Schedulability Analysis of Timed Systems

with contributions from
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PROBLEM SETTING

Real Time Systems

Scheduler (Resource management)

Real Time Software

Who is Who in Timed Systems

- Real Time Scheduling [RTSS ...]
  - Task models, Schedulability analysis
  - Real-time operating systems
- Automata/logic-based methods [CAV, TACAS ...]
  - (Timed) Automata, Petri Nets, Process Algebras ...
  - Modelling, Model checking ...
- (Real-Time) Programming Languages [...]
  - Esterel, Signal, Lustre, Ada ...
- ...
**“Classic” Real Time Scheduling**

Periodic tasks

Schedulability can be checked by solving (or UB test):

\[ R_i = B_i + C_i + \sum_{j \in HP(i)} \left( \frac{R_i}{T_j} \right) \cdot C_j \]

If \( R_i \leq D_i \) for all tasks \( P_i \), the system is schedulable.

**Rate-Monotonic Scheduling**

- \( P_1 \ldots P_n \) arrive at fixed rates
- Fixed Priority Order: higher frequency => higher priority
- Always run the task with highest priority (FPS)
- Schedulability can be checked by solving (or UB test):

**Arrival Rates (of Tasks): Periodic**

**In “real life”**

- tasks may share many resources (not only CPU time)
- tasks may have complex control structures and interactions
- tasks may not be that “regular” (often non-periodic)

**Task Arrival Patterns: Timed Traces**

**The ABB Robot Controller**

- ABB robot controller (2 500 000 loc)
- Real time tasks A,B,C,D
- Read inputs from channels write output to channels
- Task priority order: D>C>B>A (FPS)
- Buffer overflow/underflow, WCRT
Automata-Based Approaches

- Hardware
- Sensors
- Actuators
- State Machines
  e.g. Timed automata

Networks of Real-Time Components

- Scheduler
  (Resource management)
- State Machines
  e.g. Timed automata

Automata-based Approaches

A controller = a set of timed automata accepting events trigger tasks $P_i$

- How to schedule tasks/automata? Worst-case response times?

Problems to solve

- Schedulability analysis: check
  $(A_1 || A_2 || ... || A_n || $Scheduler$)$ satisfies $K$
  - A scheduler is given e.g. FPS, RMS, EDF etc.
  - $K$ is a requirement specifying e.g. safety
- Schedule synthesis: find $X$ such that
  $(A_1 || A_2 || ... || A_n || X)$ satisfies $K$

OUTLINE

- A Model for Timed Systems [1998]
  - Timed automata with tasks
- Schedulability and Decidability [TACAS 02,04]
  - Timed automata with bounded subtraction
- More Efficient Algorithms [TCS 06, IC 07]
  - Schedulability analysis using 3 clocks
  - similar to Rate-Monotonic Scheduling
- TIMES Demo
- Current/Ongoing Work

The MODEL

(Timed Automata with Tasks)
Timed Automata with Tasks

- Events
  - Discrete Transitions
- Timing constraints
  - Clocks / Guards / Resets
  - Complex arrival rates
- Tasks
  - Asynchronous execution
  - WCET, Deadline
  - Scheduling policy
  - Precedence constraints
  - Resource constraints

Example: periodic tasks

Example: Timed Automata with Tasks

Tasks = Executable Programs (e.g. C, Java)

- Task parameters:
  - C: WCET
  - D: Relative Deadline
  - (other parameters for scheduling e.g. Priority)
- Task Interface:

  Task P
  { 
  v1 := F1(v1...vn) 
  ...
  vn := Fn(v1...vn) 
  }

(a set of variables updated)

Timed Automata with Tasks (Example)

Processor 1 (event handler)
- Initially, P in the queue
- Run-to-Completion/Stabilization
- Whenever a available and x>10, Q is put in the queue
- Then
  - Whenever b available and y<=50, P is put in the queue
  - Whenever f available, R is put in the queue.

Processor 2 (task handler)
- Schedule and Compute tasks in the queue

The Execution Platform

Thread 1
Thread 2: Scheduling Policy
Thread 3
States/Configurations of automata

A state is a triple: \((m, u, q)\)

- **Location (node)**
- **clock assignment (valuation)**
- **task queue**

Run of TAT

\[(\text{Idle}, x=0, []) \rightarrow (\text{RelP}, x=0, [P(2,8)])\]
\[1.5 \rightarrow (\text{RelQ}, x=1.5, [P(0.5,6.5)]) \rightarrow (\text{RelQ}, x=1.5, [P(0.5,6.5), Q(2,20)]) \rightarrow (\text{Idle}, x=3, [Q(1,18.5)]) \rightarrow (\text{RelP}, x=0, [P(2,8), Q(1,18.5)]) \rightarrow (\text{RelQ}, x=3, [Q(1,18.5)]) \rightarrow (\text{Idle}, x=3, [Q(1,18.5)]) \rightarrow (\text{RelP}, x=2, [Q(1,16.5)])\]

