Instructions: This is a multiple-choice exam, to save you the time of tidying up the presentation of your answers. There is exactly one correct answer per question. You can keep the question sheets and should hand in only the answer sheet: you are not expected to explain your answers. Unfortunately, the teacher cannot attend this exam. Also read the instructions on the answer sheet before starting.

## 0 Warm-Up Questions: General Culture

Question -2: How is Edsger W. Dijkstra's surname pronounced in his native language?
A dəkstra
B dəikstra
C dikstra
D d3ikstra
E deikstra

Question -1: In which year was the Alan M. Turing Award not won by a scientist whose work was discussed in this course?
A 1972
B 1974
C 1976
D 1978
E 1982

## 1 String Matching

Question 1: On which of the following length- $m$ patterns $P$ does the naïve string matching algorithm reach its worst-case runtime when looking for all occurrences of $P$ in the text $T=0^{n}$ (that is, a string of $n$ occurrences of the character ' 0 '), with $n \geq m \geq 3$ ?
(A $1^{m}$
(B) $1^{m-1} 0$
C $1\left(0^{m-1}\right)$
D $0\left(1^{m-1}\right)$
E $0^{m-1} 1$

Question 2: For the Rabin-Karp string matching algorithm, let $p$ denote the fingerprint of the length- $m$ pattern $P$, and let $t_{s}$ denote the fingerprint of the length- $m$ substring $T_{s}$ for shift $s$ in text $T$ (of length at least $m$ ). On which assumption does the algorithm rely?
A $p=t_{s} \Rightarrow P=T_{s}$
D $p \neq t_{s} \Leftarrow \exists k \in 1 \ldots m: P[k] \neq T_{s}[k]$
B $p=t_{s} \Leftrightarrow P=T_{s}$
C $p=t_{s} \Leftarrow P=T_{s}$
E $p \neq t_{s} \Rightarrow \forall k \in 1 \ldots m: P[k] \neq T_{s}[k]$

Question 3: How many spurious hits does the Rabin-Karp string matching algorithm encounter in the text $T=$ " 3141512653849792 " when looking for all occurrences of the pattern $P=" 26$ ", working modulo $q=11$ and over the alphabet $\Sigma=\{0,1,2, \ldots, 9\}$ ?
A 0
(B) 1
(C 2
D 3
E 4

Question 4: On which of the following patterns $P$ does the Rabin-Karp string matching algorithm reach its best-case runtime when looking for all occurrences of $P$ in the text $T=0^{n}$ (that is, a string of $n$ occurrences of the character ' 0 '), with $n \geq 3$, working modulo $q=3$ and over the alphabet $\Sigma=\{0,1,2, \ldots, 9\}$ ?
A " 660 "
(B) "300"
C "099"
D "007"
E "000"

## 2 Greedy Algorithms

Consider a weighted directed graph $G$ with a set $V$ of $n$ vertices, a set $E$ of $m$ edges, and an edge-weight function $w: E \rightarrow \mathbb{R}$, such as in the figure to the right. In this example, some edges have negative weights: Dijkstra's single-source shortest paths algorithm is not applicable for finding shortest paths (that is, minimum-weight paths) from vertex $s$ to all vertices; for instance, a shortest path from $s$ to $d$ is $s-c-e-b-a-d$ and has weight $3+(-3)+(-2)+(-4)+6=0$.


Question 5: Let $G$ have some negative weights: which condition below is sufficient for the existence of a shortest path from a source vertex $s$ to a vertex reachable from $s$ ?
(A) if $G$ is acyclic
B if $G$ is connected
C if real weights
(D) if rational weights
E if integer weights

Question 6: What weight does Dijkstra's algorithm find for a shortest path from $s$ to $d$ ?
(A it loops
(B) 0
C 3
D 4
E it crashes forever

