Fuzzy Sets and Fuzzy Techniques

Lecture 11 – Fuzzy control

Joakim Lindblad
joakim@cb.uu.se

Centre for Image Analysis
Uppsala University

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Fuzzy Sets and Fuzzy Techniques
Joakim Lindblad
Outline
What is a control system?
How do conventional control systems work?
Fuzzy control
An example
Fuzzy expert systems

Topics of today

- Fuzzy control
  - What is a control system?
  - How do conventional control systems work?
  - Fuzzy control systems
    - Five steps...
  - A small example
- Fuzzy expert systems

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Fuzzy logic control

Fuzzy logic control (FLC) may be viewed as a branch of intelligent control which serves as an emulator of human decision-making behaviour that is approximate rather than exact (C.C.Lee in Singh: Systems and Control Encyclopedia, 1992)

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Outline
What is a control system?
Taking a shower...
How do conventional control systems work?
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Fuzzy expert systems

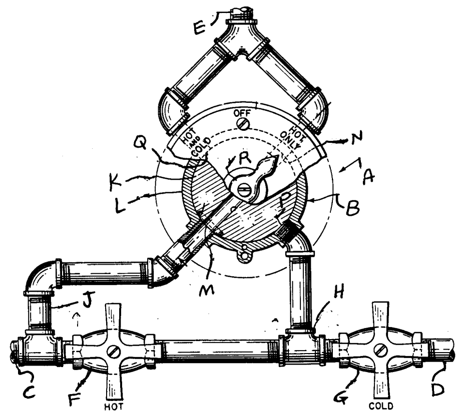
What is a control system?

A small example

Taking a shower. . .

Taking a shower. . .

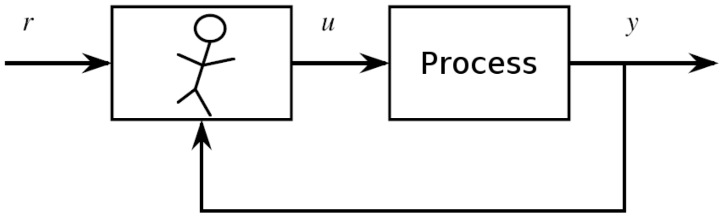
Facing this device. . .



How to adjust the temperature so that it is “lagom”?

Taking a shower. . .

A control system



Taking a shower. . .

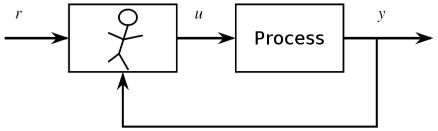
Feedback Control Loops

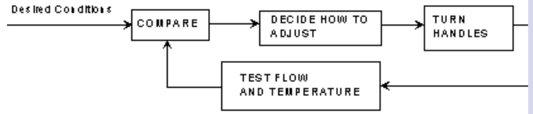
Imagine taking a shower from a two-knob faucet. You want to set the rate of water flow and its temperature so that the shower is effective and comfortable. You can control the hot water flow and the cold water flow separately. Throughout your shower, there may be ”disturbances” in the form of changes in water supply temperature and pressure.

So how do you regulate your shower?

1. You test the flow and temperature
2. Decide if it is OK
3. If not OK, decide what adjustment to make
4. Adjust the knobs
5. Repeat from the first step

Taking a shower. . .





Taking a shower. . .

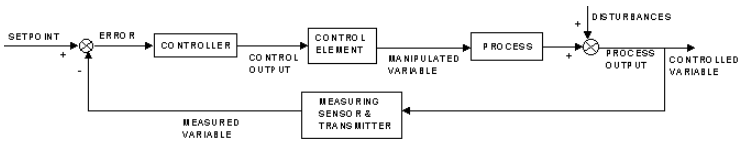
This is the structure of a basic **feedback control loop**. Information from the process (water flow and temperature) is "fed back" by a measurement device (you) through a controller (you again) to a control element (the knobs) to change the process input.

