

# Final Exam (Part 1) in Program Design and Data Structures (1DL201)

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## Instructions

Read and follow these instructions carefully to increase your chance of getting good marks.

- This is a closed book exam. You may use a standard English dictionary. Otherwise, **no notes, calculators, mobile phones, or other electronic devices are allowed**. Cheating will not be tolerated.
- This is a multiple-choice exam. Each question has exactly **one** correct answer.
- You may keep these question sheets. **Only hand in the answer sheet**. Also read the instructions on the answer sheet before you start.
- Tjark Weber will come to the exam hall around 15:30 to answer questions.

Good luck!

## Master Theorem

Given a recurrence of the form

$$T(n) = aT(n/b) + f(n)$$

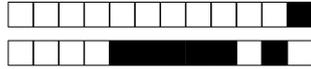
**Case 1:** If  $f(n) = O(n^c)$  where  $c < \log_b a$   
then  $T(n) = \Theta(n^{\log_b a})$ .

**Case 2:** If  $f(n) = \Theta(n^c \log^k n)$  where  $c = \log_b a$  and  $k \geq 0$   
then  $T(n) = \Theta(n^c \log^{k+1} n)$ .

**Case 3:** If  $f(n) = \Omega(n^c)$  where  $c > \log_b a$  and the regularity condition holds  
then  $T(n) = \Theta(f(n))$ .

The regularity condition is that  $a \cdot f(n/b) \leq k \cdot f(n)$  for some constant  $k < 1$   
and all sufficiently large  $n$ .





**Question 6:** The syntax of a programming language ...

- A** is usually specified only informally, e.g., in English.
- B** describes how expressions in that language are evaluated.
- C** defines what combinations of symbols constitute valid programs.
- D** defines the meaning of programs.
- E** defines the data types and operations that are available in the language.

**Question 7:** Consider the following text:

```
f x = length x + (x `div` 0)
```

Compiling this with a Haskell compiler will result in ...

- A** a run-time error.
- B** a type error.
- C** Infinity
- D** a syntax error.
- E** none of these.

**Question 8:** Which of the following patterns does **not** match the value  $(1, [2,3])$  ?

- A**  $(-, [2,x])$
- B**  $(1, [x])$
- C**  $-$
- D**  $(1, [2,3])$
- E**  $(1, x)$

**Question 9:** Consider the function

```
f x | x<1 = let x=2 in case x*x of x -> x
```

What is the value of `let x=0 in f x` ?

- A** 0
- B** 1
- C** 2
- D** 4
- E** None of these.

**Question 10:** Recall the quicksort algorithm. Suppose the algorithm is applied to the input list  $[6,8,3,11,0,8,2,9]$  and the value 6 is chosen as the pivot. What are the arguments to the immediate recursive calls of the quicksort algorithm?

- A**  $[3,0,2]$  and  $[8,11,8,9]$
- B** 6 and  $[8,3,11,0,8,2,9]$
- C**  $[6,8]$ ,  $[3,11]$ ,  $[0,8]$  and  $[2,9]$
- D**  $[8,3,11]$  and  $[0,8,2,9]$
- E**  $[0,2,3]$  and  $[8,8,9,11]$

**Question 11:** Which of the following expressions is **not** polymorphic?

- A** `[]`
- B** `(&&)`
- C** `map`
- D** `(.)`
- E** `snd`



**Question 12:** Which of the following expressions, when applied to an argument of the correct type, does **not** necessarily return the argument value unchanged?

- (A) `(1-) . (1+)`                       (D) `filter (\_ -> True)`  
 (B) `\x -> x`  
 (C) `map id`                               (E) `let f xs = [x|x <- xs] in f`

**Question 13:** What is the value of `foldl (-) 10 [3,2,1]` ?

- (A) -8                       (B) 10                       (C) 4                       (D) 0                       (E) -10

**Question 14:** Which is the **most precise** bound for the function  $n^5 + 1000n^4 + 3n^3 + n^3 \log n$ ?

- (A)  $\Omega(n^4)$                        (C)  $\Theta(n^3 \log n)$                        (E)  $\Theta(n^5)$   
 (B)  $O(n^4)$                        (D)  $O(n^5)$

**Question 15:** What is the closed form of the following recurrence?

$$\begin{aligned} T(0) &= 10 \\ T(n) &= T(n-1) + 5n + 6 \end{aligned}$$

- (A)  $T(n) = 11n + 10$                        (D)  $T(n) = \frac{5n(n+1)}{2} + 6n + 10$   
 (B)  $T(n) = 5n^3 + 6n + 10$   
 (C)  $T(n) = 5n \log n + 16$                        (E)  $T(n) = 5(2^n) + 6n + 10$

**Question 16:** Recall that  $O(g(n))$ ,  $\Theta(g(n))$  and  $\Omega(g(n))$  actually represent *sets* of functions related in the appropriate way to  $g(n)$ . What is the relationship between  $O(g(n))$ ,  $\Theta(g(n))$  and  $\Omega(g(n))$ ?

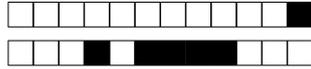
- (A)  $\Omega(g(n)) \subseteq \Theta(g(n)) \subseteq O(g(n))$                        (D)  $O(g(n)) = \Omega(g(n)) \cup \Theta(g(n))$   
 (B)  $\Omega(g(n)) \cap O(g(n)) = \Theta(g(n))$   
 (C)  $O(g(n)) \subseteq \Theta(g(n)) \subseteq \Omega(g(n))$                        (E) No relationship.

**Question 17:** Use the Master Theorem to find a closed form for the following recurrence:

$$T(n) = 2^n T(n/2) + n^n.$$

The closed form is:

- (A)  $\Theta(n^{2^n})$ .                       (C)  $\Theta(2^n)$ .                       (E) The Master Theorem does not apply.  
 (B)  $\Theta(n^n)$ .                       (D)  $\Theta(n^n \log n)$ .



**Question 18:** Consider the following recurrence:

$$\begin{aligned}T(0) &= \Theta(1) \\T(1) &= \Theta(1) \\T(n) &= T(n-1) + T(n-2) + \Theta(1)\end{aligned}$$

Which of the following Haskell functions' runtime function is given by this recurrence?

- A** `golf [] = 0`  
`golf [a] = a`  
`golf (a : b : as) = a + golf (b:as) + golf as`
- B** `dog [] = 0`  
`dog [a] = a`  
`dog l = dog left + dog right + 1`  
where  
`(left, right) = split l`  
`split l = let n = length l `div` 2 in (take n l, drop n l)`
- C** `zig [] = 1`  
`zig (a : as) = a - zag as`  
`zag [] = 0`  
`zag (a : as) = a + zig as`
- D** `foo [] = []`  
`foo (a : as) = [a] : foo as`
- E** `bar [] = 0`  
`bar [a] = a`  
`bar (a : _ : as) = a + bar as`

**Question 19:** You are required to implement a function, but are not 100% sure how to do it. You recall the design technique called dodging, which can help simplify your task. Which of the following is **not** an example of a dodge?

- A** Implement a function that returns a fixed value.
- B** Implement a function that only works for some input values.
- C** Implement a function that ignores boundary conditions.
- D** Use an obvious but inefficient algorithm.
- E** Implement all code within a single function.



**Question 20:** Which is **not** one of the purposes of the *Function Examples* step of the 8 Step Design Process?

- A To describe the data types used in the program.
- B To understand better how the function works.
- C To provide valid inputs to the function.
- D To provide some test cases.
- E To provide expected outputs from the function.

