



Final Exam (Part 1) in Program Design and Data Structures (1DL201)

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2015-12-18 / 14:00–19:00

Instructions

Read and follow these instructions carefully to increase your chance of getting good marks.

- This is a closed book exam. You may use a standard English dictionary. Otherwise, **no notes, calculators, mobile phones, or other electronic devices are allowed**. Cheating will not be tolerated.
- This is a multiple-choice exam. Each question has exactly **one** correct answer.
- You may keep these question sheets. **Only hand in the answer sheet**. Also read the instructions on the answer sheet before you start.
- Tjark Weber will come to the exam hall around 15:30 to answer questions.

Good luck!

Master Theorem

Given a recurrence of the form

$$T(n) = aT(n/b) + f(n)$$

Case 1: If $f(n) = O(n^c)$ where $c < \log_b a$
then $T(n) = \Theta(n^{\log_b a})$.

Case 2: If $f(n) = \Theta(n^c \log^k n)$ where $c = \log_b a$ and $k \geq 0$
then $T(n) = \Theta(n^c \log^{k+1} n)$.

Case 3: If $f(n) = \Omega(n^c)$ where $c > \log_b a$ and the regularity condition holds
then $T(n) = \Theta(f(n))$.

The regularity condition is that $a \cdot f(n/b) \leq k \cdot f(n)$ for some constant $k < 1$
and all sufficiently large n .



Common Material

Some of the exam questions refer to the following function:

```
{- rev acc xs
  PRE: ?PRE?
  POST: ?POST?
-}
rev :: ?TYPE?
-- VARIANT: ?VARIANT?
rev acc [] = acc
rev acc (x:xs) = rev (x:acc) xs
```

Questions

Please choose a single answer for each question. Read the questions carefully, and watch out for negations (*not*, *except*, etc.).

Question 1: What is the value of `rev [1,2] [3,4]` ?

- | | | |
|--|--|--|
| <input type="checkbox"/> [A] [3,4,1,2] | <input type="checkbox"/> [C] [1,2,3,4] | <input type="checkbox"/> [E] [4,3,2,1] |
| <input type="checkbox"/> [B] 10 | <input type="checkbox"/> [D] [4,3,1,2] | |

Question 2: What is the type (`?TYPE?`) of `rev`?

- | | | |
|---|---|---|
| <input type="checkbox"/> [A] <code>[a] -> [b] -> [a]</code> | <input type="checkbox"/> [C] <code>(([a],[b]) -> [a])</code> | <input type="checkbox"/> [E] <code>Int -> Int -> Int</code> |
| <input type="checkbox"/> [B] <code>[a] -> [a] -> [a]</code> | <input type="checkbox"/> [D] <code>[a] -> [a]</code> | |

Question 3: What is the most appropriate precondition (`?PRE?`) for `rev`?

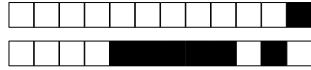
- | | | |
|---|---|--|
| <input type="checkbox"/> [A] <code>True</code> | <input type="checkbox"/> [C] <code>acc</code> and <code>xs</code> are lists | <input type="checkbox"/> [E] <code>acc</code> is empty |
| <input type="checkbox"/> [B] <code>False</code> | <input type="checkbox"/> [D] <code>xs</code> is non-empty | |

Question 4: What is the most appropriate postcondition (`?POST?`) for `rev`?

- | | |
|---|---|
| <input type="checkbox"/> [A] <code>a</code> non-empty list | <input type="checkbox"/> [D] <code>True</code> |
| <input type="checkbox"/> [B] <code>xs</code> in reverse order | <input type="checkbox"/> [E] <code>rev</code> applied to (i) the head of <code>xs</code> prepended to <code>acc</code> , and (ii) the tail of <code>xs</code> |
| <input type="checkbox"/> [C] <code>xs</code> in reverse order, followed by <code>acc</code> | |

Question 5: Which of the following is a variant (`?VARIANT?`) for the function `rev`?