All control systems have these same basic components:

- measurements of variables affecting the system
- a specified desired value or range of values for the controlled variable (the setpoint).
- a control calculation or algorithm
- a way of adjusting the system to reflect the results of the control calculation (the control element).

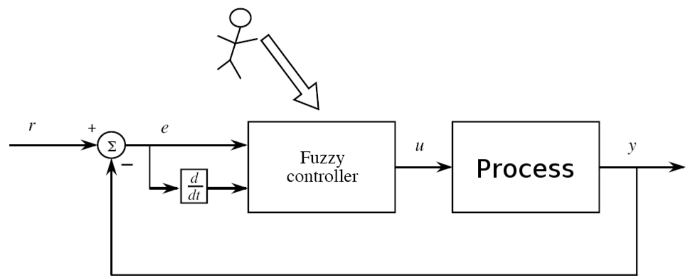
Taking a shower. . .

The system can be represented by a block diagram where lines are used to represent variables or signals and boxes for actions.



Taking a shower. . .

Stepping out of the control loop. . .



Taking a shower...
Terminology

Steady State: A steady state system does not change with time. Mathematically, this means the time derivatives in the balance equations (the accumulation terms) are zero. Often, systems will reach steady state if given a long time to settle – usually, real systems don't get the time. This leads to another mathematical approximation – steady state is the behavior of the system as time approaches infinity. Some people use the words static or stationary as synonyms for steady state.

Dynamic (or transient) systems are time dependent. All real systems are dynamic; this makes process control necessary. Dynamic systems must be **modeled using differential equations**, unlike steady state systems where algebraic systems will suffice.

Taking a shower...
Terminology

Inputs and Outputs are not necessarily material flows. An input is a variable that causes an output to change. Both inputs and outputs may be measurable or they may not. **Disturbances** are inputs that cannot be adjusted, and often they are not measurable.

Error is the difference between the measured behavior of a process output and its desired behavior or setpoint. **Never forget** that the measured values of the outputs are only representations of the real values, and may be **limited in accuracy**.

Taking a shower...
Terminology

Feedback Control: information from an output of a system is used to adjust a manipulator to change an input to the system to try and compensate for disturbances after they have changed the system.

Negative feedback reduces the difference between the actual and desired values, so it is beneficial. Positive feedback increases the difference, so it is undesired.

One of the most important things we will be watching is the **stability** of the system. The error of an unstable system becomes larger and larger (unbounded) with time, often leading to undesirable consequences.

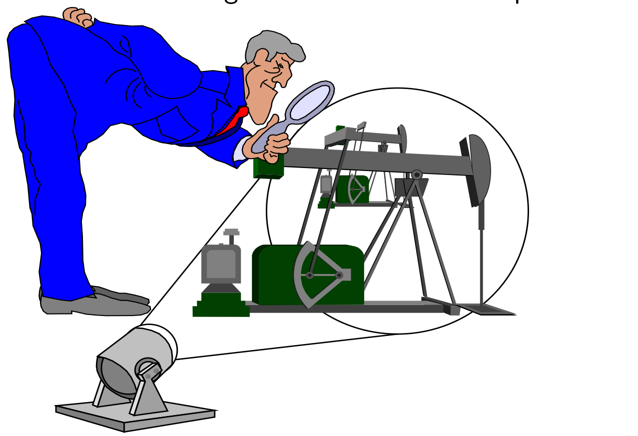
How do conventional control systems work?

Conventional control systems

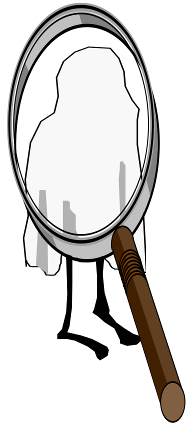
- Linear
  - easy, well explored, not too flexible.
- Non-linear
  - powerful but often sensitive to modelling errors.

Model building / system identification

Hunting for those differential equations. . .



Model building / system identification



Undiscovered variables/states leads to. . .

- Wrong actions
- Wrong results
- Annoying
- Unpredictable
- Dangerous!

