Mikael Passare 1959—2011

Christer O. Kiselman

Mikael Passare died from a sudden cardiac arrest in Oman in the evening of September 15, 2011. His next of kin are his wife Galina Passare, his son Max, and his daughter Märta.

Mikael was born in Västerås, Sweden, on January 01, 1959, and pursued a fast and brilliant career as a mathematician. He started his studies at Uppsala University in the fall of 1976 while still a high-school student, merely seventeen and a half. He finished high school in June 1978 at the Rudbeckianska skolan in Västerås, gave his first seminar talk in November 1978 at Uppsala University, where he got his Bachelor Degree in 1979, and where he also was an assistant. He was then a PhD student with me as advisor, and presented his thesis on December 15, 1984. He was a research assistant (half time) and lecturer (half time) at Stockholm University from January through June 1985, research assistant financed by the Science Research Council (NFR) July 1985 – August 1986 and later research assistant, July 1987 – 1990. He received the title of *Docent* (corresponding to the *Habilitation* in some countries) on January 28, 1988. He was a senior university lecturer (full time) from July 1988, from time to time on leave of absence. Later he was a research lecturer at the Royal Institute of Technology, July 1990 – 1994. He was appointed full professor at Stockholm University from October 01, 1994.

In addition to English and French, he studied Russian during three years in high school (Rudbeckianska skolan 1978), and later did his military service at the National Defence Radio Establishment (the Swedish national authority for signals intelligence) from June 1979 through August 1980. There he learned even more Russian.

He spent four academic years in four different countries: during the academic year 1980-81 he was at Stanford University; during 1981-82 at Lomonosov University in Moscow; during 1986-87 at Université Pierre et Marie Curie, Paris VI (he was also often at Orsay, Université Paris-Sud 11), and then, during 1992-93, he was at Humboldt-Universität zu Berlin. He was a guest professor in France on several occasions: at Toulouse (June 1988), Grenoble (April 1992), Bordeaux (May 1992), Paris VII (March 1993), Lille (April 1999), and Bordeaux again (June 2000).

Mikael was awarded the Lundström–Åman Scholarship during the Fall Semester of 1984 and the Spring Semester of 1985, the Marcus and Marianne Wallenberg Prize in 1988, the Lilly and Sven Thuréus Prize in 1991, and the Göran Gustafsson Prize in 2001.

Mikael was much appreciated as a researcher and teacher, and was very active outside the university. He was Head of the Department of Mathematics at Stockholm University from January 2005 through August 2010, and then Director of the newly created Stockholm Mathematics Center, common to Stockholm University and the Royal Institute of Technology. When Burglind Juhl-Jöricke and Oleg Viro had resigned from Uppsala University on February 08, 2007, he arranged for a guest professorship for Burglind at Stockholm University, and was one of the organizers of a big conference in honor of Oleg, *Perspectives in Analysis, Geometry, and Topology*, at Stockholm University during seven days, May 19–25, 2008.

As chair of the Swedish National Committee for Mathematics, he led the Swedish delegation to the General Assembly of the International Mathematical Union in Bangalore, Karnataka, India, in August 2010.

Mikael Passare was Deputy Director for Institut Mittag-Leffler, Djursholm, Sweden, from 2010. He was very much appreciated for his activity there, which included organizing the Felix Klein Days for teachers and a research school for high-school students.

Starting in July 2001, he served during ten years as one of the editors of the *Arkiv för matematik* (Ari Laptev, personal communication 2011-10-19). During the period 2004-04-01—2009-06-25 he was one of the Associate Editors for the *Journal of Mathematical Analysis and Applications* (Don Prince, personal communication 2011-10-13).

Mikael was a member of the Swedish Committee for Mathematics Education (SKM) from January 1997, when SKM started its activity, until December 2004. Mikael's efforts in SKM can only be explained by his firm dedication to mathematics education in the schools. He participated actively by organizing meetings, authoring reports to policy makers, and influencing politicians and officials at the Ministry of Education and the Swedish National Agency for Education. (Gerd Brandell, personal communication 2011-10-17.)

The activity of SKM to which he devoted most of his energy was the international competition *International Mathematical Kangaroo*, originally *Kangourou sans frontières*, in Swedish *Kängurun — Matematikens Hopp*. He took the initiative to start, with SKM as organizer, a Swedish version of this competition in 1999. He translated the problems, which arrived in English or French, up to 2009. He checked also that the mathematical content was correct after the necessary adaption to Swedish traditions in problem formulation and other circumstances. He participated, at least during the first five to six years, in the choice of problems to the Swedish edition, and he continued his commitment to the competition even after his time in SKM. The competition is run by SKM in cooperation with the National Centre for Mathematics Education, NCM. In 2010, more than 80,000 students at all levels participated. (Gerd Brandell, personal communication 2011-10-17; Karin Wallby, personal communication 2011-10-19.)

Mikael was a member of the Steering Group for the National Graduate School in Mathematics Education from March 2000, until it ceased in December 2006. The school, which had about twenty PhD students, was financed by the Bank of Sweden Tercentenary Foundation and the Swedish Research Council. He actively

and constructively contributed to shaping the education of the Graduate School, both in his role as member of the Steering Group and by participating in many meetings between PhD students and advisors. He was project leader for the school's activity at the Department of Mathematics at Stockholm University. Of the school's PhD students, two were at Stockholm: Kirsti Hemmi Löfwall and Andreas Ryve, who both got their PhDs in 2006. (Gerd Brandell, personal communication 2011-10-17.)

The Sonja Kovalevsky School in Stockholm, a private elementary school, started its activity in the Fall of 1999. Its profile includes chess, mathematics, and Russian. The aim was, among other things, to put to use educational experience from Russia. Mikael was a member of the school's Board from the beginning.

At the time of his death, Mikael was President of the Swedish Mathematical Society and also a member of the Committee for Developing Countries (CDC) of the European Mathematical Society. His activity for mathematics in Africa is described in a later section.

Mikael's nine PhD students

Mikael served as advisor of nine PhD students who successfully completed their degrees. They are registered in the *Mathematics Genealogy Project* and are:

Yang Xing, 1992, Stockholm University: *Zeros and Growth of Entire Functions of Several Variables, the Complex Monge–Ampère Operator and Some Related Topics.*

Mikael Forsberg, 1998, The Royal Institute of Technology: *Amoebas and Laurent Series.*

Lars Filipsson, 1999, The Royal Institute of Technology: *On Polynomial Interpolation and Complex Convexity.*

Timur Sadykov, 2002, Stockholm University: *Hypergeometric Functions in Several Complex Variables.* Although not listed in the Genealogy Project, August Tsikh served as a coadvisor for Timur (Timur Sadykov, personal communication 2011-11-26; August Tsikh, personal communication 2011-12-06).

Hans Rullgård, 2003, Stockholm University: *Topics in Geometry, Analysis and Inverse Problems.*

Johan Andersson, 2006, Stockholm University: *Summation Formulae and Zeta Functions.*

Alexey Shchuplev, 2007, Stockholm University: *Toric Varieties and Residues.* August Tsikh was second advisor.

David Jacquet, 2008, Stockholm University: *On Complex Convexity.*

Lisa Nilsson, 2009, Stockholm University: *Amoebas, Discriminants, and Hypergeometric Functions.* August Tsikh was second advisor.

Mikael's mathematics

Residue theory

Mikael soon became known as an eminent researcher in complex analysis in several variables, where his thesis was an important breakthrough with new results in residue theory. Its title was *Residues, Currents, and Their Relation to Ideals of Holomorphic Functions* [1984], and it was later published in [1988c].

Residue theory in several variables is a notoriously difficult part of complex analysis. Mikael's work was inspired by that of Miguel E. M. Herrera (1938–1984). Miguel and I were together at the Institute for Advanced Study in Princeton during the academic year 1965-66, and it was there that I learned about residues from him.[1] His results, which culminated in the paper by Herrera and Lieberman (1971) and the much quoted book by Coleff and Herrera (1978), were well known long before these publications. I could somehow serve as mediator to Mikael for this interest without doing much research on residues myself.

Also Alicia Dickenstein, who was a student of Miguel and got her PhD at Buenos Aires in 1982, knew this theory very well and soon came into contact with Mikael. As for integral formulas, Mikael took advice from Bo Berndtsson, already then a renowned expert in that field.

