Spectral Analysis of Signals

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Contents

List of Exercises

CHAPTER 5

CHAPTER 6

Preface

Spectral analysis considers the problem of determining the spectral content (i.e., the distribution of power over frequency) of a time series from a finite set of measurements, by means of either nonparametric or parametric techniques. The history of spectral analysis as an established discipline started more than a century ago with the work by Schuster on detecting cyclic behavior in time series. An interesting historical perspective on the developments in this field can be found in [Marple 1987]. This reference notes that the word "spectrum" was apparently introduced by Newton in relation to his studies of the decomposition of white light into a band of light colors, when passed through a glass prism (as illustrated on the front cover). This word appears to be a variant of the Latin word "specter" which means "ghostly apparition". The contemporary English word that has the same meaning as the original Latin word is "spectre". Despite these roots of the word "spectrum", we hope the student will be a "vivid presence" in the course that has just started!

This text, which is a revised and expanded version of *Introduction to Spectral Analysis* (Prentice Hall, 1997), is designed to be used with a first course in spectral analysis that would typically be offered to senior undergraduate or first-year graduate students. The book should also be useful for self-study, as it is largely self-contained. The text is concise by design, so that it gets to the main points quickly and should hence be appealing to those who would like a fast appraisal on the classical and modern approaches of spectral analysis.

In order to keep the book as concise as possible without sacrificing the rigor of presentation or skipping over essential aspects, we do not cover some advanced topics of spectral estimation in the main part of the text. However, several advanced topics are considered in the complements that appear at the end of each chapter, and also in the appendices. For an introductory course, the reader can skip the complements and refer to results in the appendices without having to understand their derivation in detail.

For the more advanced reader, we have included three appendices and a number of complement sections in each chapter. The appendices provide a summary of the main techniques and results in linear algebra, statistical accuracy bounds, and model order selection, respectively. The complements present a broad range of advanced topics in spectral analysis. Many of these are current or recent research topics in the spectral analysis literature.

At the end of each chapter, we have included both analytical exercises and computer problems. The analytical exercises are, more or less, ordered from least to most difficult; this ordering also approximately follows the chronological presentation of material in the chapters. The more difficult exercises explore advanced topics in spectral analysis and provide results that are not available in the main text. Answers to selected exercises are found in Appendix D. The computer problems are designed to illustrate the main points of the text and to provide the reader with first-hand information on the behavior and performance of the various spectral analysis techniques considered. The computer exercises also illustrate the relative performance of the methods and explore other topics—such as statistical accuracy, resolution properties, and the like—that are not developed analytically in the book. We have used MATLAB[1] to minimize the programming chore and to encourage the reader to "play" with other examples. We provide a set of MATLAB functions for data generation and spectral estimation that form a basis for a comprehensive set of spectral-estimation tools; these functions are available at the text website, `www.prenhall.com/stoica`.

Supplementary material may also be obtained from the text website. We have prepared a set of overhead transparencies that can be used as a teaching aid for a spectral analysis course. We believe that these transparencies are useful not only to course instructors but also to other readers, because they summarize the principal methods and results in the text. For readers who study the topic on their own, it should be a useful exercise to refer to the main points addressed in the transparencies after completing the reading of each chapter.

As we mentioned earlier, this text is a revised and expanded version of *Introduction to Spectral Analysis* (Prentice Hall, 1997). We have maintained the conciseness and accessibility of the main text; the revision has focused primarily on expanding the complements, appendices, and bibliography. Specifically, we have expanded Appendix B to include a detailed discussion of Cramér–Rao bounds for direction-of-arrival estimation. We have added Appendix C, which covers model order selection, and have added new computer exercises on order selection. We have more than doubled the number of complements from the previous book, to 32, most of which present recent results in spectral analysis. We have also expanded the bibliography to include new topics, along with recent results on more established topics.