Sticking with linear simple systems. . .
sacrificing performance and applicability.

Control System Design

Control System Design

When an engineer sets out to design a control system, the steps are:

1. determine control objectives
2. identify measurable variables, available manipulators
3. pair variables (choose controller structure)
4. select controller algorithms
5. tune controller (adjust sensitivity)

Controller Algorithms

Controller Algorithms

An automatic controller compares a process measurement to a desired "setpoint" value and produces a "controller output" in response.

The first step in most algorithms is a "comparator" which calculates an "error": The error is then used to calculate a compensating change in the controller output, which when transmitted to the control element changes the manipulator to adjust the process.

The goal of the controller is thus to **reduce the error to zero** in an appropriate fashion.

Controller Algorithms

Manual Control

Sometimes an operator will choose to adjust a manipulator based on intuition, experience, etc. Such an approach is inconsistent, and not practical in a situation where there are many control loops.

On-Off Control

The simplest control algorithm is on-off control. This approach is employed by most home thermostats. On/Off Algorithm

- "Bang-bang" action – not smooth
- Usually need to design a "dead band" to prevent chattering

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Controller Algorithms

Proportional Control (P-only Control)

On-off control takes exactly the same action for small errors as it does for huge ones. The resulting cycling might be reduced if the controller was able to make big changes when error is big, but small changes when error is small.

$$\text{Output} = \text{bias} + K_P\varepsilon$$

- Controllers are "tuned" by picking K_P to produce a desired output response. The primary effect of increasing K_P is faster response.

A P-only controller won't necessarily drive a process back to perfect setpoint. The permanent error which results is called "offset".

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Integral Control (Reset)

In many cases, would like to completely eliminate offset, so need a way to penalize persistent errors.

$$\text{Output} = \text{bias} + K_I \int_0^t \varepsilon \, dt$$

- Eliminates offset, since the integral will exist whenever there is error. Integral action is necessary to eliminate offset and any other steady state deviation from setpoint.
- Usually combined with proportional action to make a PI controller. "I only" will be very slow, since it won't act until the error has begun to accumulate.
- Integral action makes a loop more oscillatory

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Controller Algorithms

Derivative Control (Rate)

PI control is effected only by the magnitude and persistence of the error. There are times when it would be nice if could move the output faster when error is changing rapidly.

$$\text{Output} = \text{bias} + K_D \frac{d\varepsilon}{dt}$$

- A "D only" controller would be silly. It wouldn't be doing anything when the error wasn't changing.
- Derivative action is not used too much. One reason is that it tends to amplify signal noise, and so is only used when it is important to reduce oscillation.
- Should not be used if large, rapid changes in manipulators are bad.
- A sudden change in setpoint, and hence in error, can make the derivative very large.

Controller Algorithms

PID Control (Proportional-Integral-Derivative, Three Mode)

The PID controller is the workhorse of the process industries.

$$\text{Output} = \text{bias} + K_P\varepsilon + K_I\int_0^t \varepsilon\, dt + K_D\frac{d\varepsilon}{dt}$$

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Problems of crisp control...

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True love

Wife: Do you love me?

Husband (Boolean logician): Yes.

Wife: How much?

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Logic

Wife: Do you like my girlfriend?

Husband: Very much.

Wife: Then you don't love me.

Learning

Teacher (authoritative): All I say is true.

Student: OK.

Teacher (later): You shouldn't believe all I say.

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Exact truths

Precision is not truth.

- Henri Matisse

So far as the laws of mathematics refer to reality, they are not certain. And so far as they are certain, they do not refer to reality.

- Albert Einstein

As complexity rises, precise statements lose meaning and meaningful statements lose precision.

