The Centre for Image Analysis

Fuzzy Sets and Fuzzy Techniques, 7.5p.
VT 2010

Exam 2010-03-18

Time:	9.00–13.00
Place:	Seminar room CBA
Grading:	*Preliminary*
	18–27 ⇒ G (pass), 28–40 ⇒ VG (pass with distinction)
	18–25 ⇒ 3, 26–31 ⇒ 4, 32–40 ⇒ 5
Misc:	**Only** paper, pen(cil), eraser, calculator, and dictionary are allowed.
	Write your name on all papers. Do not use red ink.
	Please, use figures to complement your answers.
	Answer in Swedish or English.

1 Basics and notions

a) What is the underlying idea of the fuzzy set theory? (1p)

b) What is a fuzzy sets of type two? When can such a fuzzy set be useful? Motivate your answer. (2p)

c) What do the decomposition theorems say? (1p)

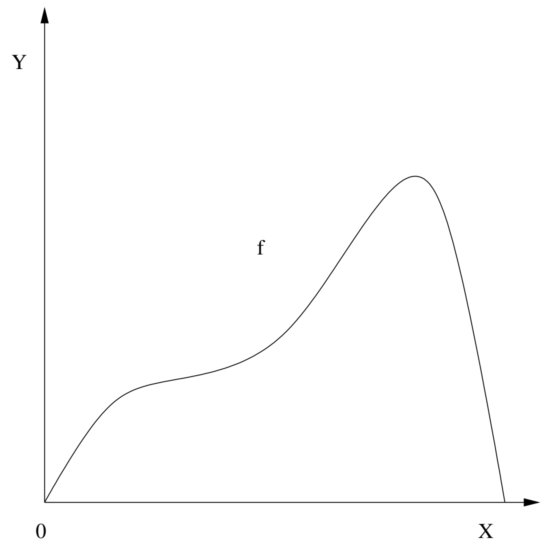
d) When is a property cutworthy? Give one example. (1p)

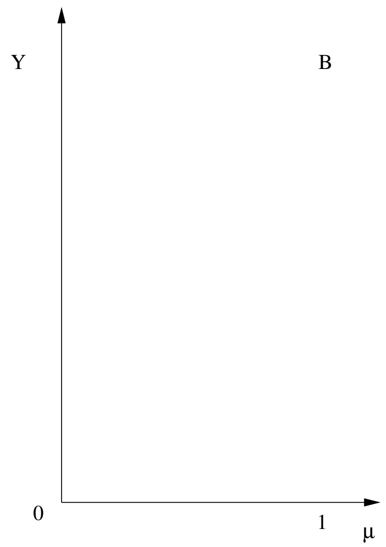
e) Give an equation that is used to measure the fuzziness of a fuzzy set. (1p)

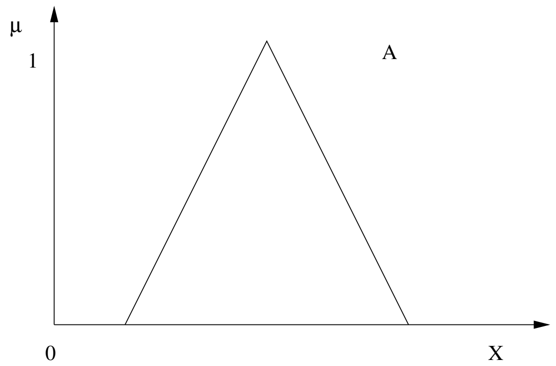
f) What is a linguistic hedge? What is the difference between a weak and a strong linguistic hedge? Give an example of a weak and a strong linguistic hedge. (2p)

2 Extension principle

Let A be a fuzzy sets defined on $\mathbb{R}$, with membership function as shown below, and f a function $f : \mathbb{R} \to \mathbb{R}$, also shown below. Extend f to work on fuzzy sets $f : \mathcal{F}(\mathbb{R}) \to \mathcal{F}(\mathbb{R})$ using the extension principle. Give the definition of $[f(A)](y)$. Also draw (approximately) $B = f(A)$ in the figure below (membership axis pointing right).