Another important person for Mikael's mathematical development was Gennadi Henkin. They met in Moscow during the academic year 1981-82, afterwards several times in the period 1984–1990, and then in Paris and Stockholm in 1991–2010. During these meetings they discussed, in particular, integral formulas of Cauchy–Leray type and applications from the papers of Gennadi and Bo (starting with Henkin 1969).

While residues in one complex variable have been well understood for a long time, the situation is quite different in several variables. There were pioneers like Henri Poincaré (1854–1912) and Jean Leray (1906–1998). Alexandre Grothendieck developed a residue theory in higher dimensions, but it was quite abstract. Through work by Miguel Herrera, François Norguet and Pierre Dolbeault, the theory could be linked to distribution theory, which had been developed by Laurent Schwartz (1915–2002), and that was the road that Mikael continued to follow. He worked intensively with August Tsikh, both on residue theory and amoebas.

Residues in one complex variable

In one complex variable we can observe that there is a lot of symmetry:

$$\int_{\varepsilon < |z| < r} z^j \bar{z}^k f(|z|) dx \wedge dy = 0, \qquad j, k \in \mathbf{Z}, \quad j \neq k.$$

This means that heavy masses are balanced, and implies that, when calculating residues, it is enough to work with the principal value, PV (*valeur principale*, VP); we

---

[1] He also introduced me to a great writer: Ursula LeGuin.

need not use the more difficult and unstable construction of the finite part, FP (*partie finie*, PF). (In real analysis, the finite part inevitably appears: the distribution on the real axis given by the function $\log|x|$, $x \in \mathbf{R}$, has the derivative $\mathrm{PV}(1/x)$ and the second derivative $-\mathrm{FP}(1/x^2)$.)

If we write a smooth function φ as $\varphi(z) = P(z) + R(z)$, where P is a polynomial of degree at most $m - 1$, and $R(z)/z^m$ is bounded near the origin, we obtain

$$\int\limits_{\varepsilon < |z| < r} \frac{\varphi(z)}{z^m} dx \wedge dy = \int\limits_{\varepsilon < |z| < r} \frac{R(z)}{z^m} dx \wedge dy \to \int\limits_{|z| < r} \frac{R(z)}{z^m} dx \wedge dy \text{ as } \varepsilon \to 0.$$

We define the *principal value* $\mathrm{PV}(1/z^m)$ of $1/z^m$ by

$$\left\langle \mathrm{PV}\left(\frac{1}{z^m}\right), \varphi \right\rangle = \mathrm{PV} \int\limits_{\mathbf{C}} \frac{\varphi(z)}{z^m} dx \wedge dy = \lim_{\varepsilon \to 0} \int\limits_{\varepsilon < |z|} \frac{\varphi(z)}{z^m} dx \wedge dy,$$

which exists for all test functions $\varphi \in \mathscr{D}(\mathbf{C})$. If now f/g is meromorphic with a pole at the origin, we obtain

$$\left\langle \mathrm{PV}\left(\frac{f}{g}\right), \varphi \right\rangle = \mathrm{PV} \int\limits_{\mathbf{C}} \frac{f(z)}{g(z)} \varphi(z) dx \wedge dy, \qquad \varphi \in \mathscr{D}(\mathbf{C}),$$

and we define the *residue* $\mathrm{res}(f/g)$ of f/g as

$$\mathrm{res}\left(\frac{f}{g}\right) = \frac{\partial}{\partial \bar{z}} \mathrm{PV}\left(\frac{f}{g}\right) \in \mathscr{D}'(\mathbf{C}).$$

Residues in several variables

Let f and g be holomorphic functions of n complex variables. The *principal value* $\mathrm{PV}(f/g)$ of f/g is a distribution defined by the formula

$$\left\langle \mathrm{PV}\left(\frac{f}{g}\right), \varphi \right\rangle = \lim_{\varepsilon \to 0} \int\limits_{|g| > \varepsilon} \frac{f\varphi}{g} = \lim_{\varepsilon \to 0} \int \frac{\chi f \varphi}{g}, \qquad \varphi \in \mathscr{D}(\mathbf{C}^n),$$

where $\chi = \chi(|g|/\varepsilon)$ and χ is a smooth function on the real axis satisfying $0 \leqslant \chi \leqslant 1$ and $\chi(t) = 0$ for $t \leqslant 1$, $\chi(t) = 1$ for $t \geqslant 2$ (in [1985:727] when the $f_j = 1$; in [1988:39] in general).

The *residue current* is $\bar{\partial} \mathrm{PV}(f/g)$. Can the products

$$(\mathrm{PV}(f_1/g_1))(\mathrm{PV}(f_2/g_2)), \quad \left(\bar{\partial}(\mathrm{PV}(f_1/g_1))\right)(\mathrm{PV}(f_2/g_2))$$

and other similar products be defined?

Schwartz proved (1954) that it is in general impossible to multiply two distributions while respecting the associative law. He indicated three distributions $u, v, w \in \mathscr{D}'(\mathbf{R})$ where uv, vw, $(uv)w$ and $u(vw)$ all have a good meaning, but where $(uv)w \neq u(vw)$. He took $u = \mathrm{PV}(1/x)$, the principal value of $1/x$; v as the

identity, i.e., the smooth function $v(x) = x$, which can be multiplied to any distribution; and $w = \delta$, the Dirac measure placed at the origin. Then we have $uv = 1$, $(uv)w = \delta$, while $vw = 0$, $u(vw) = 0$. Hence there is no associative multiplication.

Mikael's construction of residue currents goes as follows. Take $f = (f_1, \ldots, f_{p+q})$, $g = (g_1, \ldots, g_{p+q})$, two $(p+q)$-tuples of holomorphic functions, and consider the limit

$$\lim_{\varepsilon_j \to 0} \frac{f_1}{g_1} \cdots \frac{f_{p+q}}{g_{p+q}} \bar{\partial} \chi_1 \wedge \cdots \wedge \bar{\partial} \chi_p \cdot \chi_{p+1} \cdots \chi_{p+q},$$

where $\chi_j = \chi(|g_j|/\varepsilon_j)$, and the ε_j tend to zero in some way.

Coleff and Herrera (1978:35–36) took $q = 0$ or 1, and assumed that ε_j tends to zero much faster than ε_{j+1}, which in this context means that $\varepsilon_j/\varepsilon_{j+1}^m \to 0$ for all $m \in \mathbf{N}$ and $j = 1, \ldots, p+q-1$; thus it is almost an iterated limit. This gives rise to the strange situation that, in general, the limit depends on the order of the functions (and is not just an alternating product).

Mikael took instead $\varepsilon_j = \varepsilon^{s_j}$ for fixed $s_1, \ldots, s_{p+q}$. The limit, which will be written as $R^p P^q[f/g](s)$, where we now write $[\ldots]$ for the principal value, does not exist for arbitrary s_j. But he proved [1985:728] that, if we remove finitely many hyperplanes, then $R^p P^q[f/g](s)$ is locally constant in a finite subdivision of the simplex

$$\Sigma = \{s \in \mathbf{R}^{p+q};\ s_j > 0,\ \ \sum s_j = 1\},$$

so that the mean value

$$R^p P^q \left[\frac{f}{g}\right] = \fint_\Sigma R^p P^q \left[\frac{f}{g}\right](s) = \bar{\partial}\left[\frac{f_1}{g_1}\right] \wedge \cdots \wedge \bar{\partial}\left[\frac{f_p}{g_p}\right] \cdot \left[\frac{f_{p+1}}{g_{p+1}}\right] \cdots \left[\frac{f_{p+q}}{g_{p+q}}\right]$$

exists (Definition A in [1987]). This is the product of p residue currents and q principal-value distributions.

In the little paper [1993c], based on his talk when accepting the Thuréus Prize in 1991, he discusses the possibility of defining the product $\mathrm{PV}(1/x)\delta$ on the real axis, and finds that it should be $-\frac{1}{2}\delta'$, which is the mean value of $-\delta'$ and zero. This is an analogue in real analysis to the mean value over Σ which he considered in the complex case.

