Mikael Passare (1959–2011)

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Mikael Passare was a brilliant mathematician who died much too early. In this chapter we present a sketch of his work and life.

Mikael was born in Västerås, Sweden, on 1959 January 01, and pursued a fast and brilliant career as a mathematician. He started his studies at Uppsala University in the fall of 1976 while still a high-school student, merely seventeen and a half. He finished high school in June 1978 at the Rudbeckianska skolan in Västerås, gave his first seminar talk in November 1978 at Uppsala University, where he got his Bachelor Degree in 1979, and where he also was an assistant.

He was accepted as a graduate student at Uppsala University on 1980 February 14 with me as advisor, and presented his thesis on 1984 December 15. He was a research assistant (half time) and lecturer (half time) at Stockholm University from January through June 1985, research assistant financed by the Swedish Natural Science Research Council (NFR) July 1985 – August 1986 and later research assistant, July 1987 through 1990.

He received the title of *Docent* (corresponding to the *Habilitation* in some countries) on 1988 January 28. He was a senior university lecturer (full time) from July 1988, from time to time on leave of absence. Later he was a research lecturer at the Royal Institute of Technology, July 1990 – 1994. He was appointed full professor at Stockholm University from 1994 October 01.

In addition to English and French, he studied Russian during three years in high school (Rudbeckianska skolan 1978), and later did his military service at the National Defence Radio Establishment (the Swedish national authority for signals intelligence) from June 1979 through August 1980. There he learned even more Russian.

He spent four academic years in four different countries: during the academic year 1980-81 he was at Stanford University; during 1981-82 at Lomonosov University in Moscow; during 1986-87 at Université Pierre et Marie Curie (Paris VI) and Université de Paris-Sud (Paris IX), having received a post-doctoral fellowship from the Swedish Natural Science Research Council (NFR). During 1992-93 he was at Humboldt-Universität zu Berlin on an Alexander von Humboldt fellowship. He was a guest professor in France on several occasions: at Toulouse (June 1988), Grenoble (April 1992), Bordeaux (May 1992), Paris VII (March 1993), Lille (April 1999), and Bordeaux again (June 2000).

Mikael was much appreciated as a researcher and teacher, and was very active outside the university. He was Head of the Department of Mathematics at Stockholm University from January 2005 through August 2010, and then Director of the newly created Stockholm Mathematics Center, common to Stockholm University and the Royal Institute of Technology. When Burglind Juhl-Jöricke and Oleg Viro had resigned from Uppsala University on 2007 February 08, he arranged for a guest professorship for Burglind at Stockholm University, and was one of the organizers of a big conference in honor of Oleg, *Perspectives in Analysis, Geometry, and Topology*, at Stockholm University during seven days, 2008 May 19–25.

As president of the Swedish National Committee for Mathematics, he led the Swedish delegation to the General Assembly of the International Mathematical Union in Bangalore, Karnataka, India, in August 2010. In 2011 he invited, as president of the National Committee, Bernd Sturmfels to lecture in Linköping, Lund, and Göteborg.

Mikael Passare was Deputy Director for Institut Mittag-Leffler, Djursholm, Sweden, from 2010. He was very much appreciated for his activity there, which included organizing the Felix Klein Days for teachers and a research school for high-school students.

Starting in July 2001, he served during ten years as one of the editors of the *Arkiv för matematik* (Ari Laptev, personal communication 2011-10-19). During the period 2004 April 01 — 2009 June 25 he was one of the Associate Editors for the *Journal of Mathematical Analysis and Applications* (Don Prince, personal communication 2011-10-13).

Mikael was a member of the Swedish Committee for Mathematics Education (SKM) from January 1997, when SKM started its activity, until December 2004. Mikael's efforts in SKM can only be explained by his firm dedication to mathematics education in the schools. He participated actively by organizing meetings, authoring reports to policy makers, and influencing politicians and officials at the Ministry of Education and the Swedish National Agency for Education. (Gerd Brandell, personal communication 2011-10-17.)

The activity of SKM to which he devoted most of his energy was the international competition *International Mathematical Kangaroo*, originally *Kangourou sans frontières*, in Swedish *Kängurun — Matematikens Hopp*. He took the initiative to start, with SKM as organizer, a Swedish version of this competition in 1999. He translated the problems, which arrived in English or French, up to 2009. He checked also that the mathematical content was correct after the necessary adaption to Swedish traditions in problem formulation and other circumstances. He participated, at least during the first five to six years, in the choice of problems to the Swedish edition, and he continued his commitment to the competition even after his time in SKM. The competition is run by SKM in cooperation with the National Centre for Mathematics Education, NCM. In 2010, more than 80,000 students at all levels participated. (Gerd Brandell, personal communication 2011-10-17; Karin Wallby, personal communication 2011-10-19.)

Mikael was a member of the Steering Group for the National Graduate School in Mathematics Education from March 2000, until it ceased in December 2006. The school, which had about twenty PhD students, was financed by the Bank of Sweden Tercentenary Foundation (*Riksbankens Jubileumsfond*, RJ) and the Swedish Research Council (*Vetenskapsrådet*, VR). He actively and constructively contributed to shaping the education of this Graduate School, both in his role as member of the Steering Group and by participating in many meetings between PhD students and advisors. He was project leader for the school's activity at the Department of Mathematics at Stockholm University. Of the school's PhD students, two were at Stockholm: Kirsti Löfwall Hemmi and Andreas Ryve, who both got their PhDs in 2006. (Gerd Brandell, personal communication 2011-10-17.)

The Sonja Kovalevsky School in Stockholm, a private elementary school, started its activity in the Fall of 1999. Its profile includes chess, mathematics, and Russian. The aim was, among other things, to benefit from educational experience from Russia. Mikael was a member of the school's Board from the beginning.

At the time of his death, Mikael was President of the Swedish Mathematical Society and also a member of the Committee for Developing Countries (CDC) of the European Mathematical Society. His activity for mathematics in Africa is described in a later section.

Mikael died from a sudden cardiac arrest in Oman in the evening of 2011 September 15.[1] His next of kin are his wife Galina Passare, his son Max, and his daughter Märta.