Sch and Run

- **Sch** is a function sorting task queues according to a given scheduler e.g. FPS, EDF, FIFO etc
  - Example: EDF \([P(2, 10), Q(4, 7)]\) = \([Q(4, 7), P(2, 10)]\)

- **Run** is a function corresponding to running the first task of the queue for a given amount of time.
  - Examples: Run\((0.5, [Q(4, 7), P(2, 10)])\) = \([Q(3.5, 6.5), P(2, 9.5)]\)
  - Run\((5, [Q(4, 7), P(2, 10)])\) = \([P(1, 5)]\)

Semantics (as transition systems)

- **States**: \(<m, u, q>\)
  - \(m\) is a location
  - \(u\) is a clock assignment (valuation)
  - \(q\) is a queue of tasks (ready to run)

- **Transitions**:
  1. \((m, u, q) \rightarrow ((n, r(u)), \text{Sch}(M(n); q))\) if \(g(u)\)
  2. \((m, u, q) \rightarrow ((m, u+\Delta), \text{Run}(u; q))\) where \(d\) is a real

OBS: \(q\) is growing (by actions) and shrinking (by delays)

“Zenoness” = Non-Schedulability

Zeno: \(\infty\) many P’s may arrive within 1 time unit!

But after 2 copies, the queue will be non-schedulable
Schedulability of automata

A state is a triple: \((m, u, q)\)

- Location
- Clock assignment
- Task queue

• A state is schedulable if \(q\) is schedulable
• An automaton is schedulable if all reachable states are schedulable

Schedulability of Automata

Assume a scheduler \(Sch\):

• A state \((m, u, q)\) is schedulable with \(Sch\) if
  - \(Sch(q) = [P_1(c_1,d_1)P_2(c_2,d_2)\ldots P_n(c_n,d_n)]\) and
  - \((c_1 + \ldots + c_i) \leq d_i\) for all \(i \leq n\) (i.e. all deadlines met)
• An automaton is schedulable with \(Sch\) if all its reachable states are schedulable
• An automaton is schedulable with a class of scheduling policies if it is schedulable with every \(Sch\) in the class.

DECIDABILITY

Schedulability Analysis (Non-preemptive scheduling)

FACT [1998]

For Non-preemptive schedulers, the schedulability of an automaton can be checked by reachability analysis on ordinary timed automata.

Proof ideas (1):
Size of schedulable queues is bounded

- The maximal number of instances of \(P_i\) in a schedulable queue is bounded by \(M_i = \lceil \frac{D_i}{C_i} \rceil\)
- The maximal size of schedulable queues is bounded by \(M_1 + M_2 + \ldots + M_n\)
- To code the queue/scheduler, for each task instance, use 2 clocks:
  - \(c_i\) remembers the computing time
  - \(d_i\) remembers the deadline

Proof ideas (2):
The scheduler as an automaton

Start

- \(P_i\) is running
- \(\text{released}_{P_i}\)
- \(\text{released}_{P_j}\)

\(P_i = (C_i,D_i)\)
\(P_j = (C_j,D_j)\)

Proof ideas (2):
The scheduler as an automaton

Start

- \(P_i\) is running
- \(\text{released}_{P_i}\)
- \(\text{released}_{P_j}\)

\(P_i = (C_i,D_i)\)
\(P_j = (C_j,D_j)\)
The scheduler automaton

Proof Ideas (3)

- Modify the original automaton $M$: adding 'release!' to inform the scheduler
- Check reachability of the error state for $M^* || SCHEDULER$

How about preemptive scheduling?

- We may try the same idea
  - Use clocks to remember computing times and deadlines
- BUT a running task may be stopped to run a more 'urgent' task
  - Thus we need stop-watches to remember "accumulated computing times"
- Then the schedulability problem is undecidable?
  - This is wrong!!

Decidability Result [TACAS 2002]

- FACT
  - For Preemptive schedulers, the schedulability of an automaton can be checked by reachability analysis on Bounded Subtraction Timed Automata (BSA).

- NOTE
  - Reachability for BSA is decidable
    - Preemptive EDF is optimal; thus the general schedulability checking problem is decidable.