Leibniz' rule for the derivative of a product and some other rules of calculus hold; for example we have [1988d:43]:

$$\left[\frac{1}{z_1}\right]\left[\frac{z_1}{z_2}\right] = \left[\frac{1}{z_2}\right],$$

which yields

$$\left(\bar{\partial}\left[\frac{1}{z_1}\right]\right)\left\{\left[\frac{1}{z_1}\right]\left[\frac{z_1}{z_2}\right]\right\} = \left(\bar{\partial}\left[\frac{1}{z_1}\right]\right)\left[\frac{1}{z_2}\right],$$

while

$$\left\{\left(\bar{\partial}\left[\frac{1}{z_1}\right]\right)\left[\frac{1}{z_1}\right]\right\}\left[\frac{z_1}{z_2}\right] = \frac{1}{2}z_1\left(\bar{\partial}\left[\frac{1}{z_1^2}\right]\right)\left[\frac{1}{z_2}\right] = \frac{1}{2}\left(\bar{\partial}\left[\frac{1}{z_1}\right]\right)\left[\frac{1}{z_2}\right].$$

Thus the associative law does not hold.

We saw in Schwartz' example that an associative multiplication is impossible in general; the example shown here makes us wonder whether it is possible to define an associative multiplication in some algebra of principal-value distributions and residue currents. Can we find an interesting non-associative algebra of principal-value distributions and residue currents?

For complete intersections, i.e., when the set of common zeros of f_1, f_2, ..., f_p has maximal codimension, Mikael established a division formula with remainder term:

$$h = \sum_1^p g_j f_j + h \cdot \text{Res},$$

where Res is the residue current, which is a factor in the remainder term $h \cdot \text{Res}$ and has the property that $f_j \cdot \text{Res} = 0$ for all j. This implies that h belongs to the ideal generated by $f_1, \ldots, f_p$ if and only if $h \cdot \text{Res}$ vanishes. This is a beautiful characterization of the ideals of holomorphic functions and explains the choice of title in the papers [1984, 1986, 1988c]. The characterization of the ideals with the help of residues was proved independently and about the same time by Alicia Dickenstein and Carmen Sessa (1985:424).

This characterization of ideals enabled Mikael and Bo to formulate an elegant and explicit variant of Leon Ehrenpreis' Fundamental Principle; it was published in a joint paper with Bo [1989]. Later, Mats Andersson and Elizabeth Wulcan (2007) could define a residue without the assumption of a complete intersection. In their work the paper [2000b] by Mikael, August and Alain Yger played an important role.

For complete intersections we have, according to [1988d:42, Theorem 4 iii)],

$$g_j R^p P^1 [1/g] = 0, \qquad g = (g_1, \ldots, g_{p+1}), \quad j = 1, \ldots, p,$$

while the example [1988d:43, Example 3] shows that this is not true when we do not have complete intersections: with $n = 2$, $p = 2$, $q = 0$, $g_1(z) = z_1 z_2$, $g_2(z) = z_2$ we get

$$\bar{\partial} \left[\frac{1}{z_1 z_2} \right] \wedge \bar{\partial} \left[\frac{1}{z_2} \right] = \frac{1}{2} \bar{\partial} \left[\frac{1}{z_1} \right] \wedge \bar{\partial} \left[\frac{1}{z_2^2} \right];$$

when we multiply this current by g_2, we get

$$\frac{1}{2} z_2 \bar{\partial} \left[\frac{1}{z_1} \right] \wedge \bar{\partial} \left[\frac{1}{z_2^2} \right] = \frac{1}{2} \bar{\partial} \left[\frac{1}{z_1} \right] \wedge \bar{\partial} \left[\frac{1}{z_2} \right] \neq 0.$$

Other examples in this calculus are [1988d:43]:

$$\left[\frac{1}{z_1} \right] \bar{\partial} \left[\frac{1}{z_2^2} \right] = 2 \left[\frac{1}{z_1 z_2} \right] \bar{\partial} \left[\frac{1}{z_2} \right] \text{ and } \bar{\partial} \left[\frac{1}{z_1} \right] \wedge \bar{\partial} \left[\frac{1}{z_2^2} \right] = 2 \bar{\partial} \left[\frac{1}{z_1 z_2} \right] \wedge \bar{\partial} \left[\frac{1}{z_2} \right].$$

The original definition and the definition which uses meromorphic extension agree [1987:159]:

$$R^p P^q \left[\frac{1}{g} \right] = \lim_{\varepsilon \to 0} \frac{\bar{\partial} |g_1|^\varepsilon}{g_1} \wedge \cdots \wedge \frac{\bar{\partial} |g_p|^\varepsilon}{g_p} \cdot \frac{|g_{p+1}|^\varepsilon}{g_{p+1}} \cdots \frac{|g_{p+q}|^\varepsilon}{g_{p+q}}.$$

Here the left-hand side is defined according to Definition A already mentioned, while the right-hand side, called Definition B, is the one which comes from meromorphic extension (although this is not directly visible here).

In a CV which Mikael wrote in 2000 he mentions a book project with August Tsikh as coauthor and which had the title *Multidimensional Residues and Toric Varieties*. He gives a detailed table of contents of the five chapters in the book. Later they abandoned this project, since amoebas and tropical geometry became more interesting for them, and they aimed at writing a book on amoebas (August Tsikh, personal communication 2011-10-06).

Lineal convexity

André Martineau (1930–1972) gave a couple of seminars on lineal convexity (*convexité linéelle*) in Nice during the academic year 1967-68, when I was there. This is a kind of complex convexity which is stronger than pseudoconvexity and weaker than convexity. Since I was of the opinion that the results for this convexity property were too scattered in the literature and did not always have optimal proofs, I suggested that Mikael write a survey article on the topic.

On the one hand, this piece of advice was certainly very good, for he found a lot of results in cooperation with his friends Mats Andersson and Ragnar Sigurðsson (Mikael's mathematical uncle). On the other hand, it was perhaps not such a good suggestion, for the survey just kept growing, and two preprints were circulating starting in 1991[2]—and by then they had been busy writing for a long time already. The article became a book, and it did not appear until 2004 [2004b]. Anyway, it is thanks to André Martineau that lineal convexity came to be studied in the Nordic countries—and the book has become a standard reference.

In the book, the authors study in detail a property which Martineau called strong lineal convexity (*convexité linéelle forte*), and which he did not characterize geometrically. This notion, in the book called **C**-*convexity*, is not linked to any cleistomorphism (closure operator), since the intersection of two strongly lineally convex sets need not have the property. Therefore it has a different character than lineal convexity and usual convexity.

An important characterization of strong lineal convexity has been obtained recently by Gennadi Henkin and Peter Polyakov: a lineally convex compact set is strongly lineally convex if and only if it can be approximated in the Hausdorff metric by lineally convex compact sets with smooth boundaries (Henkin & Polyakov 2011: Proposition 2.4).

---

[2]I no longer possess any documentation about preprints from 1991, but in the CV that Mikael wrote in 2000, two are mentioned: Andersson, Mats; Passare, Mikael; Sigurdsson, Ragnar (1995), Complex convexity and analytic functionals I, Reykjavík, 71 pp.; and (2000), Complex convexity and analytic functionals II, Reykjavík and Sundsvall, 103 pp. The book [2004b] came to comprise xii + 160 pages.

Amoebas and tropical geometry

Mikael's later work is concerned with amoebas and coamoebas—the first publications in this field were Mikael Forsberg's thesis 1998 and their joint paper [2000a]. The spine of an amoeba—in mathematical zoology, amoebas are vertebrates—is a tropical hypersurface. Tropical mathematics is a rather new branch of mathematics, where addition and multiplication is replaced by the maximum operation and addition, somewhat similar to taking the logarithm of a sum and a product.[3] His interest in tropical mathematics was a break with his earlier work on complex analysis, which he once compared with my switching to digital geometry.

An amoeba is a set in $\mathbf{R}^n$ defined as follows. We define a mapping

$$\text{Log}\colon (\mathbf{C} \smallsetminus \{0\})^n \to \mathbf{R}^n \ \text{ by } \ \text{Log}(z) = (\log |z_1|, \log |z_2|, \ldots, \log |z_n|).$$

If f is a function defined in $(\mathbf{C} \smallsetminus \{0\})^n$, then its *amoeba* is the image under Log of its set of zeros. The term was introduced by Gelfand et al. (1994).

One can of course study the image in $\mathbf{R}^n$ of any set, but zero sets of certain functions have interesting properties. An amoeba is typically a closed semianalytic subset of $\mathbf{R}^n$ with tentacles which go out to infinity and separate the components of the complement of the amoeba. The number of such components is at most equal to the number of integer points in the Newton polytope for f if f is a Laurent polynomial; in certain cases equal to the latter number [2000a].