Mikael's nine PhD students

Mikael served as advisor of nine PhD students who successfully completed their degrees. They are registered in the *Mathematics Genealogy Project* and are:

Yang Xing, 1992, Stockholm University: *Zeros and Growth of Entire Functions of Several Variables, the Complex Monge–Ampère Operator and Some Related Topics.* Now Senior Lecturer at Lund University.

Mikael Forsberg, 1998, The Royal Institute of Technology: *Amoebas and Laurent Series.* Now Senior Lecturer at Gävle University College.

Lars Filipsson, 1999, The Royal Institute of Technology: *On Polynomial Interpolation and Complex Convexity.* Now Senior Lecturer at the Royal Institute of Technology, Stockholm.

Timur Sadykov, 2002, Stockholm University: *Hypergeometric Functions in Several Complex Variables.* Although not mentioned in the Genealogy Project, August Tsikh served as a coadvisor (Timur Sadykov, personal communication 2011-11-26; August Tsikh, personal communication 2011-12-06). Now Timur is Full Professor at the Department of Mathematics at the Russian Plekhanov University, Moscow.

Hans Rullgård, 2003, Stockholm University: *Topics in Geometry, Analysis and Inverse Problems.* Now at Comsol Group, Stockholm, a company providing software solutions for multiphysics modelling.

Johan Andersson, 2006, Stockholm University: *Summation Formulae and Zeta Functions.* Now Senior Lecturer at Mälardalen University, Campus Västerås.

[1]The cause of death has been established to be a complete occlusion of the right coronary artery leading to an acute myocardial infarction and an immediate death; there are no injuries whatsoever that would indicate a fall into a canyon (The Swedish National Board of Forensic Medicine (2011)).

Alexey Shchuplev, 2007, Stockholm University: *Toric Varieties and Residues.* August Tsikh was second advisor. Now Assistant Professor and Head of Laboratory at the Siberian Federal University in Krasnoyarsk.

David Jacquet, 2008, Stockholm University: *On Complex Convexity.* Now Specialist Consultant in quantitative analysis and CEO of his company Mathsolutions Sweden AB.

Lisa Nilsson, 2009, Stockholm University: *Amoebas, Discriminants, and Hypergeometric Functions.* August Tsikh was second advisor. Now she is employed at the insurance company If Skadeförsäkring AB in Stockholm as risk analyst within capital modelling.

Mikael's mathematics

Residue theory

Mikael soon became known as an eminent researcher in complex analysis in several variables, where his thesis was an important breakthrough with new results in residue theory. Its title was *Residues, Currents, and Their Relation to Ideals of Holomorphic Functions* [1984], and it was later published in [1988c].[2]

Residue theory in several variables is a notoriously difficult part of complex analysis. Mikael's work was inspired by that of Miguel E. M. Herrera (1938–1984). Miguel and I were together at the Institute for Advanced Study in Princeton during the academic year 1965-66, and it was there that I learned about residues from him.[3] His results, which culminated in the paper by Herrera and Lieberman (1971) and the much quoted book by Coleff and Herrera (1978), were well known long before these publications. I could somehow serve as mediator to Mikael for this interest without doing much research on residues myself.

Also Alicia Dickenstein, who was a student of Miguel and got her PhD at Buenos Aires in 1982, knew this theory very well and soon came into contact with Mikael. As for integral formulas, Mikael took advice from Bo Berndtsson, already then a renowned expert in that field.

[2] Years in brackets refer to the list of Mikael Passare's publications. Years in parentheses refer to publications listed at the end of this chapter.

[3] He also introduced me to a great writer: Ursula LeGuin.

Another important person for Mikael's mathematical development was Gennadi Henkin (1942–2016). They met in Moscow during the academic year 1981-82, afterwards several times in the period 1983–1990, and then in France and Sweden during the period 1991–2010, for example in Trosa in 1997, Saltsjöbaden in 1999, and in Uppsala in 2006. During these meetings they discussed, in particular, integral formulas of Cauchy–Leray type and applications from the papers of Gennadi and Bo (starting with Henkin (1969)).

While residues in one complex variable have been well understood for a long time, the situation is quite different in several variables. There were pioneers like Henri Poincaré (1854–1912) and Jean Leray (1906–1998). Alexandre Grothendieck (1928–2014) developed a residue theory in higher dimensions, but it was quite abstract. Through work by Miguel Herrera, François Norguet (1929–2010), and Pierre Dolbeault (1924–2015), the theory could be linked to distribution theory, developed by Laurent Schwartz (1915–2002), and that was the road that Mikael continued to follow. He worked intensively with August Tsikh, on residue theory as well as on amoebas.

Residues in one complex variable

In one complex variable we can observe that there is a lot of symmetry:

$$\int\limits_{\varepsilon<|z|<r} z^j \bar{z}^k f(|z|) dx \wedge dy = 0, \qquad j,k \in \mathbf{Z}, \quad j \neq k.$$

This means that heavy masses are balanced, and implies that, when calculating residues, it is enough to work with the principal value, PV (*valeur principale*, VP); we need not use the more difficult and unstable construction of the finite part, FP (*partie finie*, PF). (In real analysis, the finite part inevitably appears: the distribution on the real axis given by the function $\log|x|$, $x \in \mathbf{R}$, has the derivative $\mathrm{PV}(1/x)$ and the second derivative $-\mathrm{FP}(1/x^2)$.)

If we write a smooth function φ as $\varphi(z) = P(z) + R(z)$, where P is a polynomial in z and $\bar{z}$ of degree at most $m-1$, $m \in \mathbf{N}$, $m \geqslant 1$, and $R(z)/z^m$ is bounded near the origin, it follows from the symmetry mentioned above that P does not influence the following integral at all.

$$\int\limits_{\varepsilon<|z|<r} \frac{\varphi(z)}{z^m} dx \wedge dy = \int\limits_{\varepsilon<|z|<r} \frac{R(z)}{z^m} dx \wedge dy.$$