- Lotfi Zadeh

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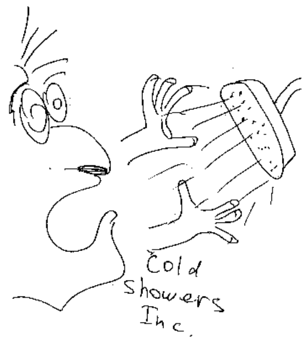
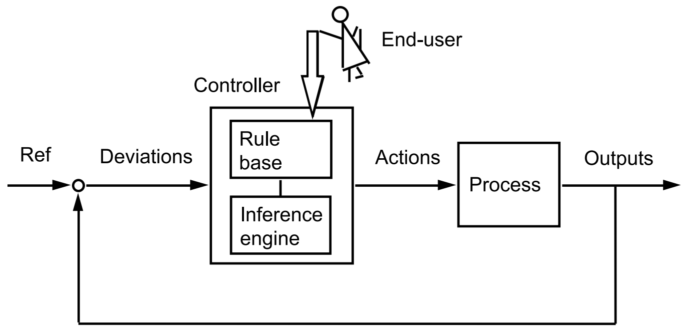
So...

How do we avoid the...

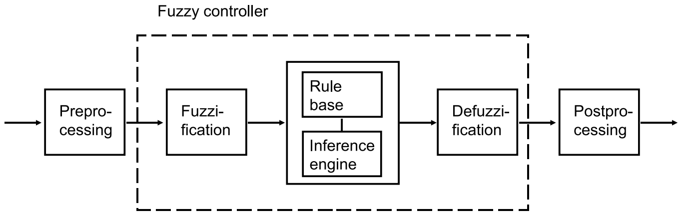


We use a Fuzzy controller! :-)

Elements of a Fuzzy Controller

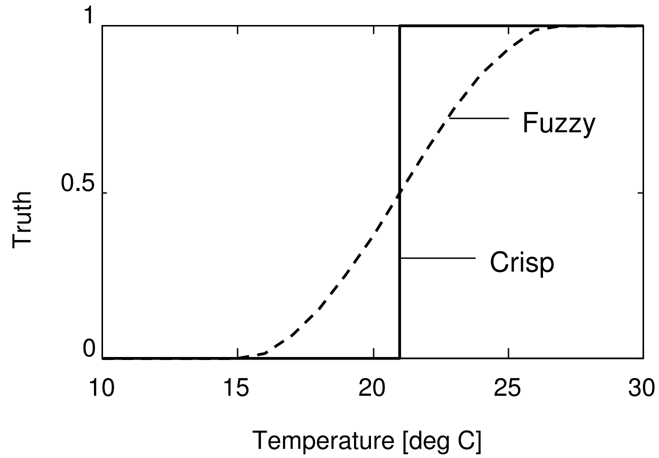


Structure of a Fuzzy Controller



A warm room

in need of cooling(!?)



A warm room

Implication

IF temperature is high

THEN full power

A warm room

Inference

If temperature is high then full power

Temperature is 25 deg C

Power = 95%

Fuzzy logic control

Methodology first developed by Mamdani in 1975 used to control a steam plant. Based on work by Zadeh (1973) on fuzzy algorithms for complex systems and decision processes.

In a manner analogous to conventional control systems, **inputs of a system are mapped to outputs using fuzzy logic rather than differential equations**.

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Motivation

- Can be used for systems that are difficult or impossible to model mathematically.
- Can also be applied to processes that are too complex or nonlinear to be controlled with traditional strategies.
- In fact, a detailed and precise mathematical description is not always necessary for optimized operation of an engineering process.
- Human operators often are capable of managing complex situations of a plant without knowing anything about differential equations.
- Such a rule based system can be used to define a controller that emulates the heuristic rule-of-thumb strategies of an expert.

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Motivation

Useful cases

1. The control processes are too complex to analyze by conventional quantitative techniques.
2. The available sources of information are interpreted qualitatively, inexactly, or uncertainly.

Advantage of Fuzzy logic control

- Flexible
  - Universal approximator
- Easy to understand
  - Powerful – yet simple
- Linguistic control
  - linguistic terms – human knowledge
- Tolerant of imprecision / Robust control
  - more than 1 control rules - an error of a rule is not fatal
  - limited trust in input data
- Parallel or distributed control
  - multiple fuzzy rules - complex nonlinear system

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Motivation

Disadvantages

- More complex than PID
- More parameters to tune (hopefully more intuitive paremeters though)
- Difficult to analyze mathematically (stability?)