- | | |
|--|---|
| <input type="checkbox"/> [A] <code>length acc</code> | <input type="checkbox"/> [D] <code>acc == xs</code> |
| <input type="checkbox"/> [B] <code>length xs</code> | |
| <input type="checkbox"/> [C] <code>length acc + length xs</code> | <input type="checkbox"/> [E] <code>0</code> |



Question 6: The syntax of a programming language ...

- ☐ A is usually specified only informally, e.g., in English.
- ☐ B describes how expressions in that language are evaluated.
- ☐ C defines what combinations of symbols constitute valid programs.
- ☐ D defines the meaning of programs.
- ☐ E defines the data types and operations that are available in the language.

Question 7: Consider the following text:

```
f x = length x + (x `div` 0)
```

Compiling this with a Haskell compiler will result in ...

- ☐ A a run-time error.
- ☐ B a type error.
- ☐ C Infinity
- ☐ D a syntax error.
- ☐ E none of these.

Question 8: Which of the following patterns does **not** match the value $(1, [2,3])$?

- ☐ A $(-, [2,x])$
- ☐ B $(1, [x])$
- ☐ C $-$
- ☐ D $(1, [2,3])$
- ☐ E $(1, x)$

Question 9: Consider the function

```
f x | x<1 = let x=2 in case x*x of x -> x
```

What is the value of `let x=0 in f x` ?

- ☐ A 0
- ☐ B 1
- ☐ C 2
- ☐ D 4
- ☐ E None of these.

Question 10: Recall the quicksort algorithm. Suppose the algorithm is applied to the input list $[6,8,3,11,0,8,2,9]$ and the value 6 is chosen as the pivot. What are the arguments to the immediate recursive calls of the quicksort algorithm?

- ☐ A $[3,0,2]$ and $[8,11,8,9]$
- ☐ B 6 and $[8,3,11,0,8,2,9]$
- ☐ C $[6,8]$, $[3,11]$, $[0,8]$ and $[2,9]$
- ☐ D $[8,3,11]$ and $[0,8,2,9]$
- ☐ E $[0,2,3]$ and $[8,8,9,11]$

Question 11: Which of the following expressions is **not** polymorphic?

- ☐ A `[]`
- ☐ B `(&&)`
- ☐ C `map`
- ☐ D `(.)`
- ☐ E `snd`



Question 12: Which of the following expressions, when applied to an argument of the correct type, does **not** necessarily return the argument value unchanged?

☐ (1-) . (1+)

☐ filter (_ -> True)

☐ \x -> x

☐ map id

☐ let f xs = [x|x <- xs] in f

Question 13: What is the value of foldl (-) 10 [3,2,1] ?

☐ -8

☐ 10

☐ 4

☐ 0

☐ -10

Question 14: Which is the **most precise** bound for the function $n^5 + 1000n^4 + 3n^3 + n^3 \log n$?

☐ $\Omega(n^4)$

☐ $\Theta(n^3 \log n)$

☐ $\Theta(n^5)$

☐ $O(n^4)$

☐ $O(n^5)$

Question 15: What is the closed form of the following recurrence?

$$T(0) = 10$$

$$T(n) = T(n-1) + 5n + 6$$

☐ $T(n) = 11n + 10$

☐ $T(n) = \frac{5n(n+1)}{2} + 6n + 10$

☐ $T(n) = 5n^3 + 6n + 10$

☐ $T(n) = 5n \log n + 16$

☐ $T(n) = 5(2^n) + 6n + 10$

Question 16: Recall that $O(g(n))$, $\Theta(g(n))$ and $\Omega(g(n))$ actually represent *sets* of functions related in the appropriate way to $g(n)$. What is the relationship between $O(g(n))$, $\Theta(g(n))$ and $\Omega(g(n))$?

