

Natural deduction rules

Bevisregler, satslogiken / Alfken

pp 5-30

Axiom Rule, p8:

$$\psi \vdash \psi$$

And-intro, p11:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \end{array} \quad \begin{array}{c} \mathcal{D}' \\ \psi \end{array}}{\psi \wedge \psi} \text{ (AI)}$$

And-elimination, p14:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \wedge \psi \end{array}}{\psi} \text{ (AE)} \quad \frac{\begin{array}{c} \mathcal{D}' \\ \psi \wedge \psi \end{array}}{\psi} \text{ (AE)} \quad \text{[Two rules!]}$$

Or-intro, p28:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \end{array}}{\psi \vee \psi} \text{ (VI)} \quad \frac{\begin{array}{c} \mathcal{D}' \\ \psi \end{array}}{\psi \vee \psi} \text{ (VI)} \quad \text{[Two rules!]}$$

Or-elimination, p29:

(Page 29: also proof that $\vdash \psi \vee (\neg \psi)$)

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \vee \psi \end{array} \quad \begin{array}{c} \mathcal{D}' \\ \chi \end{array} \quad \begin{array}{c} \mathcal{D}'' \\ \chi \end{array}}{\chi} \text{ (VE) } \textcircled{1}$$

Not-intro, p25:

$$\frac{\begin{array}{c} \mathcal{D} \\ \neg \psi \end{array} \quad \perp}{(\neg \psi)} \text{ (}\neg\text{I) } \textcircled{2}$$

Not-Reductio Ad Absurdum: $(\neg \psi)$ page 26

$$\frac{\perp}{\neg \psi} \text{ (RAA) } \textcircled{3}$$

Not-elimination, p24:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \end{array} \quad \perp}{\perp} \text{ (}\neg\text{E)}$$

Arrow-intro, p17:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \end{array}}{\psi \rightarrow \psi} \text{ (}\rightarrow\text{I) } \textcircled{4}$$

Arrow-elimination, p19:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \end{array} \quad \begin{array}{c} \mathcal{D}' \\ (\psi \rightarrow \psi) \end{array}}{\psi} \text{ (}\rightarrow\text{E)}$$

Double-arrow-intro, p22:

$$\frac{\begin{array}{c} \mathcal{D} \\ (\psi \rightarrow \psi) \end{array} \quad \begin{array}{c} \mathcal{D}' \\ (\psi \rightarrow \psi) \end{array}}{(\psi \leftrightarrow \psi)} \text{ (}\leftrightarrow\text{I)}$$

Double-arrow-elimination, p22:

$$\frac{\begin{array}{c} \mathcal{D} \\ \psi \leftrightarrow \psi \end{array}}{\psi \rightarrow \psi} \text{ (}\leftrightarrow\text{E)}$$

[Two rules!]

$$\frac{\begin{array}{c} \mathcal{D}' \\ \psi \leftrightarrow \psi \end{array}}{\psi \rightarrow \psi} \text{ (}\leftrightarrow\text{E)}$$