As ε tends to 0, the last integral tends to

$$\int_{|z|<r} \frac{R(z)}{z^m} dx \wedge dy.$$

We define the *principal value* $\mathrm{PV}(1/z^m)$ of $1/z^m$ by

$$\left\langle \mathrm{PV}\left(\frac{1}{z^m}\right), \varphi \right\rangle = \mathrm{PV}\int_{\mathbf{C}} \frac{\varphi(z)}{z^m} dx \wedge dy = \lim_{\varepsilon\to 0} \int_{\varepsilon<|z|} \frac{\varphi(z)}{z^m} dx \wedge dy,$$

which exists for all test functions $\varphi \in \mathscr{D}(\mathbf{C})$. If now f/g is meromorphic with a pole at the origin, we obtain

$$\left\langle \mathrm{PV}\left(\frac{f}{g}\right), \varphi \right\rangle = \mathrm{PV}\int_{\mathbf{C}} \frac{f(z)}{g(z)} \varphi(z) dx \wedge dy, \qquad \varphi \in \mathscr{D}(\mathbf{C}),$$

and we define the *residue* $\mathbf{res}(f/g)$ of f/g as

$$\mathbf{res}\left(\frac{f}{g}\right) = \frac{\partial}{\partial \bar{z}} \mathrm{PV}\left(\frac{f}{g}\right) \in \mathscr{D}'(\mathbf{C}).$$

Residues in several variables

Let f and g be holomorphic functions of n complex variables. The *principal value* $\mathrm{PV}(f/g)$ of f/g is a distribution defined by the formula

$$\left\langle \mathrm{PV}\left(\frac{f}{g}\right), \varphi \right\rangle = \lim_{\varepsilon\to 0} \int_{|g|>\varepsilon} \frac{f\varphi}{g} = \lim_{\varepsilon\to 0} \int \frac{\chi f \varphi}{g}, \qquad \varphi \in \mathscr{D}(\mathbf{C}^n),$$

where $\chi = \chi(|g|/\varepsilon)$ and χ is a smooth function on the real axis satisfying $0 \leqslant \chi \leqslant 1$ and $\chi(t) = 0$ for $t \leqslant 1$, $\chi(t) = 1$ for $t \geqslant 2$ (in [1985:727] when $f = 1$; in [1988:39] in general).

The *residue current* is $\bar{\partial}\,\mathrm{PV}(f/g)$. Can the products

$$(\mathrm{PV}(f_1/g_1))(\mathrm{PV}(f_2/g_2)), \quad \left(\overline{\partial}(\mathrm{PV}(f_1/g_1))\right)(\mathrm{PV}(f_2/g_2))$$

and other similar products be defined?

Schwartz proved (1954) that it is in general impossible to multiply two distributions while respecting the associative law. He indicated three distributions $u, v, w \in \mathscr{D}'(\mathbf{R})$ where uv, vw, $(uv)w$ and $u(vw)$ all have a good meaning, but where $(uv)w \neq u(vw)$. He took $u =$

PV$(1/x)$, the principal value of $1/x$; v as the identity, i.e., the smooth function $v(x) = x$, which can be multiplied to any distribution; and $w = \delta$, the Dirac measure placed at the origin. Then we have $uv = 1$, $(uv)w = \delta$, while $vw = 0$, $u(vw) = 0$. Hence there is no associative multiplication.

Mikael's construction of residue currents goes as follows. Take $f = (f_1, \dots, f_{p+q})$, $g = (g_1, \dots, g_{p+q})$, two $(p+q)$-tuples of holomorphic functions, and consider the limit

$$\lim_{\varepsilon_j \to 0} \frac{f_1}{g_1} \cdots \frac{f_{p+q}}{g_{p+q}} \bar\partial\chi_1 \wedge \cdots \wedge \bar\partial\chi_p \cdot \chi_{p+1} \cdots \chi_{p+q},$$

where $\chi_j = \chi(|g_j|/\varepsilon_j)$, and the ε_j tend to zero in some way.

Coleff and Herrera (1978:35–36) took $q = 0$ or 1, and assumed that ε_j tends to zero much faster than ε_{j+1}, which in this context means that $\varepsilon_j/\varepsilon_{j+1}^m \to 0$ for all $m \in \mathbf{N}$ and $j = 1, \dots, p+q-1$; thus it is almost an iterated limit. This gives rise to the strange situation that, in general, the limit depends on the order of the functions (and is not just an alternating product).

Mikael took instead $\varepsilon_j = \varepsilon^{s_j}$ for fixed $s_1, \dots, s_{p+q}$. The limit, which will be written as $R^pP^q[f/g](s)$, where we now write $[\dots]$ for the principal value, does not exist for arbitrary s_j. But he proved [1985:728] that, if we remove finitely many hyperplanes, then $R^pP^q[f/g](s)$ is locally constant in a finite subdivision of the simplex

$$\Sigma = \{s \in \mathbf{R}^{p+q};\ s_j > 0,\ \ \textstyle\sum s_j = 1\},$$

so that the mean value

$$R^pP^q\left[\frac{f}{g}\right] = \fint_\Sigma R^pP^q\left[\frac{f}{g}\right](s) = \bar\partial\left[\frac{f_1}{g_1}\right] \wedge \cdots \wedge \bar\partial\left[\frac{f_p}{g_p}\right] \cdot \left[\frac{f_{p+1}}{g_{p+1}}\right] \cdots \left[\frac{f_{p+q}}{g_{p+q}}\right]$$

exists (Definition A in [1987]). This is the product of p residue currents and q principal-value distributions.

In the little paper [1993c], based on his talk when accepting the Thuréus Prize in 1991, he discusses the possibility of defining the product PV$(1/x)\delta$ on the real axis, and finds that it should be $-\frac{1}{2}\delta'$, which is the mean value of $-\delta'$ and zero. This is an analogue in real analysis to the mean value over Σ which he considered in the complex case.

Leibniz' rule for the derivative of a product and some other rules of calculus hold; for example we have [1988d:43]:

$$\left[\frac{1}{z_1}\right]\left[\frac{z_1}{z_2}\right]=\left[\frac{1}{z_2}\right],$$

which yields

$$\left(\overline{\partial}\left[\frac{1}{z_1}\right]\right)\left\{\left[\frac{1}{z_1}\right]\left[\frac{z_1}{z_2}\right]\right\}=\left(\overline{\partial}\left[\frac{1}{z_1}\right]\right)\left[\frac{1}{z_2}\right],$$

while

$$\left\{\left(\overline{\partial}\left[\frac{1}{z_1}\right]\right)\left[\frac{1}{z_1}\right]\right\}\left[\frac{z_1}{z_2}\right]=\frac{1}{2}z_1\left(\overline{\partial}\left[\frac{1}{z_1^2}\right]\right)\left[\frac{1}{z_2}\right]=\frac{1}{2}\left(\overline{\partial}\left[\frac{1}{z_1}\right]\right)\left[\frac{1}{z_2}\right].$$

Thus the associative law does not hold.