Question 7: If $G$ has some negative weights, violating the precondition of Dijkstra's algorithm, then let us first modify $G$ by adding $-\ell$ to all weights, where $\ell<0$ is the smallest weight, giving us a new graph $G^{\prime}$. When does Dijkstra's algorithm find shortest paths in $G^{\prime}$ that correspond (but with exaggerated weights) 1-to-1 to shortest paths in $G$ ?
(A) always
B sometimes
(C) never
(D) undecidable
E we do not know

Question 8: Dijkstra's greedy algorithm for the single-source shortest paths problem performs $n$ iterations: does it stay correct if we drop the inner loop of relaxation checks at the $n$th iteration of the outer loop (extraction of a minimum from a min-priority queue)?
(A) always
(B) sometimes
C never
(D) undecidable
E we do not know

Question 9: Let $G$ possibly have some negative weights: what is the maximum number of edges of a shortest path that is simple (that is, it does not re-visit vertices)?
(A) $-\infty$
B $n-1$
C $n$
(D) $n+1$
E] $+\infty$

## 3 Dynamic Programming

Consider a weighted directed graph $G$ with a set $V$ of $n$ vertices, a set $E$ of $m$ edges, an edge-weight function $w: E \rightarrow \mathbb{R}$ with no negative-weight cycles, and a source vertex $s$. Consider the following recurrence, parameterised by $\left\langle\alpha, \beta_{1}, \ldots, \beta_{4}, \gamma\right\rangle$, for a quantity $T[i, v]$ :

$$
T[i, v]= \begin{cases}0 & \text { if } \alpha \wedge(v=s) \\ \beta_{1} & \text { if } \alpha \wedge(v \neq s) \\ \gamma\left(T\left[i-1, \beta_{2}\right], \min _{(x, v) \in E}\left(T\left[i-1, \beta_{3}\right]+\beta_{4}\right)\right) & \text { if } \neg \alpha\end{cases}
$$

Question 10: If $T[n-1, v]$ is returned by a correct algorithm for computing the weight of a shortest path in $G$ from $s$ to $v$, then $T[i, v]$, with $0 \leq i<n$ and $v \in V$, denotes the weight of a shortest path from $s$ to $v$ with how many edges?
A $<i$
B $\leq i$
C $=i$
D $\geq i$
E $>i$

Question 11: For the graph in the figure of Section 2, what is the sum $T[3, a]+T[3, b]$ ?
(A -8
(B) -6
(C -5
(D) -4
(E -3

Question 12: What is the logical condition $\alpha$ ?
A $i=0$
[B] $i=n-1$
C $i=m-1$
(D $i=m$
E $i \cdot v=0$

Question 13: What is the numeric expression $\beta_{1}$ ?
(A) $-\infty$
(B) -1
C 0
(D +1
(E] $+\infty$

Question 14: What is the index expression $\beta_{2}$ ?
(A) $s$
(B) $v-1$
C $v$
(D) $v+1$
E $x$

Question 15: What is the index expression $\beta_{3}$ ?
(A) $s$
(B) $v-1$
C $v$
(D) $v+1$
E $x$

Question 16: What is the numeric expression $\beta_{4}$ ?
(A) $w(s, v)$
[B] $w(s, x)$
(C) $w(v, x)$
(D) $w(x, v)$
E] 1

Question 17: What is the two-argument operator $\gamma$ (written in prefix form above)?
A +
(B) $\Sigma$
(C) П
(D) $w$
E min

Question 18: What is an ordering of the indices $i$ and $v$ under which the elements $T[i, v]$ of the table $T$ can be filled without performing any redundant computations?
A by lexicographically increasing $\langle v, i\rangle$
D by lexicographically decreasing $\langle v,-i\rangle$
[B any order
C by lexicographically increasing $\langle-i, v\rangle$
E by lexicographically decreasing $\langle-i, v\rangle$

We say that $\langle a, b\rangle$ is lexicographically smaller than $\langle c, d\rangle$ if either $a<c$ or $a=c \wedge b<d$.

## 4 Complexity

Consider a weighted directed graph $G$ with a set $V$ of $n$ vertices, a set $E$ of $m$ edges, and an edge-weight function $w: E \rightarrow \mathbb{R}$. Some of the following questions refer to the recurrence on $T[i, v]$ of Section 3, but they can be answered without knowing the correct answers to the questions of that section.

