

# Final Exam (Part 1) in Program Design and Data Structures (1DL201)

Teachers: Dave Clarke, Tjark Weber

2013-12-20 / 14:00–17:00

## Instructions

Read and follow these instructions carefully to increase your chance of getting good marks.

- This is a closed book exam. You may use a standard English dictionary. Otherwise, **no notes, calculators, mobile phones, or other electronic devices are allowed**. Cheating will not be tolerated.
- Read and follow the instructions on the front sheet.
- In the table below, clearly mark **at most one** answer for each question. (If you think that a question is ambiguous or has no correct answer, mark the question number with a ★ and explain on a separate sheet of paper what the problem is and what assumptions you have made to answer the question.)
- Tjark Weber will come to the exam hall around 15:00.

Good luck!

## Your Answers

	Question	Answer					Question	Answer				
Grade 3	1	A	B	C	D	E	2	A	B	C	D	E
	3	A	B	C	D	E	4	A	B	C	D	E
	5	A	B	C	D	E	6	A	B	C	D	E
	7	A	B	C	D	E	8	A	B	C	D	E
	9	A	B	C	D	E	10	A	B	C	D	E
Grade 4	11	A	B	C	D	E	12	A	B	C	D	E
	13	A	B	C	D	E	14	A	B	C	D	E
	15	A	B	C	D	E						
Grade 5	16	A	B	C	D	E	17	A	B	C	D	E
	18	A	B	C	D	E	19	A	B	C	D	E
	20	A	B	C	D	E						

## Master Theorem

Given a recurrence of the form

$$T(n) = aT(n/b) + f(n)$$

**Case 1:** If  $f(n) = O(n^c)$  where  $c < \log_b a$   
then  $T(n) = \Theta(n^{\log_b a})$ .

**Case 2:** If  $f(n) = \Theta(n^c \log^k n)$  where  $c = \log_b a$  and  $k \geq 0$   
then  $T(n) = \Theta(n^c \log^{k+1} n)$ .

**Case 3:** If  $f(n) = \Omega(n^c)$  where  $c > \log_b a$  and the regularity condition holds  
then  $T(n) = \Theta(f(n))$ . The regularity condition is that  $a \cdot f(n/b) \leq d \cdot f(n)$  for some  
constant  $d < 1$  and all sufficiently large  $n$ .

## Common Material

Some of the questions refer to the following function:

```
(* member (e, xs)
  TYPE: ?1?
  PRE: ?2?
  POST: ?3?
*)
(* VARIANT: ?4? *)
fun member (e, []) = false
  | member (e, x::xs) = if e=x then true else member (e, xs)
```

---

Please choose a single answer for each question. Read the questions carefully, and watch out for negations (*not*, *except*, etc.).

---

## Questions for Grade 3

If you answer at least 7 of the 10 questions in this section correctly, your exam grade will be (at least) 3. You cannot compensate incorrect answers in this section with points from questions for higher grades.

1. What is the type of `member (1, [2,1,3])`?

(A) `false`    (B) `int * int list -> bool`    (C) `bool`    (D) `int`    (E) `true`

Answer: (C) `bool`

2. What is the value of `member (1, [2,1,3])`?

(A) `false`    (B) `int * int list -> bool`    (C) `bool`    (D) `int`    (E) `true`

Answer: (E) `true`

3. What is the most appropriate precondition (?2?) for the `member` function?
- (A) `false`    (B) `xs` is a list    (C) `true`    (D) `none`    (E) `e` is contained in `xs`

Answer: (C) `true` (we will also accept (D) `none` because this answer was accepted in labs and assignments; however, you should understand that a precondition is a predicate)

4. What is the most appropriate postcondition (?3?) for the `member` function?
- (A) `true` if `e=x`, `member (e, xs)` otherwise    (B) `none`  
(C) `true` if `e` is contained in `xs`, `false` otherwise    (D) `true`    (E) `false`

Answer: (C) `true` if `e` is contained in `xs`, `false` otherwise

5. Which of the following is a variant (?4?) for the `member` function?
- (A) 1    (B) `length xs`    (C) `size x`    (D) `e`    (E) `e=x`