Remember to hand in this page together with your solution. (3p)

3 Distances

a) What is the Hamming distance between the following two sets?

$$A(x) = 0.5/(-1) + 0.8/0 + 1/1 + 0.2/2 \quad \text{and}$$
$$B(x) = 0.5/1 + 1/2 + 0.5/3 + 0.8/4$$

(2p)

b) Explain why it is problematic to generalize the Hausdorff set distance to general spatial fuzzy sets. (1p)

c) Bloch and Maître have defined a constrained distance on fuzzy sets that uses the strength of connectedness between two points. What is the used *strength of connectedness* and how is their constrained distance defined? (2p)

4 Operations on fuzzy sets

a) What is the relationship between aggregation operators, t-norms, t-conorms and averaging operators? (2p)

b) How do the standard union and intersection of fuzzy sets fit in the answer given in (a)? (1p)

c) How do Ordered Weighted Averaging operators (OWA) work? Give an example. (2p)

5 Morphology

Give at least two examples how we can define the basic morphological operations Dilation and Erosion for fuzzy sets. (2p)

6 Fuzzy arithmetics

Given the following triangular fuzzy numbers:

$$A(x) = \begin{cases} 0 & \text{for } x \leq 3 \text{ and } x > 5 \\ x - 3 & \text{for } 3 < x \leq 4 \\ 5 - x & \text{for } 4 < x \leq 5 \end{cases} \qquad B(x) = \begin{cases} 0 & \text{for } x \leq 12 \text{ and } x > 32 \\ (x - 12)/8 & \text{for } 12 < x \leq 20 \\ (32 - x)/12 & \text{for } 20 < x \leq 32\ . \end{cases}$$

Solve the following fuzzy equation: $A \cdot X = B$ (3p)

7 Fuzzy logics

a) Assume that $I = \{$ Adam, Bob, Cathy, David, Eve $\}$ is a group of students, and $\mathcal{V}$ is a variable with values in the interval [0,100], that expresses the degree of fluency in English.

Given the quantified proposition p: "About half of the students in I are highly fluent in English."

We define the fuzzy set Q representing the fuzzy quantifier "about half" as

$$Q(x) = \begin{cases} 0 & \text{for } x \leq \frac{1}{4} \text{ and } x > \frac{3}{4} \\ 4(x - \frac{1}{4}) & \text{for } \frac{1}{4} < x \leq \frac{1}{2} \\ 4(\frac{3}{4} - x) & \text{for } \frac{1}{2} < x \leq \frac{3}{4} \end{cases},$$

and the fuzzy set F, representing "High fluency" on [0,100], as

$$F(x) = \begin{cases} 0 & \text{for } x \leq 50 \\ (x - 50)/30 & \text{for } 50 < x \leq 80 \\ 1 & \text{for } x > 80 \end{cases},$$

Assume that the following fluency scores are given: $\mathcal{V}(\text{Adam}) = 35, \mathcal{V}(\text{Bob}) = 20, \mathcal{V}(\text{Cathy}) = 80, \mathcal{V}(\text{David}) = 95, \mathcal{V}(\text{Eve}) = 70$.

What is the truth value of the proposition p? (3p)

b) Let $X = \{x_1, x_2, x_3\}$ and $Y = \{y_1, y_2\}$ be the sets of values of variables $\mathcal{X}, \mathcal{Y}$.
Let implication be defined by $R(x, y) = \mathcal{J}[A(x), B(y)] = \min[1, 1 - A(x) + B(y)]$.

Given the **rule**: If $\mathcal{X}$ is A, then $\mathcal{Y}$ is B,
where $A = 0.5/x_1 + 1/x_2 + 0.2/x_3$ and $B = 1/y_1 + 0.4/y_2$,
and the **fact**: $A' = 0.6/x_1 + 0.9/x_2 + 0.4/x_3$.

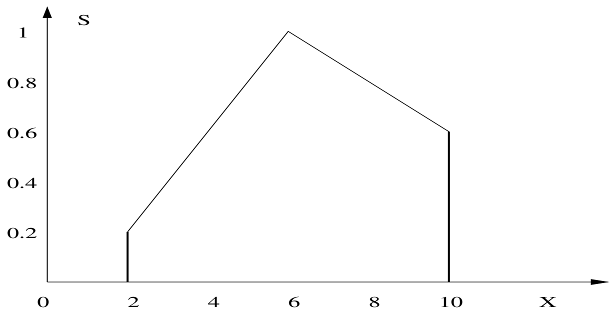
Derive the **conclusion** $\mathcal{Y}$ is B' by the compositional rule of inference. (3p)

8 Defuzzification

a) Name at least two criteria that may be of interest when performing defuzzification to a set. (1p)

b) List at least three methods to perform defuzzification to a point. (1p)

c) Compute the *average α-cut* (as given by the Kudo-Aumann integral) of the below fuzzy set S on $\mathbb{R}$.



(2p)

9 Fuzzy segmentation

a) Fuzzy connectedness is based on the concept of strength of a path. Describe what a path is, and what its strength is. What is the fuzzy connectedness between two points? Explain in words, no mathematics needed (but still allowed). (2p)

b) The fuzzy set theory is very flexible. As such, it is a powerful tool for representing objects in images. Pixel coverage segmentation is based on a specific way to assign memberships to image elements. What is the membership value of an image element according to this model? Explain why this model may be useful. (2p)

Good luck!

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