Question 19: What is the space complexity of a dynamic program obtained directly (that is, without improvements) from the recurrence on $T[i, v]$ ?
A $\Theta(n \cdot m)$
(B) $\Theta\left(n \cdot m^{2}\right)$
C $\Theta\left(n^{2}\right)$
(D $\Theta\left(n^{2} \cdot m\right)$
E $\Theta\left(n^{3}\right)$

Question 20: What is the tightest time complexity of a dynamic program for the recurrence on $T[i, v]$ ?
A $\mathcal{O}(n \cdot m)$
(B) $\mathcal{O}\left(n \cdot m^{2}\right)$
(C) $\mathcal{O}\left(n^{2}\right)$
(D) $\mathcal{O}\left(n^{2} \cdot m\right)$
E $\mathcal{O}\left(n^{3}\right)$

Question 21: A useful decision version of the shortest path problem, which is an optimisation problem, asks whether from a given source vertex to a given target vertex there exists a path whose weight, say $\omega$, is in what relationship to a given weight $W$ ?
A $\omega=n-1$
(B) $\omega=n$
C] $\omega \leq W$
(D) $\omega=W$
E $\omega \geq W$

Question 22: What is the tightest time complexity class, as currently known, of a decision version of the single-source shortest paths problem, considering the existence of a dynamic program for the recurrence on $T[i, v]$ ?
(A) P
B pseudo-
C NP-
(D) NP-hard
E we do not know

Question 23: What is the tightest time complexity class, as currently known, of a decision version of the optimisation problem of finding longest paths from a given source vertex to all other vertices of a weighted directed graph?
(A) $P$
B pseudo-
C NPpolynomial complete
(D) NP-hard
E we do not know

## Corrected

## Answer Sheet - AD2 Exam (1DL231) of 16 December 2016

Instructions: Do not alter the drawing above. Using a very dark colour, fill in entirely at most one answer box (A to E) per question: we will use an optical character recognition (OCR) system that ignores circles, crosses, ticks, etc. Transfer your answers from the question sheets to this answer sheet just before handing in; if an answer becomes ambiguous to an OCR system, then please request another answer sheet. Every correct answer gives 2 points. Every multiple answer or incorrect answer gives 0 points. Partial credit of 1 point may be given in exceptional circumstances. If you think a question is unclear or wrong, then mark its number with a $\star$ on this sheet, and explain on the backside of this sheet what your difficulty with the question is and what additional assumption underlies the candidate answer that you have chosen or the new answer that you indicate.

|  |  | Grade |  | Cond |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 5 |  | $38 \leq e$ |
| Grading: Your grade is as follows, wh | our mark is e points: | 4 |  | $30 \leq e$ |
|  |  | 3 |  | $23 \leq$ |
|  |  | U |  | $00 \leq$ |
| 1 String Matching | Question 11: A |  | D | E |
|  | Question 12: | C | D | E |
| Question 1: A 为 B C | Question 13: A |  | D |  |
| Question 2: A B D E | Question 13. A |  | D |  |
|  | Question 14: A |  | D | E |
| Question 4: A B C | Question 15: A | C | D |  |
|  | Question 16: A | B C |  | E |
| 2 Greedy Algorithms | Question 17: A | C | D |  |
| Question 5: $\square$ B $\square$ C $\square$ | Question 18: A | C | D |  |
| Question 6: A B D E | 4 Complexit |  |  |  |
| Question 7: A D C D E | 4 Complexity |  |  |  |
| Question 8: $\square$ B C D E | Question 19: A | B | D | E |
| Question 9: A $\square$ C D E | Question 20: | B C | D | E |
| 3 Dynamic Programmin | Question 21: A |  | D | E |
| 3 Dynamic Programmin | Question 22: | C |  | E |
| Question 10: A $C$ C D E | Question 23: A | B | D | E |

Again: Please use a very dark colour to fill in your chosen boxes entirely!
$\square$