Answer: (B) `length xs`

6. Which of the following is a valid expression that is equivalent to `if e=x then true else member (e, xs)`?

- (A) `if e=x orelse member (e, xs) then true`  
(B) `if e=x andalso member (e, xs) then true`  
(C) `if true then e=x else member (e, xs)`  
(D) `e=x orelse member (e, xs)`  
(E) `e=x andalso member (e, xs)`

Answer: (D) `e=x orelse member (e, xs)`

7. Suppose you want to define a data type to model student transcripts. A transcript contains the name of the student, the student's ID number, and the title of each course that the student has taken together with the (numeric) grade that the student has achieved in that course. How would you complete the following data type declaration?

```
datatype transcript = Transcript of ...
```

- (A) `string * int * (string list * int list) list`  
(B) `string * int * string * int`  
(C) `string * int * string list * int`  
(D) `(string * int * string * int) list`  
(E) `string * int * (string * int) list`

Answer: (E) `string * int * (string * int) list`

8. Consider the following function:

```
fun f8 x = (print "a"; x)
```

Which output is observed (as a side-effect) when the expression `f8 (print "b", print "c")` is evaluated?

- (A) `cba`    (B) `bca`    (C) `acb`    (D) `abc`    (E) none of these

Answer: (B) `bca`

9. What is a correct asymptotic bound on the polynomial  $4n^3 + 99n^2 + 3$ ?  
 (A)  $O(n^2)$       (B)  $\Theta(n^4)$       (C)  $O(n^4)$       (D)  $\Theta(n^2)$       (E)  $\Omega(n^4)$

Answer: (C)  $O(n^4)$ , because  $O(-)$  provides an upper bound.

10. Consider the following function, whose type is `int -> int`:

```
fun f10 n =
  if n <= 1 then
    1
  else
    f10 (n - n div 2) + f10 (n - n div 2)
```

This function can easily be optimized to avoid computing `f10 (n - n div 2)` twice in the last line:

```
fun f10_opt n =
  if n <= 1 then
    1
  else
    let
      val x = f10_opt (n - n div 2)
    in
      x + x
    end
```

What are the tight asymptotic bounds of the running time of the original function `f10` and the optimized version `f10_opt`?

- (A)  $\Theta(n \log n)$  and  $\Theta(\log n)$       (B)  $\Theta(n^2)$  and  $\Theta(n)$       (C)  $\Theta(n^2)$  and  $\Theta(\log n)$   
 (D)  $\Theta(2^n)$  and  $\Theta(n)$       (E)  $\Theta(n)$  and  $\Theta(\log n)$

Answer: (E)  $\Theta(n)$  and  $\Theta(\log n)$

## Questions for Grade 4

If you answer at least 3 of the 5 questions in this section correctly *and* you pass the criteria for grade 3, your exam grade will be (at least) 4. You cannot compensate incorrect answers in this section with points from questions for grades 3 or 5.

11. What is the type (?1?) of `member`?

- (A) `'a * 'a list -> bool`      (B) `int * int list -> bool`  
 (C) `''a * ''a list -> bool`      (D) `'a * 'a list`      (E) `bool`

Answer: (C) `''a * ''a list -> bool`

12. Consider the following pairs of test cases for the `member` function:

- (A) `member (1, [])` and `member (1, [2])`
- (B) `member (1, [2])` and `member (2, [1])`
- (C) `member (1, [])` and `member (1, [1])`
- (D) `member (1, [])` and `member (1, [2,1,3])`
- (E) `member (1, [2,1,3])` and `member (2, [2,1,3])`

Which of these pairs achieves full code coverage (i.e., all sub-expressions of `member` are evaluated at least once)?

- (A) (B) (C) (D) (E)

Answer: (D) `member (1, [])` and `member (1, [2,1,3])`

13. Consider the following function:

```
fun f13 (true, 1) = 1
  | f13 (true, _) = 2
  | f13 (_, 1)    = 3
  | f13 (false, _) = 4
  | f13 (false, 1) = 5
```

What is the value of `f13 (false, 1)`?