The text is organized as follows: Chapter 1 introduces the spectral analysis problem, motivates the definition of power spectral density functions, and reviews some important properties of autocorrelation sequences and spectral density functions. Chapters 2 and 5 consider nonparametric spectral estimation. Chapter 2 presents classical techniques, including the periodogram, the correlogram, and their modified versions to reduce variance. We include an analysis of bias and variance of these techniques and relate them to one another. Chapter 5 considers the more recent filter-bank version of nonparametric techniques, including both data-independent and data-dependent filter design techniques. Chapters 3 and 4 consider parametric techniques; Chapter 3

[1]MATLAB® is a registered trademark of The Mathworks, Inc.

focuses on continuous spectral models (Autoregressive Moving Average (ARMA) models and their AR and MA special cases), while Chapter 4 focuses on discrete spectral models (sinusoids in noise). We have placed the filter-bank methods in Chapter 5, after Chapters 3 and 4, mainly because the Capon estimator has interpretations both as an averaged AR spectral estimator and as a matched filter for line-spectral models, and we need the background of Chapters 3 and 4 to develop these interpretations. The data-independent filter-bank techniques in Sections 5.1–5.4 can equally well be covered directly following Chapter 2, if desired.

Chapter 6 considers the closely related problem of spatial spectral estimation in the context of array signal processing. Both nonparametric (beamforming) and parametric methods are considered, and both are tied into the temporal spectral estimation techniques considered in Chapters 2, 4, and 5.

The bibliography contains both modern and classical references (ordered both alphabetically and by subject). We include many historical references as well, for those interested in tracing the early developments of spectral analysis. However, spectral analysis is a topic with contributions from many diverse fields, including electrical and mechanical engineering, astronomy, biomedical spectroscopy, geophysics, mathematical statistics, and econometrics—to name a few. As such, any attempt to document the historical development of spectral analysis accurately is doomed to failure. The bibliography reflects our own perspectives, biases, and limitations; there is no doubt that the list is incomplete, but we hope that it gives the reader an appreciation of the breadth and diversity of the spectral analysis field.

The background needed for this text includes a basic knowledge of linear algebra, discrete-time linear systems, and introductory discrete-time stochastic processes (or time series). A basic understanding of estimation theory is helpful, though not required. Appendix A develops most of the needed background results on matrices and linear algebra, Appendix B gives a tutorial introduction to the Cramér–Rao bound, and Appendix C develops the theory of model order selection. We have included concise definitions and descriptions of the required concepts and results where needed. Thus, we have tried to make the text as self-contained as possible.

We are indebted to Jian Li and Lee Potter for adopting a former version of the text in their spectral estimation classes, for their valuable feedback, and for contributing to this book in several other ways. We would like to thank Torsten Söderström, for providing the initial stimulus for preparation of lecture notes that led to the book, and Hung-Chih Chiang, Peter Händel, Ari Kangas, Erlendur Karlsson, and Lee Swindlehurst, for careful proofreading and comments and for many ideas on and early drafts of the computer problems. We are grateful to Mats Bengtsson, Tryphon Georgiou, K.V.S. Hari, Andreas Jakobsson, Erchin Serpedin, and Andreas Spanias for comments and suggestions that helped us eliminate some inadvertencies and typographical errors from the previous edition of the book. We also wish to thank Wallace Anderson, Alfred Hero, Ralph Hippenstiel, Louis Scharf, and Douglas Williams, who reviewed a former version of the book and provided us with numerous useful comments and suggestions. It was a pleasure to work with the excellent staff at Prentice Hall, and we are particularly appreciative of Tom Robbins and Scott Disanno for their professional expertise.

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Notational Conventions

$\mathbf{R}$	the set of real numbers
$\mathbf{C}$	the set of complex numbers
$\mathcal{N}(A)$	the null space of the matrix A (p. 343)
$\mathcal{R}(A)$	the range space of the matrix A (p. 342)
Dn	the nth definition in Appendix A or B
Rn	the nth result in Appendix A
$\\|x\\|$	the Euclidean norm of a vector x
$*$	convolution operator
$(\cdot)^T$	transpose of a vector or matrix
$(\cdot)^c$	conjugate of a vector or matrix
$(\cdot)^*$	conjugate transpose of a vector or matrix; also used for scalars in lieu of $(\cdot)^c$
A_{ij}	the (i, j)th element of the matrix A
a_i	the ith element of the vector a
$\hat{x}$	an estimate of the quantity x
$A > 0 \ (\geq 0)$	A is positive definite (positive semidefinite) (p. 357)
$\arg\max\limits_x f(x)$	the value of x that maximizes $f(x)$
$\arg\min\limits_x f(x)$	the value of x that minimizes $f(x)$