☐ $\Omega(g(n)) \subseteq \Theta(g(n)) \subseteq O(g(n))$

☐ $O(g(n)) = \Omega(g(n)) \cup \Theta(g(n))$

☐ $\Omega(g(n)) \cap O(g(n)) = \Theta(g(n))$

☐ $O(g(n)) \subseteq \Theta(g(n)) \subseteq \Omega(g(n))$

☐ No relationship.

Question 17: Use the Master Theorem to find a closed form for the following recurrence:

$$T(n) = 2^n T(n/2) + n^n.$$

The closed form is:

☐ $\Theta(n^{2^n})$.

☐ $\Theta(2^n)$.

☐ The Master Theorem does not apply.

☐ $\Theta(n^n)$.

☐ $\Theta(n^n \log n)$.



Question 18: Consider the following recurrence:

$$\begin{aligned} T(0) &= \Theta(1) \\ T(1) &= \Theta(1) \\ T(n) &= T(n-1) + T(n-2) + \Theta(1) \end{aligned}$$

Which of the following Haskell functions' runtime function is given by this recurrence?

- ☐ **A**

```
golf [] = 0
golf [a] = a
golf (a : b : as) = a + golf (b:as) + golf as
```
- ☐ **B**

```
dog [] = 0
dog [a] = a
dog l = dog left + dog right + 1
  where
    (left, right) = split l
    split l = let n = length l `div` 2 in (take n l, drop n l)
```
- ☐ **C**

```
zig [] = 1
zig (a : as) = a - zag as
zag [] = 0
zag (a : as) = a + zig as
```
- ☐ **D**

```
foo [] = []
foo (a : as) = [a] : foo as
```
- ☐ **E**

```
bar [] = 0
bar [a] = a
bar (a : _ : as) = a + bar as
```

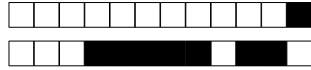
Question 19: You are required to implement a function, but are not 100% sure how to do it. You recall the design technique called dodging, which can help simplify your task. Which of the following is **not** an example of a dodge?

- ☐ **A** Implement a function that returns a fixed value.
- ☐ **B** Implement a function that only works for some input values.
- ☐ **C** Implement a function that ignores boundary conditions.
- ☐ **D** Use an obvious but inefficient algorithm.
- ☐ **E** Implement all code within a single function.



Question 20: Which is **not** one of the purposes of the *Function Examples* step of the 8 Step Design Process?

- ☐ A To describe the data types used in the program.
- ☐ B To understand better how the function works.
- ☐ C To provide valid inputs to the function.
- ☐ D To provide some test cases.
- ☐ E To provide expected outputs from the function.



Do not write above this line!

Answer Sheet — Exam 1DL201 of 2015-12-18

Instructions: Using a **dark** color, fill in **at most one** answer box (A to E) per question. Fill the answer box **entirely** (■)—we will use an optical character recognition (OCR) system that may not recognize ticks, crosses, circles, etc.

If you think that a question is ambiguous or has no correct answer, mark the question number with a ★ and explain **on the backside of this sheet** what the problem is and what assumptions you have made to answer the question.

Transfer your answers from the question sheets to this answer sheet **just before handing in**. If you want to change an answer, then please request a new answer sheet. You may keep the question sheets; at the end of the exam, **only hand in this answer sheet**.

Also fill in your **exam code** in clear handwriting at the bottom of this page.

Grading:	Correct answers	≤ 9	10 – 13	14 – 16	17 – 20
	Grade	U	3	4	5

Question 1: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 2: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 3: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 4: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 5: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 6: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 7: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 8: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 9: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 10: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 11: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 12: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 13: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 14: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 15: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 16: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 17: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 18: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 19: ☐ A ☐ B ☐ C ☐ D ☐ E

Question 20: ☐ A ☐ B ☐ C ☐ D ☐ E

Again: Please fill your chosen boxes **entirely** and in **dark** color!

Your exam code:

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