Motivation

Problems Attacked

- Nonlinear
- Multivariable
- Operator's rules
- Learning

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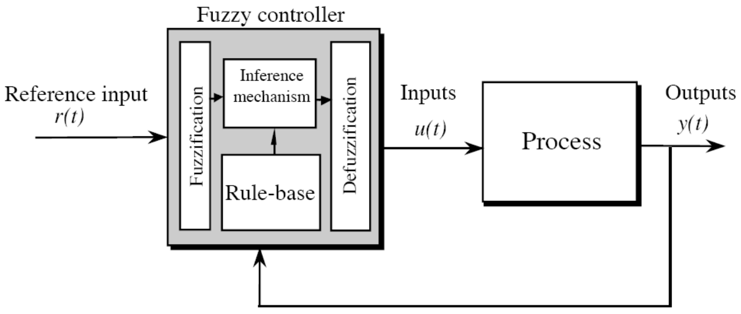

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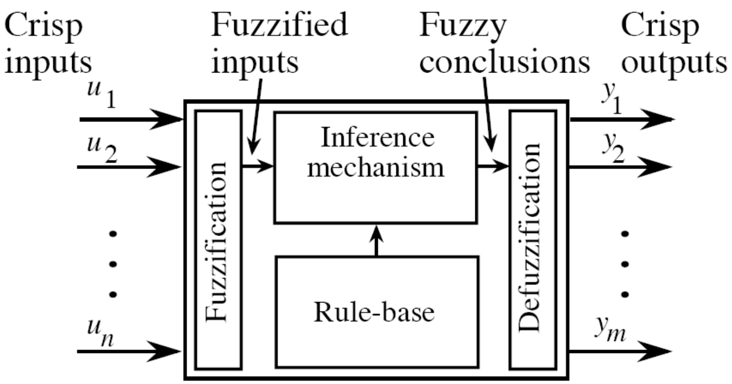

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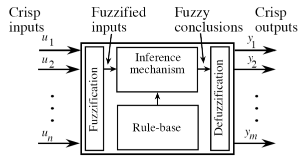

Four main components

1. The fuzzification interface : transforms input crisp values into fuzzy values
2. The knowledge base : contains a knowledge of the application domain and the control goals.
3. The decision-making logic : performs inference for fuzzy control actions
4. The defuzzification interface : provides a crisp control action out

Five steps...

How to build a fuzzy controller in five easy steps. . .

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Step 1.

Partition input and output spaces

Select meaningful **linguistic states** for each variable and express them as appropriate fuzzy sets.

In most cases the fuzzy sets are fuzzy numbers.

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Step 1.

Partition input and output spaces

Example of fuzzy partition with linguistic terms

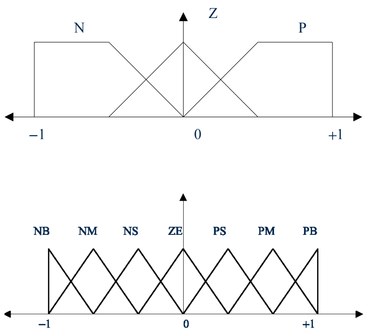

Figure: Fuzzy quantization by triangular fuzzy numbers.

7 linguistic terms are often used

- NB: negative big
- NM: negative medium
- NS: negative small
- ZE: zero
- PS: positive small
- PM: positive medium
- PB: positive big

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Step 1.

Partition input and output spaces

$e \backslash \Delta e$	NB	NM	NS	AZ	PS	PM	PB
NB							
NM							
NS							
AZ							
PS							
PM							
PB							

Figure: Fuzzy partition of input space.

Step 2.
Fuzzification of input

Introduce a **fuzzification function** for each input variable to express the associated measurement uncertainty.