We saw in Schwartz' example that an associative multiplication is impossible in general; the example shown here makes us wonder whether it is possible to define an associative multiplication in some algebra of principal-value distributions and residue currents. We may also ask if there is an interesting non-associative algebra of principal-value distributions and residue currents.

For complete intersections, i.e., when the set of common zeros of $f_1, f_2, \ldots, f_p$ has maximal codimension, Mikael established a division formula with remainder term:

$$h=\sum_1^p g_j f_j + h\cdot\mathbf{res},$$

where **res** is the residue current, which is a factor in the remainder term $h\cdot\mathbf{res}$ and has the property that $f_j\cdot\mathbf{res}=0$ for all j. This implies that h belongs to the ideal generated by $f_1,\ldots,f_p$ if and only if $h\cdot\mathbf{res}$ vanishes. This is a beautiful characterization of the ideals of holomorphic functions and explains the choice of title in the papers [1984, 1986, 1988c]. The characterization of the ideals with the help of residues was proved independently and at about the same time by Alicia Dickenstein and Carmen Sessa (1985:424).

This characterization of ideals enabled Mikael and Bo to formulate an elegant and explicit variant of Leon Ehrenpreis' Fundamental Principle; it was published in a joint paper with Bo [1989]. Later, Mats Andersson and Elizabeth Wulcan (2007) could define a residue without the assumption of a complete intersection. In their work, an important role was played by a paper by Mikael, August, and Alain Yger, viz. [2000b].

For complete intersections we have, according to [1988d:42, Theorem 4 iii)],

$$g_j R^p P^1[1/g] = 0, \qquad g = (g_1, \dots, g_{p+1}), \quad j = 1, \dots, p,$$

while the example [1988d:43, Example 3] shows that this is not true when we do not have complete intersections: with $n = 2$, $p = 2$, $q = 0$, $f_1 = f_2 = 1$, $g_1(z) = z_1 z_2$, $g_2(z) = z_2$ we get

$$R^2[1/g] = \bar\partial \left[\frac{1}{z_1 z_2}\right] \wedge \bar\partial \left[\frac{1}{z_2}\right] = \frac{1}{2}\,\bar\partial \left[\frac{1}{z_1}\right] \wedge \bar\partial \left[\frac{1}{z_2^2}\right];$$

when we multiply this current by g_2, we get

$$g_2 R^2[1/g] = \frac{1}{2} z_2 \bar\partial \left[\frac{1}{z_1}\right] \wedge \bar\partial \left[\frac{1}{z_2^2}\right] = \frac{1}{2}\,\bar\partial \left[\frac{1}{z_1}\right] \wedge \bar\partial \left[\frac{1}{z_2}\right] \neq 0.$$

Other examples in this calculus are [1988d:43]:

$$\left[\frac{1}{z_1}\right]\bar\partial\left[\frac{1}{z_2^2}\right] = 2\left[\frac{1}{z_1 z_2}\right]\bar\partial\left[\frac{1}{z_2}\right] \text{ and } \bar\partial\left[\frac{1}{z_1}\right]\wedge\bar\partial\left[\frac{1}{z_2^2}\right] = 2\bar\partial\left[\frac{1}{z_1 z_2}\right]\wedge\bar\partial\left[\frac{1}{z_2}\right].$$

The original definition and the definition which uses meromorphic extension agree [1987:159]:

$$R^p P^q \left[\frac{1}{g}\right] = \lim_{\varepsilon \to 0} \frac{\bar\partial |g_1|^\varepsilon}{g_1} \wedge \cdots \wedge \frac{\bar\partial |g_p|^\varepsilon}{g_p} \cdot \frac{|g_{p+1}|^\varepsilon}{g_{p+1}} \cdots \frac{|g_{p+q}|^\varepsilon}{g_{p+q}}.$$

Here the left-hand side is defined according to Definition A already mentioned, while the right-hand side, called Definition B, is the one which comes from meromorphic extension. In fact, when $\operatorname{Re} \varepsilon$ is sufficiently large, the expression following the lim operator defines a current. It has a meromorphic extension which is holomorphic near $\varepsilon = 0$, and the right-hand side is its value at $\varepsilon = 0$.

In a CV which Mikael wrote in 2000 he mentions a book project with August Tsikh as coauthor and which had the title *Multidimensional Residues and Toric Varieties*. He gives a detailed table of contents of the five chapters in the book. Later they abandoned this project, since amoebas and tropical geometry became more interesting for them, and they aimed at writing a book on amoebas (August Tsikh, personal communication 2011-10-06).

Lineal convexity

André Martineau (1930–1972) gave a couple of seminars on lineal convexity (*convexité linéelle*) in Nice during the academic year October 1967 through September 1968, when I was there. This is a kind of complex convexity which is stronger than pseudoconvexity and weaker than convexity. Since I was of the opinion that the results for this convexity property were too scattered in the literature and did not always have optimal proofs, I suggested that Mikael write a survey article on the topic.

On the one hand, this piece of advice was certainly very good, for he found a lot of results in cooperation with his friends Mats Andersson and Ragnar Sigurðsson (Mikael's mathematical uncle). On the other hand, it was perhaps not such a good suggestion, for the survey just kept growing, and two preprints were circulating starting in 1991[4]—and by then they had been busy writing for a long time already. The article became a book, and it did not appear until 2004 [2004b]. Anyway, it is thanks to André Martineau that lineal convexity came to be studied in the Nordic countries—and the book has become a standard reference.

In this book, the authors study in detail a property which Martineau called strong lineal convexity (*convexité linéelle forte*), and which he did not characterize geometrically. This notion, in the book called **C**-*convexity*, is not linked to any cleistomorphism (closure operator), since the intersection of two strongly lineally convex sets need not have the property. Therefore it has a different character than lineal convexity and usual convexity, which, as is well known, are the fixed points of cleistomorphisms.

