Rules

- Places and transitions are connected via arcs.
- Arcs can not connect a place to a place or a transition to a transition.
- Tokens can only be put in places.

Lecture 2

Terminology, formal representation, and analysis of Petri-Nets.

Enabling and firing transitions.

- A transition is enabled when there is a token in each input place to that transition.
- A transition is chosen non-deterministically from among the set of enabled transitions for firing.
- Firing a transition consumes a token from each place from which an arc leads to the transition, and generates a token onto each output arc which is deposited in the place to which the arc leads.

Terminology

- Place:
- Transition:
- Token:
- Arc:

Formal representation

P = {P1, P2, P3, P4}

T = {T1, T2, T3}

I(T1) = {P3} O(T1) = {P4}

I(T2) = {P1, P4} O(T2) = {P3}

I(T3) = {P1, P1} O(T3) = {P2}

μ = {2, 1, 0, 1}

Formal representation of Petri-Nets

- A Petri-Net is represented as a 5-tuple

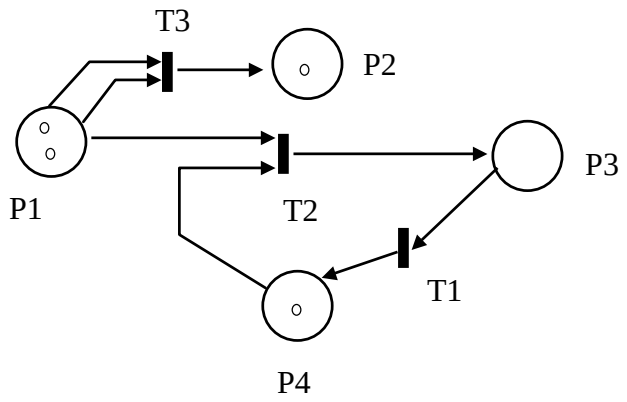
  C = { P, T, I, O, μ }

- P is the set of places,
- T is the set of transitions,
- I is a function that yields the set of input places to each transition.
- O is a function that yields the set of output places for a transition,
- μ records the distribution of tokens througout the network.

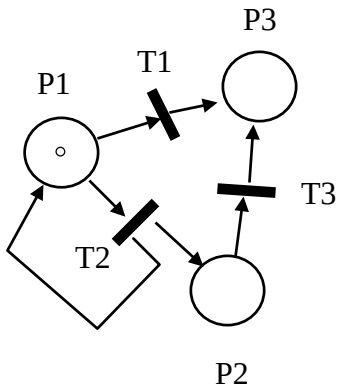
Analysing Petri-Nets

- To prove properties of Petri-Nets it we can use a technique called "reachability analysis".
- A "reachability tree" shows all the possible states that a petri-net can achieve by firing transitions begining from the initial marking.
- By enumerating all the possible "reachable" states of the net, we can quickly determine safety, liveness or deadlock properties of the net by inspection.

Example



An example of folding



A reachability tree

The reachability tree

Dealing with infinite nets.

- To deal with infinite firing sequences we introduce an infinite token marker ω.
- Putting ω in a place represents the occurence of a possibly infinite number if tokens in that place.
- If a marking is greater than or equal to a previous marking in terms of the number of tokens in each place, and their sets of potentially fireable transitions are identical and unchanged by introducing ω, then the places which hold strictly greater numbers of tokens can be labeled with an ω.

Useful properties of a reachability tree

- A leaf node represents a deadlock state.
- An ω means that the net is infinite.
- The transitions connecting a node to its children are the potentially fireable transitions from the state represented by that node.