For each measurement $e = x_0$, the fuzzy set $f_e(x_0)$ enters into the inference process (Step 4) as a fact.

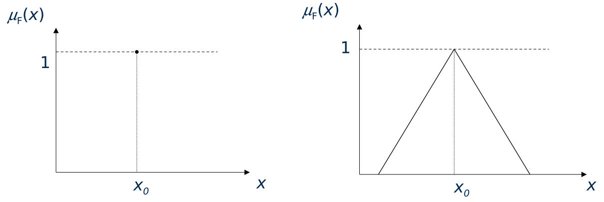

Figure: Fuzzification examples, singleton and triangular.

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Step 3.
Rule base

Formulate a set of **inference rules**.

- Elicit them from experienced human operators.
- Obtain the from empirical data by suitable learning methods.

$$\text{If } \epsilon = A \text{ and } \frac{d\epsilon}{dt} = B, \text{ then } C.$$

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Step 3.
Rule base

$e \backslash \Delta e$	NB	NM	NS	AZ	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	AZ
NM	NB	NB	NB	NM	NS	AZ	PS
NS	NB	NB	NM	NS	AZ	PS	PM
AZ	NB	NM	NS	AZ	PS	PM	PB
PS	NM	NS	AZ	PS	PM	PB	PB
PM	NS	AZ	PS	PM	PB	PB	PB
PB	AZ	PS	PS	PB	PB	PB	PB

Figure: An example of a fuzzy rule base.

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Step 3.
Rule base

4 modes of derivation of fuzzy control rules

- Expert experience and control engineering knowledge:
  - operating manual and questionnaire
- Based on operators' control actions :
  - observation of human controller's actions in terms of input-output operating data
- Based on the fuzzy model of a process :
  - linguistic description of the dynamic characteristics of a process
- Based on learning :
  - ability to modify control rules such as self-organizing controller

Step 4.

Inference

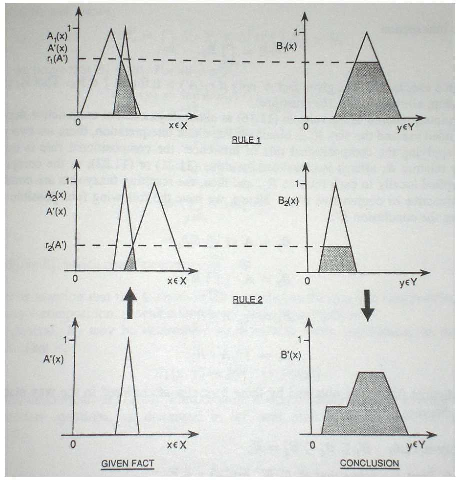
Design an **inference engine**.

Use method of interpolation, explained in Lecture 10.

- minimum operator for a fuzzy implication
- max-min operator for the composition

Multiconditional approximate reasoning

Method of interpolation-Example



An application

Region growing based on fuzzy rule based system

A. Steudel and M.Glesner: "Fuzzy segmented image coding using orthonormal bases and derivative chain coding", *Pattern Recognition*, 32, 1999.

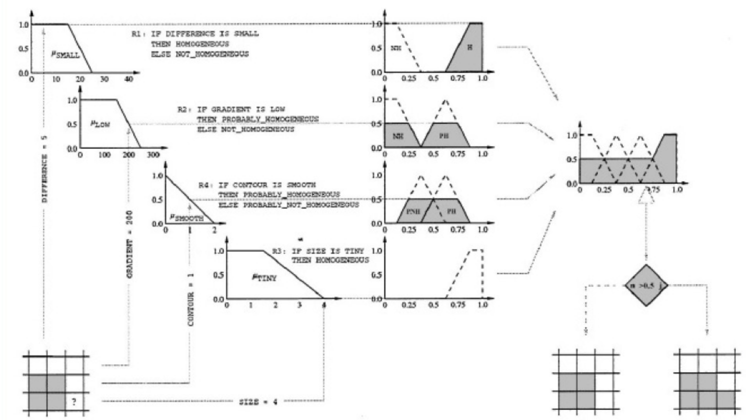


Fig. 7. An example for the evaluation of the fuzzy rule-based homogeneity criterion.