Timed automata with subtraction (i.e. Subtraction Automata, [McManis and Varaiya, CAV’94])

- Subtraction automata are timed automata extended with subtraction on clocks
- That is, in addition to reset $x:=0$, it is also allowed to update a clock $x$ with $x:=x-n$ where $n$ is a natural number

Bounded Subtraction Automata

- A subtraction automaton is bounded if its clocks are non-negative and bounded with a maximal constant (or subtraction is only allowed in the bounded zone).
Schedulability Checking as a reachability problem for Bounded Subtraction Automata

Proof ideas (no stop but subtraction :-)
- Model the scheduler as a subtraction automaton
  - Do not stop the computing clock $c_2$ when a new task $P_1$ is released
  - Let $c_2$ for $P_2$ (preempted) run until the task $P_1$ (with higher priority) finishes, then perform $c_2 := c_2 - C_1$ (note: $C_1$ is the computing time for $P_1$).

Proof ideas (clocks are bounded):
- $c_2$ can never be negative.
- $c_2$ is bounded by $D_2$.

END of proof

Schedulability analysis using DBM’s
Subtraction on Clocks, added to DBM-library (UPPAAL)

Complexity
- $\#\text{clocks (needed)} = 2 \times \#\text{instances}$ (maximal number of schedulable task instances)
- $= 2 \times \sum \frac{D_i}{C_i}$

This is a huge number in the worst case
But the run-time complexity is not so bad!
It works anyway !!!

- #active tasks in the queue is normally small, and the run-time complexity is only related to #active clocks
- If Too many active tasks in the queue (i.e. Too many active clocks), the check will stop sooner and report "non-schedulable"
- AND the analysis can be done symbolically!

WE CAN DO BETTER! [TACAS 03, TCS 06]

For fixed priority scheduling strategies (FPS), we need only 2 clocks (and ordinary timed automata)!

The 2-CLOCK ENCODING
(for fixed-priority scheduling strategies)

Main Idea

- Check the schedulability of tasks one by one according to priority order (highest priority first)
- This is similar to response time analysis in RMS

To code the queue/scheduler, we need:

- 1 integer variable for Pi:
  - \( r \) denotes the response time as in RMS (the total computing time needed before Pi finishes)
- 2 clocks for Pi:
  - \( c \) remembers the accumulated computing time (so much has been computed so far)
  - \( d \) remembers the "deadline"

Intuition of the encoding:

\[
R_I = \tau_I + \sum_{\text{priority}(P_j) > \text{priority}(P_i)} C_j
\]

- Assume: priority(P_j) > priority(P_i) and Pi is analyzed

When Pi finishes, \( r = R_I \)
The “FPS scheduler”: analyzing Pi

Waiting for Pi

Check Pi

Initial

Error

Note that it is not clear that c and r are bounded!

The “FPS scheduler”: analyzing Pi

(we need the boundedness)

Waiting for Pi

Check Pi

Initial

Error

OBS: c is the only interesting info, so M can be any integer! Let M=C

SUMMARY: Decidability

• For Non-preemptive schedulers, the problem can be solved using standard TA.
• For preemptive schedulers, the problem can be solved using BSA (Bounded Subtraction Automata).
• For fixed-priority schedulers, the problem can be solved using TA with only 2 extra clocks – similar to the classic RMA technique (Rate-Monotonic Analysis).

Undecidability

Unfortunately, the problem will be undecidable if the following conditions hold together:
1. Preemptive scheduling
2. Interval computation times
3. Feedback i.e. the finishing time of tasks may influence the release times of new tasks.

Conclusions/Remarks

• Unification of model-checking, real time scheduling, and synchronous programming: a unified model for timed systems (can express complex temporal and resource constraints).
• The first decidability result (and efficient algorithms) for preemptive scheduling in dense time models:
  – The analysis is symbolic (using DBM’s in the UPPAAL tool)
• Implementation: TIMES
TIMES demo

An Overview of TIMES

The INPUT LANGUAGE is very much like "guarded commands"

OBS: guard and update may contain data variables (integer, array)

- guard, update: "synchronous" computation which takes "no time"
- task: "asynchronous" computation which takes time

Tasks = Executable Programs (e.g. C, Java)

- Task Type
  - Synchronous or Asynchronous
  - Non-Periodic (triggered by events) or Periodic
- Task parameters: C, D etc
  - C: Computing time and D: Relative Deadline
  - other parameters for scheduling e.g. priority, period
- Task Interface (variables updated 'atomically')
  - X_i :=F(X_1...X_n)
- Tasks may have shared variables
  - with automata
  - with other tasks (priority ceiling protocols)
- Tasks with precedence constraints

Functionality/Features of TIMES

- GUI
  - modeling: automata with asynchronous tasks
  - editing, task library, visualization etc
- Simulation
  - symbolic execution as MSC's and Gant Charts
- Verification
  - all you do with UPPAAL
  - Schedulability analysis
- Code Generation

Code Generation in TIMES

- Run Time Systems
  - Event Handler, OS interrupt processing system or Polling
  - Task scheduler, generated from task parameters
- Application Tasks = threads (or processes)
  - Already there! (written in C)
  - Current version of TIMES support LegoOS