An easy example, which Mikael himself used in his lectures, is the zero set of the polynomial $P(z, w) = 1 + z + w$ of degree one. A zero $(z, w) \in \mathbf{C}^2$ must satisfy $1 \leqslant |z| + |w|$, $|z| \leqslant |w| + 1$ and $|w| \leqslant 1 + |z|$. It is easy to see that any point $(p, q) \in \mathbf{R}^2$ which satisfies the inequalities $1 \leqslant p + q$, $p \leqslant q + 1$ and $q \leqslant 1 + p$ is equal to $(|z|, |w|)$ for some zero (z, w) of P. (A useful observation here is the fact that the corresponding strict inequalities are the exact conditions under which there exists a triangle with side lengths 1, p and q.) The amoeba of P is then given by the three inequalities $1 \leqslant e^x + e^y$, $e^x \leqslant e^y + 1$ and $e^y \leqslant 1 + e^x$.

Of course one can study the zero set directly in ω without moving to Ω. That it nevertheless has interesting consequences to take the logarithm Mikael showed in [2008a]: it is about area preserving.

A *coamoeba* is defined analogously but with the mapping Log replaced by the mapping $\text{Arg}(z) = (\arg z_1, \arg z_2, \ldots, \arg z_n)$. Mikael wanted to establish formally the duality between amoebas and coamoebas, and he started to write a paper with Mounir Nisse [MS], which Mounir is now finishing (Mounir Nisse, personal communication 2011-11-13).

A straight line in the plane can be described by an equation

$$ax + by + c = 0,$$

and hence as the fold line of the convex function

$$f(x, y) = (ax + by + c) \vee 0, \qquad (x, y) \in \mathbf{R}^2,$$

---

[3]It seems that the first use of the adjective *tropical* in this sense in the title of a publication was in Simon (1988).

where the maximum operation is denoted by $\vee$: $s \vee t = \max(s, t)$, $s, t \in \mathbf{R}$. If we replace multiplication by addition and addition by the maximum operation, we obtain

$$g(x, y) = (a + x) \vee (b + y) \vee c, \qquad (x, y) \in \mathbf{R}^2.$$

A tropical straight line can therefore be defined as the fold lines for the function g, which consists of three rays. They emanate from the point $(c - a, c - b)$ in the directions $(1, 1)$, $(-1, 0)$ and $(0, -1)$. For example, the amoeba of the polynomial $P(z, w) = 1 + z + w$ mentioned earlier contains the tropical line emanating from $(0, 0)$, which is its spine.

If $p = (p_1, p_2)$ and $q = (q_1, q_2)$ are two points in the plane with

$$q_1 \neq p_1, \ q_2 \neq p_2 \text{ and } q_2 - q_1 \neq p_2 - p_1,$$

then it is easy to see that there is one and only one tropical straight line through p and q. If one of the conditions is not satisfied, there exist infinitely many tropical straight lines through p and q, but Mikael explained that one should accept only those lines that are stable under small perturbations; then you get a single line. In the same way, two distinct tropical straight lines meet in a single point if we only accept intersections that are stable under small perturbations.

Just like in spherical geometry there do not exist any distinct parallel lines. We can go on and ask about all the axioms of Euclidean geometry.

The similarity with taking the logarithm is based on the formulas

$$\log(x \times y) = \log x + \log y, \qquad x, y > 0, \text{ and}$$

$$\log x \vee \log y \leqslant \log(x + y) \leqslant \log 2 + (\log x \vee \log y), \qquad x, y > 0.$$

In the little paper [2008a], which is indeed a gem, Mikael shows how the concept of an amoeba can be used to show the well-known formula $\zeta(2) = \sum_1^\infty 1/n^2 = \pi^2/6 \approx 1.644934$ (the so called Basel problem).

The Pluricomplex Seminar

I started a seminar series in Uppsala in the 1970s. In the beginning it was more like a study group, and had no name, since I thought that a name could be hampering. But later I discovered that almost everything was about several complex variables, and during a visit to Strasbourg I saw that Jean-Pierre Ramis had used the name *Séminaire pluricomplexe*. That sounded mysterious enough, and I borrowed it to Uppsala. During the Fall Semester of 1980 the title was *Pluricomplex Analysis and Geometry*; in the Spring Semester of 1981 it was *Pluricomplex Analysis*, and from the Spring Semester of 1982 on, the name was *The Pluricomplex Seminar*.

Mikael's gave his first lecture in the seminar during the Fall Semester of 1978. He reported on chosen sections of the little book by Lev Isaakovič Ronkin (1931–1998), *The Elements of the Theory of Analytic Functions of Several Variables* (1977), which had been published in Russian in 2,700 copies in Kiev the year before and cost 93 kopecks. The task was a part of the examination for the course *Mathematics D*.

Seminar lectures held by Mikael Passare

1978-11-13: *Analytisk fortsättning* [*Analytic continuation*]. (Report on a special project for the advanced course *Mathematics D*.)

1982-11-01: *Henkin–Ramirez formulas for weight factors (according to Bo Berndtsson and Mats Andersson).*

1983-01-24: *Godtyckliga områden som projektioner av pseudokonvexa områden* [*Arbitrary domains as projections of pseudoconvex domains*].

1983-04-18: *Integraloperatorer för att lösa Cauchy–Riemanns ekvationer (efter R. Michael Range)* [*Integral operators for solving the Cauchy–Riemann equations (after R. Michael Range)*].

1983-06-15: *Samband mellan mängder av Newtonkapacitet noll och pluripolära mängder (efter Azim Sadullev)* [*Links between sets of Newton capacity zero and pluripolar sets (after Azim Sadullaev)*].

1984-05-14: *Ideal i ringen av holomorfa funktioner definierade medelst strömmar, I* [*Ideals in the ring of holomorphic functions defined by means of currents, I*].

1984-05-21: *Ideal i ringen av holomorfa funktioner definierade medelst strömmar, II* [*Ideals in the ring of holomorphic functions defined by means of currents, II*].

1984-12-10: *Residuer, strömmar och deras relation till ideal av holomorfa funktioner* [*Residues, currents, and their relation to ideals of holomorphic functions*] (cf. [1984], which was defended five days later).

1985-03-20: *Produkter av residuströmmar* [*Products of residue currents*] (cf. [1985]).

1986-01-17: *A new proof for integral representation formulas without boundary term.*

1986-04-14: *Principal values of meromorphic functions.*

1986-09-18: *1. Shortcut to weighted representation formulas for holomorphic functions* (cf. [1988a]). *2. Impressions from the International Congress of Mathematicians, Berkeley.*

1988-11-07: *Kergin interpolation of entire functions* (cf. [1991a, 1991b]).

1989-02-22: *Continuity of residue integrals in codimension two.*

1989-05-24: *Integral formulas and residues on complex manifolds.*

1989-10-04: *Kergin interpolation on* **C**-*convex sets.*

1991-02-18: *Mathematical impressions from Krasnoyarsk: 1. Holomorphic extension from a part of the boundary. 2. Toric varieties.*

1993-05-05: *Projektiv konvexitet* [*Projective convexity*].

1994-05-19: *Holomorphic differential forms on analytic sets.*

1998-09-07: *Amoebas and Laurent determinants* (cf. [2000a, 2004a]).

2000-03-14: *Constant terms in powers of a Laurent polynomial.*

2001-10-16: *Complex convexity—recent results of Kiselman and Hörmander.*

2002-05-07: *Discriminant amoebas.*

2002-11-19: *Algebraic equations and hypergeometric functions.*

2003-03-04: *The Lee–Yang circle theorem and geometry of amoebas.*

2003-10-21: *Amöbor, polytoper och tropisk geometri* [*Amoebas, polytopes, and tropical geometry*].

2004-01-10: *Koamöbor och Mellin-transformer av rationella funktioner* [*Coamoebas and Mellin transforms of rational functions*].

2010-03-09: *(Co)amoebas of linear spaces.*

2010-10-19: *Mellin transforms and hypergeometric functions.*

Originally, the seminars took place at Uppsala with a lecture in general every week. From the Spring Semester of 1999 on, when Mikael had become well established as a professor at Stockholm, they became a joint activity for Uppsala University, Stockholm University, and the Royal Institute of Technology (KTH). To minimize travel we had two lectures every second week. From 2007, when I had switched to digital geometry, mathematical morphology and discrete optimization, and Burglind Juhl-Jöricke had left Uppsala University, it became an activity exclusively in Stockholm.

