1 Replace, Reverse and Delete

Define a function *replace*, such that *replace x y zs* yields *zs* with every occurrence of *x* replaced by *y*.

consts *replace* :: $'a \Rightarrow 'a \Rightarrow 'a\ list \Rightarrow 'a\ list$

Prove or disprove (by counterexample) the following theorems. You may have to prove some lemmas first.

theorem *rev* (*replace x y zs*) = *replace x y* (*rev zs*)
theorem *replace x y* (*replace u v zs*) = *replace u v* (*replace x y zs*)
theorem *replace y z* (*replace x y zs*) = *replace x z zs*

Define two functions for removing elements from a list: *del1 x xs* deletes the first occurrence (from the left) of *x* in *xs*, *delall x xs* all of them.

consts *del1* :: $'a \Rightarrow 'a\ list \Rightarrow 'a\ list$
delall :: $'a \Rightarrow 'a\ list \Rightarrow 'a\ list$

Prove or disprove (by counterexample) the following theorems.

theorem *del1 x* (*delall x xs*) = *delall x xs*
theorem *delall x* (*delall x xs*) = *delall x xs*
theorem *delall x* (*del1 x xs*) = *delall x xs*
theorem *del1 x* (*del1 y zs*) = *del1 y* (*del1 x zs*)
theorem *delall x* (*del1 y zs*) = *del1 y* (*delall x zs*)
theorem *delall x* (*delall y zs*) = *delall y* (*delall x zs*)
theorem *del1 y* (*replace x y xs*) = *del1 x xs*
theorem *delall y* (*replace x y xs*) = *delall x xs*
theorem *replace x y* (*delall x zs*) = *delall x zs*
theorem *replace x y* (*delall z zs*) = *delall z* (*replace x y zs*)
theorem *rev* (*del1 x xs*) = *del1 x* (*rev xs*)
theorem *rev* (*delall x xs*) = *delall x* (*rev xs*)