An important characterization of strong lineal convexity has been obtained recently by Gennadi Henkin and Peter Polyakov: a lineally convex compact set is strongly lineally convex if and only if it can be approximated in the Hausdorff metric by lineally convex compact sets with smooth boundaries (Henkin & Polyakov (2012: Proposition 2.4)). For related question see also Kiselman (2016) and the references mentioned there.

[4] I no longer possess any documentation about preprints from 1991, but in the CV that Mikael wrote in 2000, two are mentioned: Andersson, Mats; Passare, Mikael; Sigurdsson, Ragnar (1995), Complex convexity and analytic functionals I, Reykjavík, 71 pp.; and (2000), Complex convexity and analytic functionals II, Reykjavík and Sundsvall, 103 pp. The book [2004b] came to comprise xii + 160 pages.

Amoebas and tropical geometry

Mikael's later work is concerned with amoebas and coamoebas—the first publications in this field were Mikael Forsberg's thesis (1998) and their joint paper [2000a]. The spine of an amoeba—in mathematical zoology, amoebas are vertebrates—is a tropical hypersurface. Tropical mathematics is a rather new branch of mathematics, where addition and multiplication is replaced by the maximum operation and addition, somewhat similar to taking the logarithm of a sum and a product.[5] His interest in tropical mathematics was a break with his earlier work on complex analysis, which he once compared with my switching to digital geometry.

An amoeba is a set in $\mathbf{R}^n$ defined as follows. We define a mapping

$$\operatorname{Log}\colon (\mathbf{C} \smallsetminus \{0\})^n \to \mathbf{R}^n \quad \text{by} \quad \operatorname{Log}(z) = (\log|z_1|, \log|z_2|, \dots, \log|z_n|).$$

If f is a function defined in $(\mathbf{C} \smallsetminus \{0\})^n$, then its *amoeba* is the image under Log of its set of zeros. The term was introduced by Gelfand et al. (1994). For more recent developments see. e.g., Viro (2011).

One can of course study the image in $\mathbf{R}^n$ of any set, but zero sets of certain functions have interesting properties. An amoeba is typically a closed semianalytic subset of $\mathbf{R}^n$ with tentacles which go out to infinity and separate the components of the complement of the amoeba. The number of such components is at most equal to the number of integer points in the Newton polytope for f if f is a Laurent polynomial; in certain cases equal to the latter number [2000a].

An easy example, which Mikael himself used in his lectures, is the zero set of the polynomial $P(z,w) = 1 + z + w$ of degree one. A zero $(z,w) \in \mathbf{C}^2$ must satisfy $1 \leqslant |z| + |w|$; $|z| \leqslant |w| + 1$; and $|w| \leqslant 1 + |z|$. It is easy to see that any point $(p,q) \in \mathbf{R}^2$ which satisfies the three inequalities $1 \leqslant p + q$; $p \leqslant q + 1$; and $q \leqslant 1 + p$ is equal to $(|z|, |w|)$ for some zero (z,w) of P. (A useful observation here is the fact that the corresponding strict inequalities are the exact conditions under which there exists a triangle with side lengths 1, p and q.) The amoeba of P is then given by the three inequalities $1 \leqslant e^x + e^y$; $e^x \leqslant e^y + 1$; and $e^y \leqslant 1 + e^x$.

Of course one can study the zero set directly without taking the logarithm. That it nevertheless has interesting consequences to do so Mikael showed in [2008a]: it is about area preserving.

[5]It seems that the first use of the adjective *tropical* in this sense in the title of a publication was in Simon (1988).

A *coamoeba* is defined analogously but with the mapping Log replaced by the mapping $\operatorname{Arg}(z) = (\arg z_1, \arg z_2, \ldots, \arg z_n)$. Mikael wanted to establish formally the duality between amoebas and coamoebas, and he started to write a paper with Mounir Nisse, which Mounir has now finished (this volume). For other relevant papers, see Nisse (2009) and Nisse & Sottile (2013a; 2013b).

Jens Forsgård and Petter Johansson have continued the work on coamoebas and published two papers (2014; 2015) on the subject.

A straight line in the plane can be described by an equation

$$ax + by + c = 0,$$

and hence as the fold line of the convex function

$$f(x, y) = (ax + by + c) \vee 0, \qquad (x, y) \in \mathbf{R}^2,$$

where the maximum operation is denoted by $\vee$: $s \vee t = \max(s, t)$, $s, t \in \mathbf{R}$. If we replace addition by the maximum operation and multiplication by addition, we obtain

$$g(x, y) = (a + x) \vee (b + y) \vee c, \qquad (x, y) \in \mathbf{R}^2.$$

A tropical straight line can therefore be defined as the fold lines for the function g, which consists of three rays. They emanate from the point $(c-a, c-b)$ in the directions $(1, 1)$, $(-1, 0)$ and $(0, -1)$. For example, the amoeba of the polynomial $P(z, w) = 1 + z + w$ mentioned earlier contains the tropical line emanating from $(0, 0)$, which is its spine.

If $p = (p_1, p_2)$ and $q = (q_1, q_2)$ are two points in the plane with

$$q_1 \neq p_1,\ q_2 \neq p_2 \text{ and } q_2 - q_1 \neq p_2 - p_1,$$

then it is easy to see that there is one and only one tropical straight line through p and q. If one of the conditions is not satisfied, there exist infinitely many tropical straight lines through p and q, but Mikael explained that one should accept only those lines that are stable under small perturbations; then you get a single line. In the same way, two distinct tropical straight lines meet in a single point if we only accept intersections that are stable under small perturbations.

Just like in spherical geometry there do not exist any distinct parallel lines. We can go on and ask about all the axioms of Euclidean geometry.

The similarity with taking the logarithm is based on the formulas

$$\log(x \times y) = \log x + \log y, \qquad x, y > 0, \text{ and}$$

$$\log x \vee \log y \leqslant \log(x + y) \leqslant \log 2 + (\log x \vee \log y), \qquad x, y > 0.$$

In the little paper [2008a], which is indeed a gem, Mikael shows how the concept of an amoeba can be used to show the well-known formula $\zeta(2) = \sum_1^\infty 1/n^2 = \pi^2/6 \approx 1.644934$ (the so called Basel problem).