The *Nordan* Meetings

Together with Mats Andersson and Peter Ebenfelt, Mikael Passare initiated a series of encounters on complex analysis in the five Nordic countries. Mikael and Peter organized the first conference, which took place in Trosa, Sweden, March 14–16, 1997; Mats the second, in Marstrand, Sweden, April 24–26, 1998. Following a voting procedure at the end of the first meeting, these yearly meetings were named *Nordan*[4]—a clear reference to *Les Journées complexes du Sud*, which during a long time have taken place in the south of France.

Mikael edited abstracts in Swedish of the lectures—which had all been given in English. These brochures were published with a delay of a few years. Twelve of them have come out; he was preparing the thirteenth, which was to report on *Nordan 13* held in Borgarfjordur in 2009, and asked Ragnar Sigurðsson on September 10, 2011, to write a preface in Icelandic (Ragnar Sigurðsson, personal communication 2011-10-04).

Lars Filipsson emphasizes (personal communication 2011-10-06) that Mikael wrote these brochures in Swedish to develop Swedish terms in higher mathematics, especially in complex analysis—otherwise the Swedish mathematical terms reach up to the first, possibly the second, university year only.

Nordic meetings like these were something that Mikael and Mats had discussed and planned during many years; both of them wanted to create a forum with a more relaxed atmosphere, where Nordic complex analysts, in particular the young ones, could feel more at home than at international conferences, and which would give

---

[4]This is the name in Swedish of a chilly wind from the north, but also reminds us of the original purpose: to promote <u>Nord</u>ic <u>An</u>alysis.

those that worked in the Nordic countries occasion to get to know each other better. And the initiative turned out to be a long-lasting success: the fifteenth encounter took place in Röstånga in southern Sweden, May 06–08, 2011; the sixteenth is planned to take place in Kiruna in northern Sweden, May 11–13, 2012.

Africa

Mikael Passare was a Member of the Board of the International Science Programme (ISP), Uppsala, and a Member of the Board of the Pan-African Centre for Mathematics (PACM) in Dar es-Salaam, Tanzania. He was a driving force in the creation of this Pan-African Centre, which is a collaborative project between Stockholm University and the University of Dar es-Salaam.

Mohamed E. A. El Tom, Chairman of the Board of PACM and a member of the Reference Group for Mathematics of ISP, says that he is confident that had it not been for Mikael, PACM would have remained a mere idea in the head of its initiator, i.e., in Mohamed's head; see El Tom (2011). For, he started working with great conviction and enthusiasm on the idea when Mohamed first suggested it to him while they were walking on a Meroetic archeological site near Khartoum in April 2004. (Mohamed El Tom, personal communication 2011-10-17.)

Mikael took an early, informal contact with the Vice-Chancellor (*Rektor*) of Stockholm University, who expressed his approval in principle (Mohamed El Tom, personal communication 2011-10-20).

In October 2008, Mikael and Mohamed discussed the idea with Anders Karlhede, Dean of the Division, and asked whether Stockholm University could be a partner in the project. Anders immediately took the question to Stefan Nordlund, Dean of the Faculty. The latter proved to be very positive, which was decisive for the coming commitment of Stockholm University to PACM. (Anders Karlhede, personal communication 2011-10-19.)

Mikael then presented the idea to the Department of Mathematics at Stockholm University. While the department did not object to the idea, it was only natural that some members raised many significant issues that required clarification. Mikael maintained correspondence with Mohamed about these and related issues for more than two years, at the end of which he managed to secure the approval of the department to collaborate in establishing the Centre at some suitable university in Africa. Later he was an influential member of the committee that short-listed African universities for hosting PACM. Subsequently, he was a member of a delegation led by Stefan Nordlund which visited some of the short-listed universities and made appropriate recommendations to the Vice-Chancellor of Stockholm University.

Mikael never ceased devoting of his precious time to the Centre. His last assignment was to chair and constitute a search committee for the Director of the Centre, a process he initiated before he was asked by the Board of the Centre to undertake it. Such was Mikael, ahead of others in thinking, and working to realize important objectives without being asked to do so. When Mohamed conveyed to him the Board's decision regarding the search committee, he responded promptly,

accepting the charge, and promised to respond with detailed ideas upon his return from his trip to Dubai, Oman and Iran.

Mikael's commitment and enthusiasm for the Centre was unsurpassed. He was confident that the grand objective of establishing a world-class Centre of Mathematics in Africa is attainable. (Mohamed El Tom, personal communication 2011-10-17; this applies to the three last paragraphs.)

Sonja Kovalevsky

The chair which Mikael Passare held was the one which was created for Sonja Kovalevsky (January 03/15, 1850 — February 10, 1891). An earlier incumbent during seven years, 1957–1964, was Lars Hörmander, Mikael's mathematical grandfather. Mikael was proud of having been given Sonja's chair. He is buried not far from her grave.

Exactly 150 years after Sonja's birth, on January 15, 2000, Mikael organized a symposium to her memory. It was held in the *Aula Magna* of Stockholm University. Among the invited speakers were Agneta Pleijel, Roger Cooke and Ragni Piene.

Languages

In the section on the *Nordan* meetings, I have already mentioned that Mikael was interested in developing Swedish mathematical terms. He knew many languages. His Russian was "really perfect!" according to Timur Sadykov (personal communication 2011-10-13); "he spoke Russian perfectly, so it was totally impossible to recognize his Swedish origin" (Andrei Khrennikov, personal communication 2011-11-26). He took a course in French corresponding to 30 ECTS credits at Stockholm University before going to Paris in 1986-87 (diploma 1985-09-03). He learned some Fijian when he visited the Republic of Fiji (Timur Sadykov, personal communication 2011-10-16).

His knowledge of German was very good although he had not studied that language in high school. He studied also Finnish and spoke the language so well that he was interviewed in the Finnish-language *Sisuradio* in Sweden.

Spanish and Italian he knew enough to get along. He was recently in Italy and Spain with Anders Wändahl, and never talked English when visiting a restaurant or when asking for directions in the street. He could also speak some Polish and Bulgarian.

Finally, he studied Arabic and could at least read that langugage. Maybe Arabic would have been his next project. (Anders Wändahl, personal communication 2011-10-19; this applies to this paragraph and the preceding one.)

An extraordinary curiosity

Andrei Khrennikov writes:

> I would like mention Mikael's extraordinary curiosity, which was extended to a large variety of fields. In particular, he discussed with excitement the possibility of mathematical modeling of cognition, human psychological behavior, and consiousness. I met Mikael and Galina the last time in July 2010, in Stockholm, and during one evening we discussed a large variety of topics: complex and *p*-adic analysis, mathematical foundations of quantum physics, quantum nonlocality, Bell's inequality and experiments [...] (Andrei Khrennikov, personal communication 2011-11-26)

Music

Mikael loved classical music; in his teens he sold his bicycle in order to buy a piano. He played clarinet and flute. He composed a piece for clarinet, which was played in a theater in Stockholm. His last love was an instrument called theremin.[5] He dreamed about being able to play it.[6]

He also loved to sing and was a member in a choir and learned to sing solo both in Stanford and in Moscow. (Galina Passare, personal communication 2011-10-17; this applies to all of this section.)

A "Swedish Classic"

Mikael swam several times a week, at least 2 km. He loved the mountains and skied long distances (sometimes 90–130 km) spending the night in cottages. He swam between islands in Lake Mälaren close to Stockholm. (Galina Passare, personal communication 2011-10-17.)

He ran the Stockholm Marathon. To Yûsaku Hamada's guest lectures in Uppsala on September 10, 2002, he went by bike from Stockholm (Yûsaku Hamada, personal communication 2011-09-19). Another time he skated on Lake Mälaren to the seminar in Uppsala. (After the seminar, however, he went back to Stockholm by train.)

The Viking Run (in Swedish: *Vikingarännet*) is the world's biggest regular skating event on natural ice, and is arranged yearly since 1999. It usually starts at Skarholmen in Uppsala and finishes at some place in or near Stockholm (depending on ice conditions; sometimes the ice is so bad that the competition has to be canceled). Mikael participated in the Viking Run several times.

