Algorithms & Data Structures 2 (1DL231)   **Exam of 17 December 2015**

Instructions: This is a multiple-choice exam, to save you the time of tidying up the presentation of your answers. There is exactly *one* correct answer per question. You can keep the question sheets and should *hand in only the answer sheet* at the end: you are *not* expected to explain your answers. Normally, a teacher will attend this exam from 15:00 to 16:00. *Also read the instructions on the answer sheet before starting*.

1   Greedy Graph Algorithms

Consider the following connected weighted graph, provided twice for your convenience:
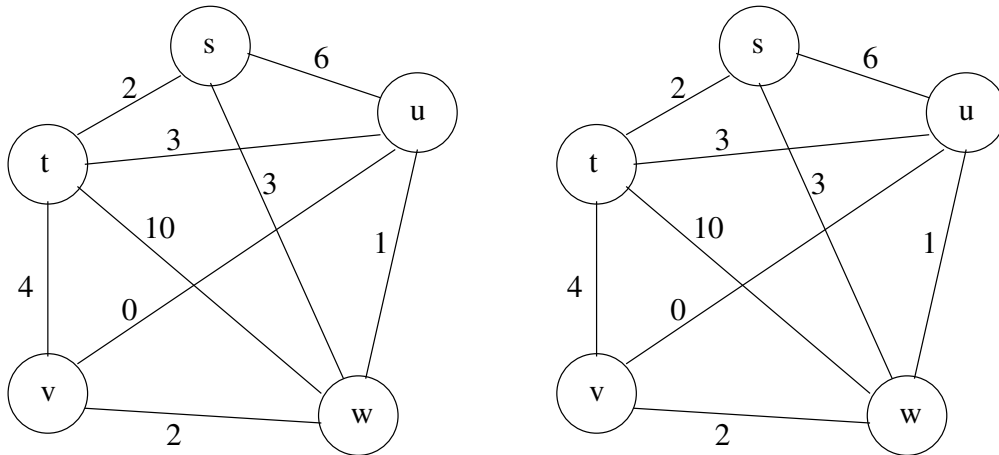
Recall that Prim's minimum spanning tree algorithm and Dijkstra's single-source shortest paths algorithm both take $\Theta(V) \cdot \mathrm{T_{EXTRACT\text{-}MIN}} + \Theta(E) \cdot \mathrm{T_{DECREASE\text{-}KEY}}$ time on a graph with vertex set V and edge/arc set E. Answer the following questions after performing both algorithms (all edges being bidirectional when seen as arcs) starting from vertex 's' and, when dequeueing, breaking ties by choosing the vertex that has had the minimum priority for the longest time:

Question 1: What is the sum of the priorities of vertex 'v' after Prim's 2nd and 3rd iterations?

|A| 0 |B| 2 |C| 4 |D| 6 |E| 8

Question 2: What is the sum of the priorities of vertex 'v' after Dijkstra's 2nd and 3rd iterations?

|A| 9 |B| 10 |C| 11 |D| 12 |E| 14

Answer the following general questions about Prim's and Dijkstra's algorithms for a graph with vertex set V and edge/arc set E:

Question 3: Under what condition on V and E should one implement the min-priority queue of these algorithms with a binary heap or a binomial heap, rather than with an array, indexed by the vertices, which is generally worse?

|A| always |C| if $|E| = |V| - 1$ |E| never
|B| if $|E| < |V|^2 / \lg |V|$ |D| if $|E| > |V|^2 / \lg |V|$
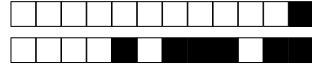
Question 4: Assuming distinct weights, under what condition on V and E does the edge with the *lowest* weight belong to a minimum spanning tree?

A always

B if $|E| < |V|^2 / \lg |V|$

C if $|E| > |V| - 1$

D if $|E| > |V|^2 / \lg |V|$

E never

Question 5: Assuming distinct weights and at least three edges, under what condition on V and E does the edge with the *highest* weight belong to a minimum spanning tree?

A always

B if $|E| < |V|^2 / \lg |V|$

C if $|E| = |V| - 1$

D if $|E| > |V|^2 / \lg |V|$

E never

Question 6: Under what condition do the shortest paths computed by Dijkstra's algorithm form a spanning tree?

A always

B if there is optimal substructure

C if the graph is strongly connected

D if all arc weights satisfy the triangle inequality

E never

2 Dynamic Programming

Given n objects of known volumes $v_1, \ldots, v_n$ and prices $p_1, \ldots, p_n$, as well as a bag of known volume V, we would like to determine the total price of the most expensive subset of the given objects that fits into the given bag. All numbers are positive integers. For example, given four objects of volumes $2, 1, 3, 2$ and prices $12, 10, 20, 15$ respectively, the most expensive subset fitting into a bag of volume $V = 5$ includes all except the third object; it has total volume $2 + 1 + 2 = 5 \leq V$ and total price $12 + 10 + 15 = 37$. Consider the following recurrence for a quantity $T[i, v]$, parameterised by $\alpha, \beta_1, \ldots, \beta_5$, and γ:

$$T[i, v] = \begin{cases} 0 & \text{if } \alpha \\ T[i-1, \beta_1] & \text{if } \neg\alpha \text{ and } v < \beta_2 \\ \gamma(T[i-1, \beta_3], \ T[i-1, \beta_4] + \beta_5) & \text{if } \neg\alpha \text{ and } v \geq \beta_2 \end{cases}$$

Question 7: If $T[n, V]$ is returned by a correct algorithm for the specification above, then the integer $T[i, v]$, with $0 \leq i \leq n$ and $0 \leq v \leq V$, denotes the total price of ...