The Pluricomplex Seminar

I started a seminar series in Uppsala in the 1970s. In the beginning it was more like a study group, and had no name, since I thought that a name could be hampering. But later I discovered that almost everything was about several complex variables, and during a visit to Strasbourg I saw that Jean-Pierre Ramis had used the name *Séminaire pluricomplexe*. That sounded mysterious enough, and I borrowed it to Uppsala. During the Fall Semester of 1980 the title was *Pluricomplex Analysis and Geometry*; in the Spring Semester of 1981 it was *Pluricomplex Analysis*, and from the Spring Semester of 1982 on, the name was *The Pluricomplex Seminar*.

Mikael's gave his first lecture in the seminar during the Fall Semester of 1978. He reported on chosen sections of the little book by Lev Isaakovič Ronkin (1931–1998) entitled *The Elements of the Theory of Analytic Functions of Several Variables* (1977), which had been published in Russian in 2,700 copies in Kiev the year before and cost 93 kopecks. The task was a part of the examination for the course *Mathematics D*.

Lectures held by Mikael Passare

Except in four cases, the lectures listed here were given by Mikael at the Pluricomplex Seminar.

1978-11-13. *Analytisk fortsättning* [*Analytic continuation*]. (Report on a special project for the advanced course *Mathematics D*.)

1982-11-01. *Henkin–Ramirez formulas for weight factors (according to Bo Berndtsson and Mats Andersson).*

1983-01-24. *Godtyckliga områden som projektioner av pseudokonvexa områden* [*Arbitrary domains as projections of pseudoconvex domains*].

1983-04-18. *Integraloperatorer för att lösa Cauchy–Riemanns ekvationer (efter R. Michael Range)* [*Integral operators for solving the Cauchy–Riemann equations (after R. Michael Range)*].

1983-06-15. *Samband mellan mängder av Newtonkapacitet noll och pluripolära mängder (efter Azim Sadullev)* [*Links between sets of Newton capacity zero and pluripolar sets (after Azim Sadullaev)*].

1984-05-14. *Ideal i ringen av holomorfa funktioner definierade medelst strömmar, I* [*Ideals in the ring of holomorphic functions defined by means of currents, I*].

1984-05-21. *Ideal i ringen av holomorfa funktioner definierade medelst strömmar, II* [*Ideals in the ring of holomorphic functions defined by means of currents, II*].

1984-12-10. *Residuer, strömmar och deras relation till ideal av holomorfa funktioner* [*Residues, currents, and their relation to ideals of holomorphic functions*] (cf. [1984], the thesis which was to be defended five days later).

1985-03-20. *Produkter av residuströmmar* [*Products of residue currents*] (cf. [1985]).

1986-01-17. *A new proof for integral representation formulas without boundary term.*

1986-04-14. *Principal values of meromorphic functions.*

1986-09-18. *1. Shortcut to weighted representation formulas for holomorphic functions* (cf. [1988a]). *2. Impressions from the International Congress of Mathematicians, Berkeley.*

1988-11-07. *Kergin interpolation of entire functions* (cf. [1991a; 1991b]).

1989-02-22. *Continuity of residue integrals in codimension two.*

1989-05-24. *Integralformler och residuer på komplexa mångfalder* [*Integral formulas and residues on complex manifolds*].

1989-10-04. *Kergin interpolation on* **C**-*convex sets.*

1991-02-18. *Mathematical impressions from Krasnoyarsk: 1. Holomorphic extension from a part of the boundary. 2. Toric varieties.*

1993-05-05. *Projektiv konvexitet* [*Projective convexity*].

1993-09-24. A lecture at a meeting of the Swedish Math Society at the Royal Institute of Technology.

1994-05-19. *Holomorphic differential forms on analytic sets.*

1998-09-07. *Amoebas and Laurent determinants* (cf. [2000a; 2004a]).

2000-03-14. *Constant terms in powers of a Laurent polynomial.*

2001-05-04. *Amoebas, Monge–Ampère measures, and triangulations of the Newton polytope.* Lecture at the Nordan Meeting in Oslo.

2001-10-16. *Complex convexity—recent results of Kiselman and Hörmander.*

2002-05-07. *Discriminant amoebas.*

2002-11-19. *Algebraic equations and hypergeometric functions.*

2003-03-04. *The Lee–Yang circle theorem and geometry of amoebas.*

2003-10-21. *Amöbor, polytoper och tropisk geometri* [*Amoebas, polytopes, and tropical geometry*].

2004-01-10. *Koamöbor och Mellin-transformer av rationella funktioner* [*Coamoebas and Mellin transforms of rational functions*].

2006-05-19. A lecture at the Nordan Meeting in Sundsvall.

2006-07. A minicourse on amoebas given at Institut de Mathématiques de Jussieu, Paris.

2010-03-09. *(Co)amoebas of linear spaces.*

2010-10-19. *Mellin transforms and hypergeometric functions.*

Originally, the seminars took place at Uppsala with a lecture in general every week. From the Spring Semester of 1999 on, when Mikael had become well established as a professor at Stockholm, they became a joint activity for Uppsala University, Stockholm University, and the Royal Institute of Technology (KTH), with an alternating venue. To minimize travel we had two lectures every second week. From 2007, when I had switched to digital geometry, mathematical morphology, and discrete optimization, and Burglind Juhl-Jöricke had left Uppsala University, it became an activity exclusively in Stockholm.

The *Nordan* Meetings

Together with Mats Andersson and Peter Ebenfelt, Mikael Passare initiated a series of encounters on complex analysis in the five Nordic countries. Mikael and Peter organized the first conference, which took place in Trosa, Sweden, March 14–16, 1997; Mats the second, in Marstrand, Sweden, April 24–26, 1998. Following a voting procedure at the end of the first meeting, these yearly meetings were named *Nordan*[6]—a clear reference to *Les Journées complexes du Sud*, which during a long time have taken place in the south of France.

Mikael edited abstracts in Swedish of the lectures—which had all been given in English. These brochures were published with a delay of a few years. Twelve of them have come out; he was preparing the thirteenth, which was to report on *Nordan 13* held in Borgarfjordur in 2009, and asked Ragnar Sigurðsson on 2011 September 10, to write a preface in Icelandic (Ragnar Sigurðsson, personal communication 2011-10-04).

