmentation. SCIA 2009

3. Computation of partial pixel coverage values

3.1 Estimate the spectral properties c_k of the pure classes locally.

- The mean values of the (assumed) completely covered pixels in a local Gaussian neighbourhood.

3.2 Compute the mixture proportions α_k of the pixels selected in step 2.

- The intensity values of a mixed pixel $p = (p_1, p_2, \dots, p_n)$ (n is number of channels of the image) are assumed, in a noise-free environment, to be a convex combination of the pure classes c_k:

$$p = \sum_{k=1}^{m} \alpha_k c_k \,, \quad \sum_{i=k}^{m} \alpha_k = 1 \,, \quad \alpha_k \geq 0 \,. \tag{1}$$

  where each coefficient α_k corresponds to the coverage of the pixel p by an object of a class c_k.

3. Computation of partial pixel coverage values

In the presence of noise, it is not certain that there exists a (convex) solution to the linear system (1). Therefore we reformulate the problem:

Find a point p^* of the form $p^* = \sum_{k=1}^{m} \alpha_k^* c_k$, such that p^* is a *convex* combination of c_k and the distance $d(p, p^*)$ is minimal.

We solve the constrained optimization problem by using Lagrange multipliers, and minimize the function

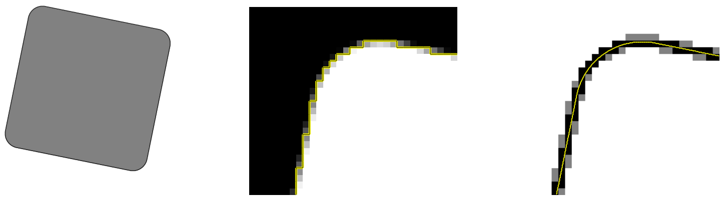
$$F(\alpha_1, \dots, \alpha_m, \lambda) = \|p - \sum_{k=1}^{m} \alpha_k c_k\|_2^2 + \lambda(\sum_{k=1}^{m} \alpha_k - 1)$$

over all $\alpha_k \geq 0$. This leads to a least squares type of computation.

The obtained solution provides estimated partial coverage of the pixel p by each of the observed classes c_k.

4. Ordered thinning

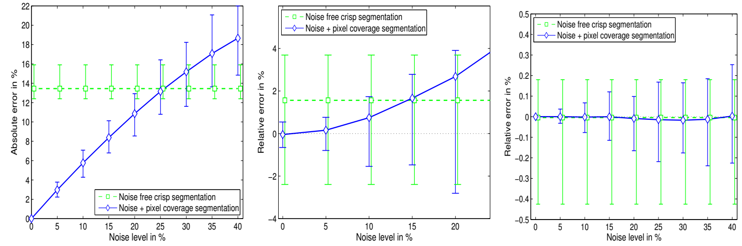
To ensure one pixel thick boundaries, the “least” mixed pixels are one at a time assigned to their most prominent class, until only one pixel thick mixed boundaries remain.



(a) Test object (b) Part of pixel coverage segm. (c) Part of re-evaluated set

Evaluation

How does this work in a noisy environment?

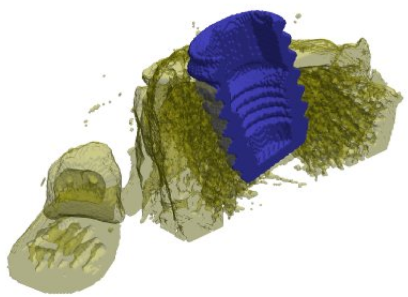


(a) Coverage values (b) Perimeter estimate (c) Area estimate

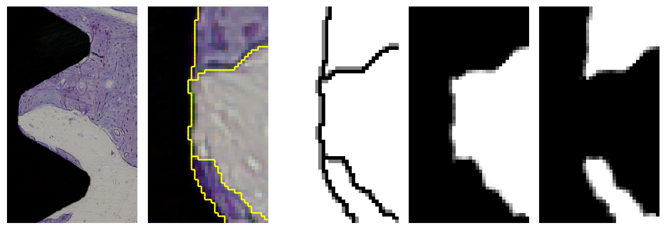
Estimation errors for increasing levels of noise. Green is noise free crisp reference. Bars represent max and min.

Application 2

Measure bone implant integration for the purpose of evaluating new surface coatings which are stimulating bone regrowth around the implant.



Application 2



(a) (b) (c) (d) (e)

Figure: (a): The screw-shaped implant (black), bone (purple) and soft tissue (light grey). (b) Part of a crisp (manual) segmentation of (a). (c) The set of re-evaluated pixels. (d) and (e) Pixel coverage segmentations of the soft tissue and the bone region, respectively.

Result:
Approximately a **30% reduction of errors** as compared to when using estimates from the crisp starting segmentation.

Summary

- Pixel coverage representations are shown to be superior to crisp image object representations for many reasons.
- By suitably utilizing information available in images it is possible to perform **pixel coverage segmentations**.
- We observe that for reasonable noise levels, the achieved pixel coverage representation provides a more accurate representation of image objects than a perfect, noise free, crisp representation.

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Thanks to the people involved

- Dr. Nataša Sladoje
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