Step 5.

Defuzzification

Select a suitable **defuzzification method**.

- In many practical applications, a control command is given as a crisp value.
- a process to get a non-fuzzy control action that best represents the possibility distribution of an inferred fuzzy control action.
- no systematic procedure for choosing a good defuzzification strategy,
- select one considering the properties of the application case

Step 5.
Defuzzification

Commonly used defuzzification methods for FLC are

- Center of Area/Bisector of Area
- Center of Gravity
- Center of Maxima
- Mean of Maxima

Confusion in the book (p. 336) between COA and COG.

Five steps...
In addition

Scale input data and output data to $[0, 1]$ (normalization).
Apply saturator on the output, limiting the output response.

Overview

1. Determination of state variables and control variables
2. Determination of inference method
3. Determination of fuzzification method
4. Discretization and normalization of state variable space
5. Partition of variable space.
6. Determination of the shapes of fuzzy sets
7. Construction of fuzzy rule base
8. Determination of defuzzification strategy
9. Test and tuning
10. Construction of lookup table

An example

The control problem of Servomotors

- Servomotors are used in many automatic system including drivers for printers, floppy disks, tape recorders, and robot manipulations.
- The servomotor process shows nonlinear properties
- We apply the fuzzy logic control to the motor control.
- The task of the control is to rotate the shaft of the motor to a set point without overshoot.

An example

Design procedure for servomotor control

1) Determination of state variables and control variable
   1. State variables (input variable of FLC):
      - Error : the set point minus the process output (e).
      - Change of error (ce) : the error from the process output minus the error from the last process output.
   2. Control variable(output variable of FLC):
      - Control input (v) : the voltage applied to the process.
2) Determination of inference method
   The Mamdani inference method is selected
3) Determination of fuzzification method
   fuzzy singleton : measure the state variables without uncertainty

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An example

4) Discretization and normalization

   The shaft encoder of the motor has a resolution of 1000.

   The universes of discourse are as follows:
   $-1000 \leq e \leq 1000, \;\; -100 \leq ce \leq 100$

   The servo amplifier has an output range of 30 V and thus the control variables (v) are in the range $-30 \leq v \leq 30$

   We discretize and normalize the input variables in the range [-1, 1].
   The control variable v is normalized in the range [-1, 1].

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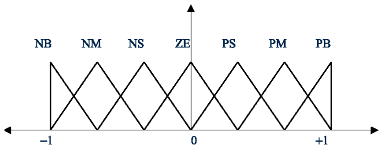
5) Partition of input space and output space
6) Determination of the shapes of fuzzy sets

   We normalize input and output variables to $[0, 1]$.
   We partition space of each input and output variable into seven regions, Each region is associated with a linguistic term as shown below.
   The maximum number of possible fuzzy rules is 49.



Fuzzy Sets and Fuzzy Techniques
Joakim Lindblad
Outline
What is a control system?
How do conventional control systems work?
Fuzzy control
An example
Fuzzy expert systems

An example

7) Construction of fuzzy rules
   We interviewed with an expert of the servomotor control, and we collect knowledge such as:
   "If the error is zero and the error change is positive small, then the control input is negative small"

(1)	If e is PB and ce is any,	then v is PB.
(2)	If e is PM and ce is NB, NM, or NS,	then v is PS.
(3)	If e is ZE and ce is ZE, PS, or PM,	then v is ZE.
(4)	If e is PS and ce is NS, ZE, or PS,	then v is ZE.
(5)	If e is NS and ce is NS, ZE, PS, or PM,	then v is NS.
(6)	If e is NS or ZE and ce is PB,	then v is PS.