- (A) 1 (B) 2 (C) 3 (D) 4 (E) 5

Answer: (C) 3

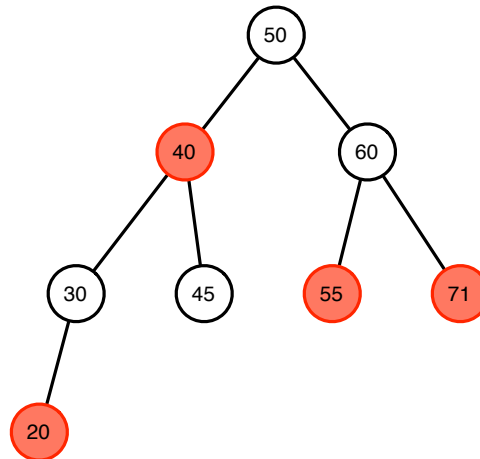
14. What is the value of the following expression?

```
let
  val a = 1
  fun f b = a + b
  val b = f a
  val a = f b
  val b = f a
in
  a + b
end
```

- (A) 1 (B) 3 (C) 5 (D) 7 (E) 9

Answer: (D) 7

15. Consider the following red-black tree.



Insert 25 into this tree.

The root of the resulting tree is

- (A) 25                      (B) 50                      (C) 40                      (D) 30                      (E) 60

Answer: (C) 40

## Questions for Grade 5

If you answer at least 3 of the 5 questions in this section correctly *and* you pass the criteria for grade 4, your exam grade will be 5. You cannot compensate incorrect answers in this section with points from questions for lower grades.

16. Recall the data type of generic trees

```
datatype 'a tree = Node of 'a * 'a tree list
```

and consider the following mutually recursive functions:

```
fun f16 (Node (x, ts)) = 1 + f16_list ts
and f16_list [] = 0
    | f16_list (t::ts) = f16 t + f16_list ts
```

What does `f16` compute?

- (A) the height of a tree                      (B) the post-order list of all tree elements  
(C) the sum of all elements in a tree                      (D) the pre-order list of all tree elements  
(E) the number of nodes in a tree

Answer: (E) the number of nodes in a tree

17. Consider the following data type declaration:

```
datatype 'a t15 = X of 'a | Y of ('a * 'a) t15
```

Which of the following is *not* a value of (some instance of) this data type?

(A) X 1   (B) Y (X (1,2))   (C) Y (X 1)   (D) X (1,2)   (E) X (Y (X (1,2)))

Answer: (C) Y (X 1)

18. Suppose the function `hd` is declared as follows:

```
fun hd [] = raise Empty
  | hd (x::_) = x
```

The expression `hd [1] = 1 div 0 orelse hd [] = [1.0]`

(A) evaluates to `false`.   (B) raises exception `Empty`.   (C) evaluates to `true`.  
(D) is not type-correct.   (E) raises exception `Div`.

Answer: (D) — is not type-correct.

19. Use the master theorem to find a closed form for the following recurrence:

$$T(n) = 4T\left(\frac{n}{2}\right) + n^2 + \sin n$$

The closed form is:

(A)  $\Theta(n^2)$    (B)  $\Theta(n^2 \log n)$    (C)  $\Theta(n^2 \sin n)$    (D)  $\Theta(n^{2.32})$   
(E) the master theorem is not applicable

Answer: (B)  $\Theta(n^2 \log n)$

20. Consider the following function whose type is `'a list -> 'a list list`:

```
fun suffixes [] = [[]]
  | suffixes (x::xs) = (suffixes xs) @ [x::xs]
```

What is the *strict* asymptotic run-time of this function, where  $n$  is the number of elements in the argument list?

(A)  $\Theta(n)$    (B)  $\Theta(n^2)$    (C)  $\Theta(n \log n)$    (D)  $\Theta(n^3)$    (E)  $\Theta(\log n)$

Answer: (B)  $\Theta(n^2)$