A ... the most expensive subset of exactly i objects that fits into a bag of volume v.

B ... the most expensive subset of at least i objects that fits into a bag of volume v.

C ... the most expensive subset of at most i objects that fits into a bag of volume v.

D ... the most expensive subset of the first i objects that fits into a bag of volume v.

E ... the most expensive subset of i random objects that fits into a bag of volume v.

Question 8: For the instance in the example above, what is the sum $T[3, 3] + T[3, 4]$?

A 42

B 44

C 50

D 52

E other

Question 9: What is the logical condition α?

A $i = 0$

B $v = 0$

C $v = V$

D $v = i$

E $v + i = 0$
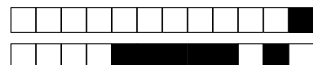
Question 10: What is the sum of the numeric expressions $\beta_1, \ldots, \beta_5$?

$\boxed{\text{A}}$ $3v - 2v_i + p_i$ $\qquad$ $\boxed{\text{C}}$ $3v + p_i$ $\qquad$ $\boxed{\text{E}}$ $3v + v_i + p_i - 3$

$\boxed{\text{B}}$ $3v - v_i + p_i$ $\qquad$ $\boxed{\text{D}}$ $3v + p_i - 2$

Question 11: What is the two-argument operator γ (written in infix form above)?

$\boxed{\text{A}}$ $+$ $\qquad$ $\boxed{\text{B}}$ $-$ $\qquad$ $\boxed{\text{C}}$ $\cdot$ $\qquad$ $\boxed{\text{D}}$ max $\qquad$ $\boxed{\text{E}}$ min

Question 12: What is an ordering of the indices i and v under which the elements $T[i, v]$ of the array T can be filled without performing any redundant computations?

$\boxed{\text{A}}$ any order $\qquad\qquad\qquad\qquad$ $\boxed{\text{D}}$ by lexicographically increasing $\langle -i, v\rangle$

$\boxed{\text{B}}$ by lexicographically increasing $\langle v, i\rangle$

$\boxed{\text{C}}$ by lexicographically decreasing $\langle i, v\rangle$ $\qquad$ $\boxed{\text{E}}$ by lexicographically increasing $\langle -v, i\rangle$

We say that $\langle a, b\rangle$ is lexicographically smaller than $\langle c, d\rangle$ if either $a < c$ or $a = c \land b < d$.

3 Disjoint Sets

Consider the forest on the right, representing disjoint sets for the integers 0 to 8. Assume the ranks of the trees *initially* are *equal* to their heights, measured as numbers of arcs. Assume the FIND-SET operation performs **path compression**. Assume the UNION operation follows the **union-by-rank** strategy. (Ignore the fact that this forest cannot be obtained using this UNION operation when starting from singleton sets.) Answer the following questions, **continuing at each question from the result of performing the operation of the previous question**:
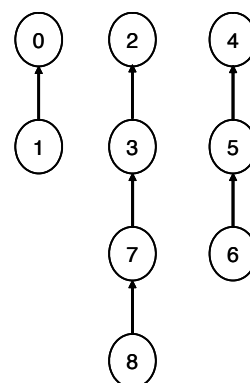
Question 13: Which element is the parent of element 0 after UNION$(0, 2)$?

$\boxed{\text{A}}$ 0 is a root $\qquad$ $\boxed{\text{B}}$ 1 $\qquad$ $\boxed{\text{C}}$ 2 $\qquad$ $\boxed{\text{D}}$ 3 $\qquad$ $\boxed{\text{E}}$ another element

Question 14: What is the rank of the tree with element 4 after UNION$(2, 4)$?

$\boxed{\text{A}}$ 0 $\qquad$ $\boxed{\text{B}}$ 1 $\qquad$ $\boxed{\text{C}}$ 2 $\qquad$ $\boxed{\text{D}}$ 3 $\qquad$ $\boxed{\text{E}}$ another value

Question 15: Which element is the parent of element 6 after FIND-SET(6)?

$\boxed{\text{A}}$ 6 is a root $\qquad$ $\boxed{\text{B}}$ 2 $\qquad$ $\boxed{\text{C}}$ 4 $\qquad$ $\boxed{\text{D}}$ 5 $\qquad$ $\boxed{\text{E}}$ another element

Question 16: What is the rank of the tree with element 8 after FIND-SET(8)?