An example

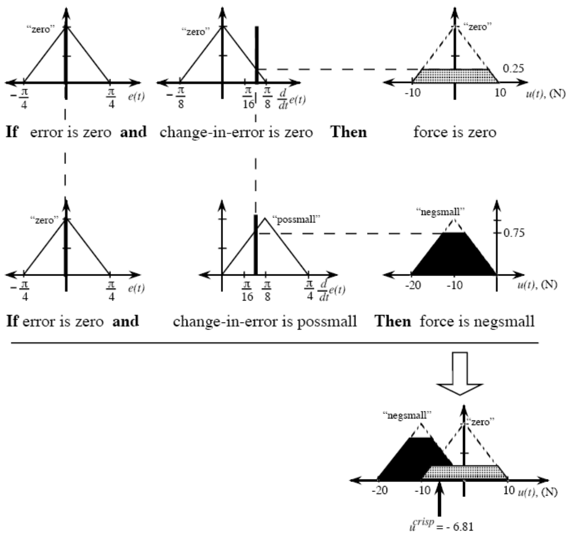
e \ ce	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NB	NB	NB
NM	NM	NM	NM	NB	NB	NB	NB
NS	NM	NM	NS	NS	NS	NS	PS
ZE	NS	NS	NS	ZE	ZE	ZE	PS
PS	NS	NS	ZE	ZE	ZE	PM	PM
PM	PS	PS	PS	PM	PM	PM	PM
PB	PB	PB	PB	PB	PB	PB	PB

A set of rules for controlling a servomotor.

An example

8) Determination of defuzzification strategy
   We take the COA (center of area) method because it is most commonly used.
9) Test and tuning
   We check the performance of the developed controller and refined some fuzzy rules.
10) Construction of lookup table
   For example,
   for c = -0.2 and ce = 0, v is -0.4
   for c = -0.4 and ce = 0.4, v is 0.2
   this lookup table can be used in order to save the inference time and defuzzification time

An example



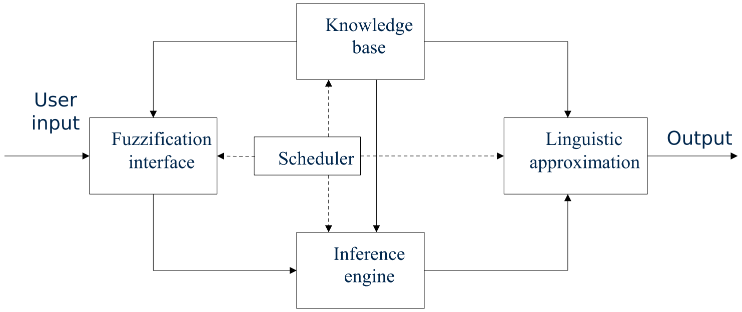
Fuzzy expert systems

Fuzzy expert systems

An expert system, also known as a knowledge based system, is a computer program that contains some of the subject-specific knowledge of one or more human experts. This class of program was first developed by researchers in artificial intelligence during the 1960s and 1970s and applied commercially throughout the 1980s. The most common form of expert systems is a program made up of a set of rules that analyze information (usually supplied by the user of the system) about a specific class of problems, as well as providing mathematical analysis of the problem(s), and, depending upon their design, recommend a course of user action in order to implement corrections. It is a system that utilizes what appear to be reasoning capabilities to reach conclusions.

Fuzzy expert systems

We can model a **fuzzy expert system** in almost the same way as a fuzzy controller.



Fuzzy expert systems

1 fuzzification interface

The fuzzy expert system receive linguistic terms, this module has to have an ability to handle such fuzzy information.

Each rule can have its certainty factor. This certainty factor is used in the aggregation of the results from each rule.

2 Inference engine

The fuzzy expert systems can use the same type of inference methods as the fuzzy logic controller.

Fuzzy expert systems

3 Linguistic approximation

Instead of the defuzzification module, a linguistic approximation module is needed.

Finds a linguistic term which is closest to the obtained fuzzy set.
Use a measuring technique of distance between fuzzy sets.

4 Scheduler

Controls all the processes in the fuzzy expert system.
Determines the rules to be executed and sequence of their executions.
Also provides an explanation function for the result.