Lars Filipsson emphasizes (personal communication 2011-10-06) that Mikael wrote these brochures in Swedish to develop Swedish terms in higher mathematics, especially in complex analysis—otherwise the Swedish mathematical terms reach up to the first, possibly the second, university year only.

Nordic meetings like these were something that Mikael and Mats had discussed and planned during many years; both of them wanted to create a forum with a more relaxed atmosphere, where Nordic complex analysts, in particular the young ones, could feel more at home than at big international conferences, and which would give those that worked in the Nordic countries occasion to get to know each other better.

And the initiative turned out to be a long-lasting success: the fifteenth encounter took place in Röstånga in southern Sweden, 2011 May 06–08; the sixteenth in Kiruna in northern Sweden, 2012 May 11–13; the seventeenth in Svolvær, Norway, 2013 May 24–26; the eighteenth at CIRM, Luminy, France, 2014 March 24–29, as a joint session of Nordan and the Komplex Analysis Winter school And workshop (KAWA); and the nineteenth in Reykjavík, Iceland, 2015 April 25–26. The twentieth Nordan took place in Stockholm, 2016 March 16–20 as a session of the 27th Nordic Congress of Mathematicians.

[6] This is the name in Swedish of a chilly wind from the north, but also reminds us of the original purpose: to promote Nordic Analysis.

Africa

Mikael Passare was a Member of the Board of the International Science Programme (ISP), Uppsala, and a Member of the Board of the Pan-African Centre for Mathematics (PACM) in Dar es-Salaam, Tanzania. He was a driving force in the creation of this Pan-African Centre, which is a collaborative project between Stockholm University and the University of Dar es-Salaam.

Mohamed E. A. El Tom, Chairman of the Board of PACM and a member of the Reference Group for Mathematics of ISP, says that he is confident that had it not been for Mikael, PACM would have remained a mere idea in the head of its initiator, i.e., in Mohamed's head; see El Tom (2011). Mikael started working with great conviction and enthusiasm on the idea when Mohamed first suggested it to him while they were walking on a Meroetic archeological site near Khartoum in April 2004. (Mohamed El Tom, personal communication 2011-10-17.)

Mikael took an early, informal contact with the Vice-Chancellor (*Rektor*) of Stockholm University, who expressed his approval in principle (Mohamed El Tom, personal communication 2011-10-20).

In October 2008, Mikael and Mohamed discussed the idea with Anders Karlhede, Dean of the Division, and asked whether Stockholm University could be a partner in the project. Anders immediately took the question to Stefan Nordlund, Dean of the Faculty. The latter proved to be very positive, which was decisive for the coming commitment of Stockholm University to PACM. (Anders Karlhede, personal communication 2011-10-19.)

Mikael then presented the idea to the Department of Mathematics at Stockholm University. While the department did not object to the idea, it was only natural that some members raised many significant issues that required clarification. Mikael maintained correspondence with Mohamed about these and related isues for more than two years, at the end of which he managed to secure the approval of the department to collaborate in establishing the Centre at some suitable university in Africa. Later he was an influential member of the committee that short-listed African universities for hosting PACM. Subsequently, he was a member of a delegation led by Stefan Nordlund which visited some of the short-listed universities and made appropriate recommendations to the Vice-Chancellor of Stockholm University.

Mikael never ceased devoting of his precious time to the Centre. His last assignment was to chair and constitute a search committee for the Director of the Centre, a process he initiated before he was asked by the Board of the Centre to undertake it. Such was Mikael, ahead of others in thinking, and working to realize important objectives without being asked to do so. When Mohamed conveyed to him the Board's decision regarding the search committee, he responded promptly, accepting the charge, and promised to respond with detailed ideas upon his return from the trip to Dubai, Oman and Iran that he was planning to undertake.

Mikael's commitment and enthusiasm for the Centre was unsurpassed. He was confident that the grand objective of establishing a world-class Centre of Mathematics in Africa is attainable. (Mohamed El Tom, personal communication 2011-10-17; this note applies to the last three paragraphs.)

Sonja Kovalevsky

The chair which Mikael Passare held was the one which was created for Sonja Kovalevsky (1850 January 03/15 — 1891 February 10). An earlier incumbent during seven years, 1957–1964, was Lars Hörmander (1931–2012), Mikael's mathematical grandfather. Mikael was proud of having been appointed to Sonja's chair. He is buried not far from her grave.

Exactly 150 years after Sonja's birth, on 2000 January 15, Mikael organized a symposium to her memory. It was held in the *Aula Magna* of Stockholm University. Among the invited speakers were Agneta Pleijel, Roger Cooke and Ragni Piene.

Languages

In the section on the *Nordan* meetings, I have already mentioned that Mikael was interested in developing Swedish mathematical terms. He knew many languages. His Russian was "really perfect!" according to Timur Sadykov (personal communication 2011-10-13); "he spoke Russian perfectly, so it was totally impossible to recognize his Swedish origin" (Andrei Khrennikov, personal communication 2011-11-26). He took a course in French corresponding to 30 ECTS credit points at Stockholm University before going to Paris in 1986-87 (diploma dated 1985-09-03). He learned some Fijian when he visited the Republic of Fiji (Timur Sadykov, personal communication 2011-10-16).

His knowledge of German was very good although he had not studied that language in high school. He studied also Finnish and spoke the language so well that he was interviewed in the Finnish-language *Sisuradio* in Sweden.

Spanish and Italian he knew enough to get along. He was in Italy and Spain with Anders Wändahl, and never talked English when visiting a restaurant or when asking for directions in the street. He could also speak some Polish and Bulgarian.

Finally, he studied Arabic and could at least read that langugage. Maybe Arabic would have been his next project. (Anders Wändahl, personal communication 2011-10-19; this remark applies to this paragraph and the preceding one.)