Mikael also performed what is known as a "Swedish Classic" in 1989. It consists of four parts, which have to be done within a twelve-month period: (1) One of the ski runs, the Engelbrekt Run, 60 km, and the Vasa Run / Open Track, 90 km; (2) Going around Lake Vättern on bicycle, 300 km; (3) The Vansbro Swim, 3 km; (4)

---

[5] Терменвокс, which was invented by Лев Сергеевич Термен, Léon Theremin (1896–1993).

[6] At his funeral on October 28, 2011, *Dance in the Moon* was played on CD; the performer was Lydia Kavina, a leading thereminist.

The Lidingö Run, running, 30 km. Mats Andersson (personal communication 2011-10-12) remembers that he claimed the cycling to be the most painful of the four, noting the chafing after so many hours on the saddle.

He loved bandy (a sport similar to ice hockey but played with a ball and on an ice field the size of a soccer field) and missed only one single Swedish Championship Final since 1980, viz. the one in 1983 (Anders Wändahl, personal communication 2011-10-12). He was considerate also of other bandy fans. When Magnus Carlehed, his mathematical nephew, was traveling in 1990 around the globe in areas where ice rinks are not so common, he sent to Magnus a big envelope to the address *Poste restante*, Denpasar (Bali), Indonesia. It contained a video recording showing the Swedish Bandy Championship Final. (Magnus Carlehed, personal communication 2011-12-09.) He went to Arkhangelsk to see the Bandy World Championship there (probably in 1999, when Russia won over Sweden in the final). He also traveled to Oulu with Björn Ivarsson, his mathematical younger brother, to see the Championship in 2001 (when Russia won over Sweden again in the final; Björn Ivarsson, personal communication 2011-12-14).

A passionate traveler

Mikael saw at least three total solar eclipses: the one which took place on August 11, 1999, he saw in Turkey (although it would perhaps have been easier to see it in Bulgaria); the eclipse of December 04, 2002, he saw in Mozambique; and on March 29, 2006, he was in Niger with Anders Wändahl (although the southern coast of Turkey would have been easier to reach from Sweden and had a greater chance of clear sky without sand storms). After that they continued to Chad.

Mikael was a passionate traveler. He visited 152 countries. When he and I, together with several other Swedish mathematicians, were invited in September 2006 to celebrate the twentieth anniversary of the *Groupe Inter-Africain de Recherche en Analyse, Géométrie et Applications* (GIRAGA) and after that to participate in the *First African-Swedish Conference on Mathematics*, both in Yaoundé, Cameroon, he first visited the Central African Republic and continued afterwards to Equatorial Guinea and Gabon (Anders Wändahl, personal communication 2011-11-14); thus he got four new countries on his list—assuming that he had not been in any of these before—while I got only one.

The United Arab Emirates and Oman turned out to be the last ones. Land number 153 should have been Iran: he planned to arrive at Tehran Imam Khomeini International Airport on September 17 at 21:25 (Mikael Passare, electronic letter 2011-09-15 to mathematicians in Tehran). Siamak Yassemi, Head of the School of Mathematics, University of Tehran, was ready to meet him there.

Finally

Mikael's significance goes much beyond his own research. Many persons have testified to his positive view of life, his humor, and to his genuine interest in people he

met. He was an unusually stimulating partner in discussions; listening, inspiring, and supportive, in professional situations as well as private ones.

For Mikael's friends and colleagues around the world his unexpected departure is a severe loss.

For me personally, Mikael's disappearance seems unreal. He was always there for me. I shall remember him with joy and gratitude as long as I live. His death is a terrible blow—I feel like I, as an advisor, failed to give him that last piece of advice: do not go that close to the rim of the canyon ...[7]

Two proposals

At a meeting at Stockholm University to Mikael's memory on September 28, 2011, arranged by Tom Britton, I ended my speech by presenting two proposals.

The first proposal was that Stockholm University organize a conference to his memory, where his many mathematical achievements could be presented and discussed.

Since, as far as I know, Mikael has not published all his ideas on tropical geometry, I proposed, secondly, that his former students write a survey article about these ideas (and of course other mathematical ideas). Alicia Dickenstein (2011-09-24), August Tsikh (2011-10-03), Alexey Shchuplev (2011-10-06), and Hans Rullgård (2011-10-11) have all spontaneously approved of this and want to contribute to this project.

The second proposal can of course be realized as a part of the first, viz. if the survey article is published in the conference proceedings.

See also my paper "Questions inspired by Mikael Passare's mathematics" (ms).

Mikael Passare's publications

[1984]. Pettersson,[8] Mikael. *Residues, Currents, and Their Relation to Ideals of Holomorphic Currents.* Uppsala: Uppsala University, Department of Mathematics. Report No. 10, November 1984, 94 pp. (PhD Thesis defended on December 15, 1984. Opponent: Nils Øvrelid.)

[1985]. Passare, Mikael. Produits des courants résiduels et règle de Leibniz. *C. R. Acad. Sci. Paris Sér. I Math.* **301**, no. 15, 727–730.

---

[7]This sentiment must be understood in view of the early, erroneous, information according to which Mikael fell by accident into a canyon. Now the cause of death has been established to be a complete occlusion of the right coronary artery leading to an acute myocardial infarction and an immediate death; there are no injuries whatsoever that would indicate a fall into a canyon (The Swedish National Board of Forensic Medicine (2011)).

[8]On December 18, 1984, the Swedish Patent and Registration Office approved the family name *Passare* for Kjell Alrik Mikael Pettersson and Galina Pettersson, née Lepjosjkina. The diploma for the Degree of Doctor of Philosophy was issued on December 19, 1984, to Kjell Alrik Mikael Passare.

[1986]. Passare, Mikael. Ideals of holomorphic functions defined by residue currents. *Complex analysis and applications '85 (Varna, 1985)*, pp. 511–514. Publ. House Bulgar. Acad. Sci., Sofia.

[1987]. Passare, Mikael. Courants méromorphes et égalité de la valeur principale et de la partie finie. *Séminaire d'Analyse P. Lelong – P. Dolbeault – H. Skoda, Années 1985/1986*, pp. 157–166. Lecture Notes in Math. 1295. Berlin et al.: Springer-Verlag. (Reviewed by Salomon Ofman.)

[1988a]. Andersson, Mats; Passare, Mikael. A shortcut to weighted representation formulas for holomorphic functions. *Ark. mat.* **26**, no. 1, 1–12. (Reviewed by Bo Berndtsson.)

[1988b]. Passare, Mikael. Residue solutions to holomorphic Cauchy problems. *Seminar in Complex Analysis and Geometry 1987 (Rende, 1987)*, pp. 99–105, Sem. Conf., 1. Rende: EditEl. (Reviewed by E. J. Akutowicz.)

[1988c]. Passare, Mikael. Residues, currents, and their relation to ideals of holomorphic functions. *Math. Scand.* **62**, no. 1, 75–152. (Reviewed by Alicia Dickenstein.)

[1988d]. Passare, Mikael. A calculus for meromorphic currents. *J. reine angew. Math.* **392**, 37–56. (Reviewed by Alicia Dickenstein.)

[1989]. Berndtsson, Bo; Passare, Mikael. Integral formulas and an explicit version of the fundamental principle. *J. Funct. Anal.* **84**, no. 2, 358–372. (Reviewed by R. Michael Range.)

[1991a]. Andersson, Mats; Passare, Mikael. Complex Kergin interpolation. *J. Approx. Theory* **64**, no. 2, 214–225. (Reviewed by A. G. Law.)

[1991b]. Andersson, Mats; Passare, Mikael. Complex Kergin interpolation and the Fantappiè transform. *Math. Z.* **208**, no. 2, 257–271. (Reviewed by Harold P. Boas.)

[1991c]. Passare, Mikael. A new division formula for complete intersections. *Proceedings of the Tenth Conference on Analytic Functions (Szczyrk, 1990). Ann. Polon. Math.* **55**, 283–286. (Reviewed by Gerd Müller.)

[1992, 1993a]. Passare, M.; Tsikh, A. On the relations between the local structure of holomorphic mappings, multidimensional residues and generalized Mellin transforms (Russian). *Dokl. Akad. Nauk* **325**, no. 4, 664–667; translation in *Russian Acad. Sci. Dokl. Math.* **46** (1993), no. 1, 88–91. (Reviewed by Alexandr M. Kytmanov.)

