

The Centre for Image Analysis

Fuzzy Sets and Fuzzy Techniques, 5p.
VT 2007

Exam 2007-03-20

Time: 9.00–14.00

Place: Seminar room CBA

Grading: *Preliminary*

Points	Grade
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18–27	G – pass
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28–40	VG – pass with distinction
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Misc: **Only** paper, pen(cil), eraser, calculator, and dictionary are allowed.
Write your name on all papers. Do not use red ink.
Please, use figures to complement your answers.
Answer in Swedish or English.

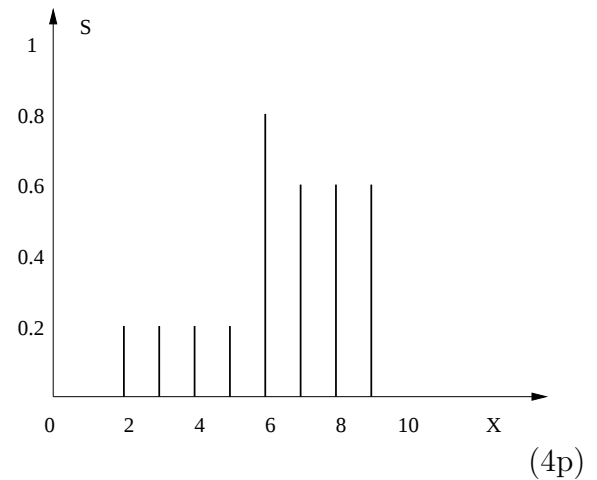
1 Motivation

- a) What is the underlying idea of the fuzzy set theory? (1p)
- b) Give a few important reasons to apply the concepts of the fuzzy set theory? When are approaches based on fuzzy set theory the most useful (or needed)? (2p)
- c) Probability theory has existed since, at least, 1930. Why do we need fuzzy set theory in addition? Is it not all covered by probability theory? What are the main differences between these two theories? Can we just replace probability theory with the theory of fuzzy sets? or do the two theories complement each other? Give explanation/examples for your opinion. (2p)

2 Basic notions

A fuzzy set S on $\mathbb{Z}$ is shown to the right.

- What is the Support of S ?
- Height of S ?
- Is S convex?
- Is S normal?
- What is the cardinality of S ?
- What is the level set of S ?
- What is the α -cut at level 0.5?
- What is the non-specificity of S (in bits)?



3 Extension principle

Let A and B be fuzzy sets defined on $\mathbb{Z}$, with membership functions given by:

$$A(x) = 0.5/(-1) + 0.8/0 + 1/1 + 0.2/2 \quad \text{and}$$

$$B(x) = 0.5/1 + 1/2 + 0.5/3 + 0.8/4$$

Let a function $f : X \times X \rightarrow X$ be defined for all $x_1, x_2 \in X$ by $f(x_1, x_2) = x_1 \cdot x_2$. Calculate $f(A, B)$. (3p)

4 Fuzzy c -means

List some properties of the fuzzy c -means algorithm. What kind of algorithm is it? (I.e., what is it good for?) What parameters are required to set? What is the output? What is required to use it for multi-dimensional data? Give a brief description on how the algorithm works. (4p)

5 Properties of fuzzy sets

- How can crisp concepts, defined for crisp sets, be generalize to fuzzy sets? Write at least two methods. (2p)
- How are area and perimeter (usually) defined for fuzzy spatial sets? (1p)
- Try to explain why the isoperimetric inequality does not hold for fuzzy sets using the standard definitions of area and perimeter. (1p)
- Örjan Smedby is facing the task of making measurements of very thin bone structures in images of limited resolution. Do you think that fuzzy techniques could help him? Give arguments why and, if so, in what ways? (2p)

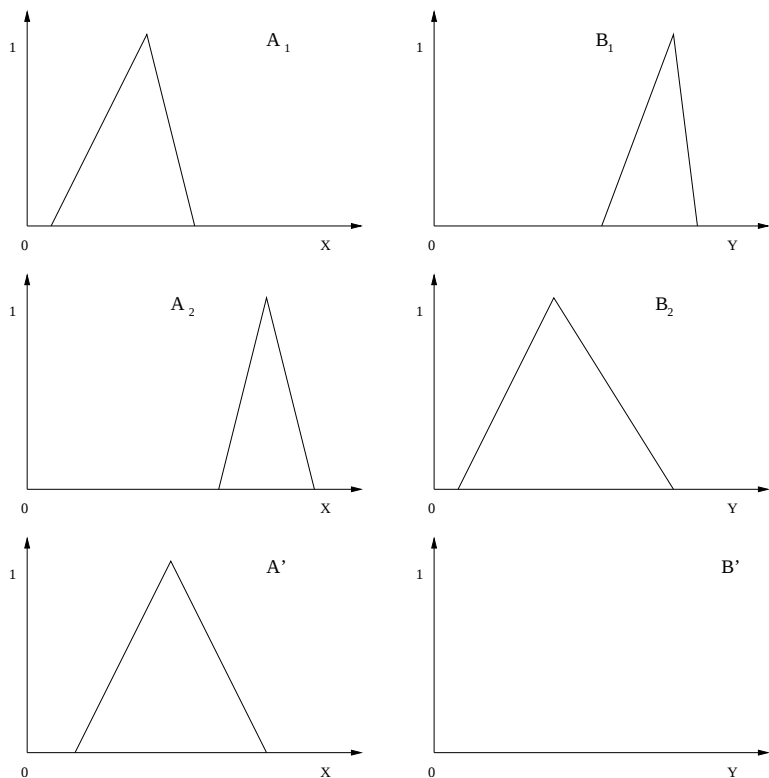
6 Operations on fuzzy sets

- a) Definitions for set complement, intersection and union are based on an axiomatic approach. Write down the axiomatic skeleton (main axioms) for defining fuzzy intersections. (2p)
- b) The so-called *standard set operations* have a special position among set operations on fuzzy set. Give definitions of standard complement, intersection, and union. (1p)
- c) Give at least two properties that make the standard intersection and union “special”? (2p)
- d) What is a *dual triple*? Give three examples of dual triples. (2p)

7 Approximate reasoning

A popular method for approximate reasoning is the *method of interpolation*. Use the method of interpolation to infer a conclusion from the following situation. Draw the fuzzy conclusion set B' and explain the steps that you took to find that result.

Rule 1:	If $\mathcal{X}$ is A_1 , then $\mathcal{Y}$ is B_1
Rule 2:	If $\mathcal{X}$ is A_2 , then $\mathcal{Y}$ is B_2
Fact:	$\mathcal{X}$ is A'
Conclusion:	$\mathcal{Y}$ is B'



Remember to hand in this page together with your solution, if you did draw in the figure to the right.

(3p)

8 Fuzzy control

List the different parts of a fuzzy controller. Describe the roles and purposes of the different parts. (4p)

9 Fuzzy connectedness

a) Describe the following fuzzy connectedness concepts:

(a) Fuzzy adjacency

(b) Fuzzy affinity

(c) Fuzzy connectedness

(d) A path, and the strength of a path

(2p)

b) How can global hanging-togetherness be computed from local hanging-togetherness using the method of fuzzy connectedness?

(2p)

Good luck!

Joakim, Nataša, and László