An extraordinary curiosity

Andrei Khrennikov writes:

> I would like mention Mikael's extraordinary curiosity, which was extended to a large variety of fields. In particular, he discussed with excitement the possibility of mathematical modeling of cognition, human psychological behavior, and consciousness. I met Mikael and Galina the last time in July 2010, in Stockholm, and during one evening we discussed a large variety of topics: complex and p-adic analysis, mathematical foundations of quantum physics, quantum nonlocality, Bell's inequality and experiments [...] (Andrei Khrennikov, personal communication 2011-11-26)

Music

Mikael loved classical music; in his teens he sold his bicycle in order to buy a piano. He played clarinet and flute. He composed a piece for clarinet, which was played in a theater in Stockholm. His last love was an instrument called theremin.[7] He dreamed about being able to play it.[8]

[7] Терменвокс, which was invented by Лев Сергеевич Термен, Léon Theremin (1896–1993).

[8] At his funeral on 2011 October 28, *Dance in the Moon* was played on CD; the performer was Lydia Kavina, a leading thereminist.

He also loved to sing and was a member in a choir and learned to sing solo both in Stanford and in Moscow. (Galina Passare, personal communication 2011-10-17; this applies to all of this section.)

A "Swedish Classic"

Mikael swam several times a week, at least 2 km. He loved the mountains and skied long distances (sometimes 90–130 km) spending the night in cottages. He swam between islands in Lake Mälaren close to Stockholm. (Galina Passare, personal communication 2011-10-17.)

He ran the Stockholm Marathon. To Yûsaku Hamada's guest lectures in Uppsala on 2002 September 10, he went by bike from Stockholm (Yûsaku Hamada, personal communication 2011-09-19). Another time he skated on Lake Mälaren to the seminar in Uppsala. (After the seminar, however, he went back to Stockholm by train.)

The Viking Run (in Swedish: *Vikingarännet*) is the world's biggest regular skating event on natural ice, and is arranged yearly since 1999. It usually starts at Skarholmen in Uppsala and finishes at some place in or near Stockholm (depending on ice conditions; sometimes the ice is so bad that the competition has to be canceled). Mikael participated in the Viking Run several times.

Mikael also performed what is known as a "Swedish Classic" in 1989. It consists of four parts, which have to be done within a twelve-month period: (1) A ski run, either the Engelbrekt Run, 60 km, or the Vasa Run / Open Track, 90 km; (2) Going around Lake Vättern on bicycle, 300 km; (3) The Vansbro Swim, 3 km; and (4) The Lidingö Run, 30 km. Mats Andersson (personal communication 2011-10-12) remembers that he claimed the cycling to be the most painful of the four, noting the chafing after so many hours on the saddle.

He loved bandy (a sport similar to ice hockey but played with a ball and on an ice field the size of a soccer field) and missed only one single Swedish Championship Final since 1980, viz. the one in 1983 (Anders Wändahl, personal communication 2011-10-12). He was considerate also of other bandy fans. When Magnus Carlehed, his mathematical nephew, was traveling in 1990 around the globe in areas where ice rinks are not so common, he sent to Magnus a big envelope to the address *Poste restante*, Denpasar (Bali), Indonesia. It contained a video recording showing the Swedish Bandy Championship Final. (Magnus Carlehed, personal communication 2011-12-09.) He went to Arkhangelsk to see the Bandy World Championship there (probably in 1999, when Russia won over Sweden in the final). He also traveled to Oulu with Björn Ivarsson, his mathematical younger brother, to see the Championship in 2001 (when Russia won over Sweden again in the final; Björn Ivarsson, personal communication 2011-12-14).

A passionate traveler

Mikael saw at least three total solar eclipses: the one which took place on 1999 August 11 he saw in Turkey (although it would perhaps have been easier to see it in Bulgaria); the eclipse of 2002 December 04 he saw in Mozambique; and on 2006 March 29 he was in Niger with Anders Wändahl (although the southern coast of Turkey would have been easier to reach from Sweden and had a greater chance of a clear sky without sand storms). After that they continued to Chad.

Mikael was a passionate traveler. He visited 152 countries. When he and I, together with several other Swedish mathematicians, were invited in September 2006 to celebrate the twentieth anniversary of the *Groupe Inter-Africain de Recherche en Analyse, Géométrie et Applications* (GIRAGA) and after that to participate in the *First African-Swedish Conference on Mathematics*, both in Yaoundé, Cameroon, he first visited the Central African Republic and continued afterwards to Equatorial Guinea and Gabon (Anders Wändahl, personal communication 2011-11-14); thus he got four new countries on his list—assuming that he had not been in any of these before—while I got only one.

The United Arab Emirates and Oman turned out to be the last ones. Land number 153 should have been Iran: he planned to arrive at Tehran Imam Khomeini International Airport on September 17 at 21:25 (Mikael Passare, electronic letter 2011-09-15 to mathematicians in Tehran). Siamak Yassemi, Head of the School of Mathematics, University of Tehran, was ready to meet him there.

Finally

Mikael's significance goes much beyond his own research. Many persons have testified to his positive view of life, his humor, and to his genuine interest in people he met. He was an unusually stimulating partner in discussions; listening, inspiring, and supportive, in professional situations as well as private ones.

For Mikael's friends and colleagues around the world his unexpected departure is a severe loss.

For me personally, Mikael's disappearance seems unreal. He was always there for me. I shall remember him with joy and gratitude as long as I live.

Two proposals

At a meeting at Stockholm University to Mikael's memory on 2011 September 28, arranged by Tom Britton, I ended my speech by presenting two proposals.

The first proposal was that Stockholm University organize a conference to his memory, where his many mathematical achievements could be presented and discussed. It has now been realized (at least in part) as a yearly event, *Mikael Passare's Day*, organized at Stockholm University in September or October each of the six years 2011 through 2016.

Since, as far as I know, Mikael has not published all his ideas on tropical geometry, I proposed, secondly, that his former students write a survey article about these ideas (and of course other mathematical ideas). Alicia Dickenstein (2011-09-24), August Tsikh (2011-10-03), Alexey Shchuplev (2011-10-06), and Hans Rullgård (2011-10-11) have all spontaneously approved of this and want to contribute to this project.

The second proposal can of course be realized as a part of the first, viz. if the survey article is published in the conference proceedings.

See also my paper "Questions inspired by Mikael Passare's mathematics" (2012, 2014).

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Sources

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The sources are in certain cases messages that I have received during the writing process, and, if so, reported as a personal communication at the end of a sentence or a paragraph. Otherwise I have relied on documents that I have saved, notes that I have made—and my memory.

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