$\boxed{\text{A}}$ 0 $\qquad$ $\boxed{\text{B}}$ 1 $\qquad$ $\boxed{\text{C}}$ 2 $\qquad$ $\boxed{\text{D}}$ 3 $\qquad$ $\boxed{\text{E}}$ another value

4   P versus NP

Complete the two sentences and answer the next question, under **current** knowledge:

Question 17: A decision problem is in NP if and only if its answer takes . . .

- $\boxed{A}$ . . . non-polynomial time to check.
- $\boxed{B}$ . . . non-polynomial time to compute.
- $\boxed{C}$ . . . polynomial time to check.
- $\boxed{D}$ . . . polynomial time to compute.
- $\boxed{E}$ . . . possibly forever to compute.

Question 18: A decision problem is NP-hard if and only if . . .

- $\boxed{A}$ . . . it can be reduced in polynomial time to every problem in NP.
- $\boxed{B}$ . . . it can be reduced in polynomial time to every NP-complete problem.
- $\boxed{C}$ . . . every problem in P can be reduced in polynomial time to it.
- $\boxed{D}$ . . . every problem in NP can be reduced in polynomial time to it.
- $\boxed{E}$ . . . every NP-complete problem can be reduced in polynomial time to it.

Question 19: When can all instances of any NP-complete problem be solved in polytime?

- $\boxed{A}$ always
- $\boxed{B}$ if and only if $P \subsetneq NP$
- $\boxed{C}$ if and only if $P = NP$
- $\boxed{D}$ never
- $\boxed{E}$ we do not know

Give the **smallest** complexity class, as **currently** known, for each of the next problems:

Question 20: The decision version of the problem of Section 2, when the bag volume V must be exactly reached and the notion of price is dropped.

- $\boxed{A}$ P
- $\boxed{B}$ NP-complete
- $\boxed{C}$ NP-hard
- $\boxed{D}$ un-decidable
- $\boxed{E}$ we do not know

Question 21: The problem of determining the existence of a flow of at least a given value within a flow network with rational capacities.

- $\boxed{A}$ P
- $\boxed{B}$ NP-complete
- $\boxed{C}$ NP-hard
- $\boxed{D}$ un-decidable
- $\boxed{E}$ we do not know

Question 22: The problem of determining the existence of an occurrence of a given string within a given longer string.

- $\boxed{A}$ P
- $\boxed{B}$ NP-complete
- $\boxed{C}$ NP-hard
- $\boxed{D}$ un-decidable
- $\boxed{E}$ we do not know

Question 23: The problem of determining the existence of a plan for a robotic arm to drill holes at given points on a computer chip and then to return to its starting point, all this within a given amount of time.

- $\boxed{A}$ P
- $\boxed{B}$ NP-complete
- $\boxed{C}$ NP-hard
- $\boxed{D}$ un-decidable
- $\boxed{E}$ we do not know

Answer Sheet — Exam 1DL231 of 17 December 2015

Instructions: Do **not** alter the drawing above. Using a **dark** colour, **fill in** entirely **at most one** answer box (A to E) per question: we will use an optical character recognition (OCR) system that ignores crosses, circles, etc. Transfer your answers from the question sheets to this answer sheet **just before handing in**; if an answer becomes ambiguous to an OCR system, then please request another answer sheet. Every correct answer gives 2 points. Every multiple answer or incorrect answer gives 0 points. Partial credit of 1 point may be given in exceptional circumstances. If you think a question is unclear or wrong, then mark its number with a ⋆ on **this sheet**, and explain **on the backside of this sheet** what your difficulty with the question is **and** what additional assumption underlies the candidate answer that you have chosen or the new answer that you have indicated.

Grading: Your grade is as follows, when your mark is e points:

Grade	Condition
5	$38 \le e \le 46$
4	$30 \le e \le 37$
3	$23 \le e \le 29$
U	$00 \le e \le 22$

1 Greedy Graph Algorithms

Question 1: A B C ■ E

Question 2: A B ■ D E

Question 3: A ■ C D E

Question 4: ■ B C D E

Question 5: A B ■ D E

Question 6: A B ■ D E

2 Dynamic Programming

Question 7: A B C ■ E

Question 8: A B C ■ E

Question 9: ■ B C D E

Question 10: A B ■ D E

Question 11: A B C ■ E

Question 12: A ■ C D E

3 Disjoint Sets

Question 13: A B ■ D E

Question 14: A B C ■ E

Question 15: A ■ C D E

Question 16: A B C ■ E

4 P versus NP

Question 17: A B ■ D E

Question 18: A B C ■ E

Question 19: A B ■ D E

Question 20: A ■ C D E

Question 21: ■ B C D E

Question 22: ■ B C D E

Question 23: A ■ C D E

Again: Please use a **dark** colour to **fill in** your chosen boxes **entirely**!

Your anonymous exam code: ☐ ☐ ☐ ☐ ☐ ☐