[1993b]. Passare, Mikael. On the support of residue currents. *Several complex variables (Stockholm, 1987/1988)*, pp. 542–549. Math. Notes, 38, Princeton Univ. Press, Princeton, NJ. (Reviewed by Salomon Ofman.)

[1993c]. Passare, Mikael. Halva sanningen om en viktig produkt. Residyer i flera variabler. Föredrag vid Kungl. Vetenskaps-Societetens högtidsdag den 8 november 1991 [Half of the truth about an important product. Residues in several variables. Lecture at the Solemnity of the Royal Society of Sciences, November 08, 1991]. **In:** *Kungl. Vetenskaps-Societens i Uppsala årsbok 1992*, pp. 17–20. Uppsala: The Royal Society of Sciences.

[1994]. Passare, Mikael; Tsikh, August; Zhdanov, Oleg. A multidimensional Jordan residue lemma with an application to Mellin–Barnes integrals. *Contributions to complex analysis and analytic geometry*, pp. 233–241. Aspects Math., E26. Braunschweig: Vieweg. (Reviewed by Aleksandr G. Aleksandrov.)

[1995a]. Demailly, Jean-Pierre; Passare, Mikael. Courants résiduels et classe fondamentale. *Bull. Sci. Math.* **119**, no. 1, 85–94. (Reviewed by Mongi Blel.)

[1995b]. Passare, Mikael; Tsikh, August. Residue integrals and their Mellin transforms. *Canad. J. Math.* **47**, no. 5, 1037–1050. (Reviewed by Mongi Blel.)

[1996a]. Passare, Mikael; Tsikh, August. Defining the residue of a complete intersection. *Complex Analysis, Harmonic Analysis and Applications (Bordeaux, 1995)*, pp. 250–267. Pitman Res. Notes Math. Ser., 347, Harlow: Longman. (Reviewed by Carlos A. Berenstein.)

[1996b, 1997]. Passare, M.; Tsikh, A. K.; Cheshel′, A. A. Iterated Mellin–Barnes integrals as periods on Calabi–Yau manifolds with two modules (Russian). *Teoret. Mat. Fiz.* **109** (1996), no. 3, 381–394; translation in *Theoret. and Math. Phys.* **109**, no. 3, 1544–1555 (1997). (Reviewed by V. V. Chueshev.)

[1998]. Bykov, Valery; Kytmanov, Alexander; Lazman, Mark. *Elimination Methods in Polynomial Computer Algebra*. Translated from the 1991 Russian original by Kytmanov and revised by the authors. Translation edited and with a preface by Mikael Passare. Mathematics and its Applications, 448. Dortrecht: Kluwer Academic Publishers. xii + 237 pp. ISBN: 0-7923-5240-8. (Reviewed by Harold P. Boas.)

[1999]. Henkin, Gennadi; Passare, Mikael. Abelian differentials on singular varieties and variations on a theorem of Lie–Griffiths. *Invent. Math.* **135**, no. 2, 297–328. (Reviewed by Reinhold Hübl.)

[2000a]. Forsberg, Mikael; Passare, Mikael; Tsikh, August. Laurent determinants and arrangements of hyperplane amoebas. *Adv. Math.* **151**, no. 1, 45–70. (Reviewed by Guangfeng Jiang.)

[2000b]. Passare, Mikael; Tsikh, August; Yger, Alain. Residue currents of the Bandner–Martinelli type. *Publ. Mat.* **44**, no. 1, 85–117. (Reviewed by Harold P. Boas.)

[2000c]. Aizenberg, Lev; Passare, Mikael. **C**-convexity, convexity in complex analysis. **In:** *Encyclopædia of Mathematics*, Supplement, vol. II, pp. 102–104. Dordrecht: Kluwer Academic Publishers.

[2001a]. Passare, Mikael. Sesam öppna dig [Open Sesame]. *Nämnaren* **28**(4), 37–39.

[2001b]. Passare, Mikael. *Complex geometry. Minicourse, June 30 – July 3, 2001.* Lectures at the *Première École d'Été Franco-Nordique de Mathématiques*, EEFN, Lake Erken, Sweden, 2001-06-26—07-03. Institut Mittag-Leffler, Lecture Notes No. 3, 2000/2001, 8 pp.

[2002]. Passare, Mikael; Rullgård, Hans. Multiple Laurent series and polynomial amoebas. *Actes des Rencontres d'Analyse Complexe (Poitiers-Futuroscope, 1999)*, pp. 123–129. Poitiers: Atlantique. (Reviewed by Guangfeng Jiang.)

[2003a]. [Passare, Mikael (Ed.)]. *Nordan Ett.* [Abstracts from *Nordan 1*, held in] Trosa 1997-03-14—16, 15 pp. [Stockholm: Stockholm University 2003.]

[2003b]. [Passare, Mikael (Ed.)]. *Nordan Två.* [Abstracts from *Nordan 2*, held in] Marstrand 1998-04-24—26, 15 pp. [Stockholm: Stockholm University 2003.]

[2004a]. Passare, Mikael; Rullgård, Hans. Amoebas, Monge–Ampère measures, and triangulations of the Newton polytope. *Duke Math. J.* **121**, no. 3, 481–507. (Reviewed by A. Yu. Rashkovskiĭ.)

[2004b]. Andersson, Mats; Passare, Mikael; Sigurdsson, Ragnar. *Complex Convexity and Analytic Functionals*. Progress in Mathematics, 225. Basel: Birkhäuser Verlag. xii + 160 pp. ISBN: 3-7643-2420-1. (Reviewed by Sergey Ivashkovich.)

[2004c]. Passare, Mikael; Tsikh, August. Algebraic equations and hypergeometric series. **In:** *The Legacy of Niels Henrik Abel*, pp. 653–672. Berlin: Springer. (Reviewed by Allen R. Miller.)

[2004d]. Passare, Mikael. Amoebas, convexity and the volume of integer polytopes. *Complex analysis in several variables — Memorial Conference of Kiyoshi Oka's Centennial Birthday*, pp. 263–268. Adv. Stud. Pure Math., 42. Tokyo: Math. Soc. Japan. (Reviewed by A. Yu. Rashkovskiĭ.)

[2004e]. [Passare, Mikael (Ed.)]. *Nordan Tre.* [Abstracts from *Nordan 3*, held in] Saltsjöbaden 1999-04-22—25, 19 pp. [Stockholm: Stockholm University 2004.]

[2004f]. Passare, Mikael. Amöbor och Laurentserier (Amoebas and Laurent series). **In:** [2004e:19]. (A report on [2000a].)

[2005a]. Passare, Mikael; Sadykov, Timur; Tsikh, August. Singularities of hypergeometric functions in several variables. *Compos. Math.* **141**, no. 3, 787–810. (Reviewed by A. Yu. Rashkovskiĭ.)

[2005b]. Passare, Mikael; Tsikh, August. Amoebas: their spines and their contours. **In:** *Idempotent Mathematics and Mathematical Physics*, pp. 275–288. Contemp. Math., 377. Providence, RI: Amer. Math. Soc. (Reviewed by Eugenii Shustin.)

[2005c]. Leĭnartas, E. K.; Passare, M.; Tsikh, A. K. Asymptotics of multidimensional difference equations (Russian). *Uspekhi Mat. Nauk* **60**, no. 5(365), 171–172; translation in *Russian Math. Surveys* **60**, no. 5, 977–978.

[2005d]. [Passare, Mikael (Ed.)]. *Nordan Fyra.* [Abstracts from *Nordan 4*, held in] Örnsköldsvik 2000-05-05—07, 18 pp. [Stockholm: Stockholm University 2005.]

[2006a]. [Passare, Mikael (Ed.)]. *Nordan Fem.* [Abstracts from *Nordan 5*, held in] Voksenåsen, Oslo, 2001-05-04—06, 17 pp. [Stockholm: Stockholm University 2006.]

[2006b]. Passare, Mikael. Amöbor, Monge–Ampère-mått och trianguleringar av Newton-polytopen (Amoebas, Monge–Ampère measures, and triangulations of the Newton polytope). **In:** [2006a:6]. (A report on [2004a].)

[2007a]. [Passare, Mikael (Ed.)]. *Nordan Sex.* [Abstracts from *Nordan 6*, held in] Reykjavík 2002-03-08—10, 17 pp. [Stockholm: Stockholm University 2007.]

[2007b]. [Passare, Mikael (Ed.)]. *Nordan Sju.* [Abstracts from *Nordan 7*, held in] Visby 2003-05-23—25, 17 pp. [Stockholm: Stockholm University 2007.]

[2008a]. Passare, Mikael. How to compute $\sum 1/n^2$ by solving triangles. *Amer. Math. Monthly* **115**, no. 8, 745–752.

[2008b]. Leĭnartas, E. K.; Passare, M.; Tsikh, A. K. Multidimensional versions of Poincaré's theorem for difference equations (Russian). *Mat. Sb.* **199**, no. 10, 87–104; translation in *Sb. Math.* **199**, no. 9-10, 1505–1521. (Reviewed by Victor I. Tkachenko.)

[2008c]. [Passare, Mikael (Ed.)]. *Nordan Åtta.* [Abstracts from *Nordan 8*, held in] Nösund, Orust 2004-05-14—16, 15 pp. [Stockholm: Stockholm University 2008.]

[2008d]. [Passare, Mikael (Ed.)]. *Nordan Nio.* [Abstracts from *Nordan 9*, held in] Sigtuna 2005-04-22—24, 17 pp. [Stockholm: Stockholm University 2008.]

[2008e]. Passare, Mikael. Mormors glasögon och räkning modulo nio [Grandma's glasses, and counting modulo nine]. *Nämnaren* **35**(1), 31.

[2009a]. Passare, Mikael (Ed.). *Complex Analysis and Digital Geometry. Proceedings from the Kiselmanfest, 2006.* Acta Universitatis Upsaliensis. Skrifter rörande Uppsala universitet. C. Organisation och Historia [Publications concerning Uppsala University. C. Organization and History] 86. Uppsala: Uppsala University. 364 pp. ISBN: 978-91-554-7672-4.

[2009b]. Passare, Mikael. Preface. **In:** Passare (Ed.) [2009a:7–8].

[2009c]. Passare, Mikael. Christer Kiselman's mathematics. **In:** Passare (Ed.) [2009a:9–26]. (Reviewed by Norman Levenberg.)

[2009d]. [Passare, Mikael (Ed.)]. *Nordan Tio.* [Abstracts from *Nordan 10*, held in] Sundsvall 2006-05-19—21, 14 pp. [Stockholm: Stockholm University 2009.]

[2009e]. Passare, Mikael. Hypergeometriska serier och integraler (Hypergeometric series and integrals). **In:** [2009d:6].

[2010a]. Nilsson, Lisa; Passare, Mikael. Discriminant coamoebas in dimension two. *J. Commut. Algebra* **2**, no. 4, 447–471. (Reviewed by Eugenii Shustin.)

[2010b]. [Passare, Mikael (Ed.)]. *Nordan Elva.* [Abstracts from *Nordan 11*, held in] Oscarsborg, Drøbak, 2007-05-18—20. [Stockholm: Stockholm University 2010.]

[2011a]. [Passare, Mikael (Ed.)]. *Nordan Tolv.* [Abstracts from *Nordan 12*, held in] Mariehamn 2008-04-18—20. [Stockholm: Stockholm University 2011.]

[2011b]. Passare, Mikael; Risler, Jean-Jacques. On the curvature of the real amoeba. *Proceedings of the Gökova Geometry-Topology Conference 2010*, pp. 129–134. Somerville, MA: International Press.

[2011c]. Passare, Mikael; Rojas, J. Maurice; Shapiro, Boris. New multiplier sequences via discriminant amoebae. *Mosc. Math. J.* **11**, No. 3, 547–560, 631.

[MS]. Nisse, Mounir; Passare, Mikael. (Co)amoebae of linear spaces. Manuscript, 2011.

[MS]. Passare, Mikael; Sottile, Frank. Discriminant coamoebas through homology. Manuscript, 2012-01-16, 20 pp.

[MS]. Passare, Mikael; Pochekutov, Dmitry; Tsikh, August. Amoebas of complex hypersurfaces in statistical thermodynamics. Manuscript, 2011-12-19, 18 pp; arXiv:1112.4332.

[2012]. Ilia Itenberg, Ilia; Jöricke, Burglind; Passare, Mikael (Eds.). *Perspectives in analysis, geometry, and topology. On the occasion of the 60th birthday of Oleg Viro.* Progress in Mathematics, 296. New York: Birkhäuser/Springer. xxxii + 464 pp. ISBN: 978-0-8176-8276-7.

References

Andersson, Mats; Wulcan, Elizabeth (2007). Residue currents with prescribed annihilator ideals. *Ann. Sci. École Norm. Sup.* (4) **40**, no. 6, 985–1007.

Coleff, Nicolás R.; Herrera, Miguel E. (1978). *Les courants résiduels associés à une forme méromorphe.* Lecture Notes in Mathematics 633. X + 211 pp. Berlin: Springer.

Dickenstein, A[licia]; Sessa, C[armen] (1985). Canonical representatives in moderate cohomology. *Invent. Math.* **80**, no. 3, 417–434.

El Tom, Mohamed E. A. (2011). Towards a new collaborative model for strengthening capacity in mathematical research in Africa. **In:** Kiselman, Christer (Ed., 2011), *Regional and Interregional Cooperation to Strengthen Basic Sciences in Developing Countries. Addis Ababa, 1–4 September 2009*, pp. 173–187. Acta Universitatis Upsaliensis. Skrifter rörande Uppsala universitet. C. Organisation och historia [Publications concerning Uppsala University. C. Organization and History] 88. Uppsala: Uppsala University. 423 pp. ISBN: 978-91-554-7910-7.

Gelfand, I. M.; Kapranov, M. M.; Zelevinsky, A. V. (1994). *Discriminants, Resultants, and Multidimensional Determinants.* Math. Theory Appl., x + 523 pp. Boston: Birkhäuser.

Henkin, G. M. (1969). Integral representation of functions which are holomorphic in strictly pseudoconvex regions, and some applications (Russian). *Mat. Sb. (N.S.)* **78** (120), 611–632.

Henkin Gennadi M.; Polyakov Peter L. (2011). Residual $\bar{\partial}$-cohomology and the complex Radon transform on subvarieties of $\mathbf{C}P^n$. *Math. Ann.*, to appear (Springer online 2011-11-03; DOI 10.1007/s00208-011-0737-1).

Herrera, M.; Lieberman, D. (1971). Residues and principal values on complex spaces. *Math. Ann.* **194**, 259–294.

Kiselman, Christer O. (2011). Mikael Passare 1959–2011. *Medlemsutskicket från Svenska Matematikersamfundet*, October 2011, pp. 17–35. (Obituary in Swedish, published by the Swedish Mathematical Society on 2011-10-25.)

Kiselman, Christer O. (ms). Questions inspired by Mikael Passare's mathematics. Manuscript, 2011, 18 pp.

Persson, Ulf (2011). Mikael Passare 1959–2011 [in Swedish with a summary in English]. *Normat* **59**, No. 2, 49–50, 96.

Ронкин, Лев Исаакович (1977). *Элементы теории аналитических функций многих переменных*. Kiev: Издательство «Наукова Думка». 168 pp.

Rudbeckianska skolan (1978). *Avgångsbetyg för Mikael Pettersson 1978-06-09. Elevkort. Terminsbetyg.*

Schwartz, Laurent (1954). Sur l'impossibilité de la multiplication des distributions. *C. R. Acad. Sci. Paris* **239**, 847–848. Citations

Simon, Imre (1988). Recognizable sets with multiplicities in the tropical semiring. **In:** *Mathematical foundations of computer science*, (Carlsbad, 1988), pp. 107–120. Lecture Notes in Computer Science, **324**, Berlin: Springer.

The Swedish National Board of Forensic Medicine (2011). Statement A11-1041 of 2011-12-13.

Sources

El Tom, Lars Filipsson, Gennadi Henkin, Anders Karlhede, Karin Wallby and Anders Wändahl.

The sources are in certain cases messages that I have received during the writing process, and, if so, reported as a personal communication at the end of a sentence or a paragraph. Otherwise I have relied on documents that I have saved, notes that I have made—and my memory.

Author's address: Uppsala University, Department of Mathematics,
P. O. Box 480, SE-751 06 Uppsala, Sweden
E-mail: `kiselman@math.uu.se`