$\text{cov}\{x, y\}$	the covariance between x and y
$\lvert x \rvert$	the modulus of the (possibly complex) scalar x
$\lvert A \rvert$	the determinant of the square matrix A
$\text{diag}(a)$	the square diagonal matrix whose diagonal elements are the elements of the vector a
$\delta_{k,l}$	Kronecker delta: $\delta_{k,l} = 1$ if $k = l$; $\delta_{k,l} = 0$ otherwise
$\delta(t - t_0)$	Dirac delta: $\delta(t - t_0) = 0$ for $t \neq t_0$; $\int_{-\infty}^{\infty} \delta(t - t_0) dt = 1$
$E\{x\}$	the expected value of x (p. 5)
f	(discrete-time) frequency: $f = \omega/2\pi$, in cycles per sampling interval (p. 8)
$\phi(\omega)$	a power spectral density function (p. 6)
$\text{Im}\{x\}$	the imaginary part of x
$\mathcal{O}(x)$	on the order of x (p. 35)
$p(x)$	probability density function
$\Pr\{A\}$	the probability of event A
$r(k)$	an autocovariance sequence (p. 5)
$\text{Re}\{x\}$	the real part of x
t	discrete-time index
$\text{tr}(A)$	the trace of the matrix A (p. 346)
$\text{var}\{x\}$	the variance of x
$w(k),\ W(\omega)$	a window sequence and its Fourier transform
$w_B(k),\ W_B(\omega)$	the Bartlett (or triangular) window sequence and its Fourier transform (p. 31)
$w_R(k),\ W_R(\omega)$	the rectangular (or Dirichlet) window sequence and its Fourier transform (p. 32)
ω	radian (angular) frequency, in radians per sampling interval (p. 3)
z^{-1}	unit delay operator: $z^{-1}x(t) = x(t-1)$ (p. 11)

Abbreviations

ACS	autocovariance sequence (p. 5)
APES	amplitude and phase estimation (p. 256)
AR	autoregressive (p. 92)
ARMA	autoregressive moving-average (p. 92)
BSP	beamspace processing (p. 337)
BT	Blackman–Tukey (p. 39)
CM	Capon method (p. 232)
CCM	constrained Capon method (p. 313)
CRB	Cramér–Rao bound (p. 373)
DFT	discrete Fourier transform (p. 27)
DGA	Delsarte–Genin algorithm (p. 99)
DOA	direction of arrival (p. 276)
DTFT	discrete-time Fourier transform (p. 3)
ESP	elementspace processing (p. 337)
ESPRIT	estimation of signal parameters by rotational invariance techniques (p. 174)
EVD	eigenvalue decomposition (p. 345)
FB	forward–backward (p. 176)
FBA	filter-bank approach (p. 219)

FFT	fast Fourier transform (p. 27)
FIR	finite impulse response (p. 18)
flop	floating-point operation (p. 27)
GAPES	gapped amplitude and phase estimation (p. 259)
GS	Gohberg–Semencul (formula) (p. 128)
HOYW	high-order Yule–Walker (p. 162)
i.i.d.	independent, identically distributed (p. 331)
LDA	Levinson–Durbin algorithm (p. 99)
LS	least squares (p. 367)
MA	moving average (p. 92)
MFD	matrix fraction description (p. 143)
ML	maximum likelihood (p. 375)
MLE	maximum likelihood estimate (p. 375)
MSE	mean squared error (p. 30)
MUSIC	multiple signal classification (or characterization) (p. 166)
MYW	modified Yule–Walker (p. 101)
NLS	nonlinear least squares (p. 151)
PARCOR	partial correlation (p. 101)
PSD	power spectral density (p. 5)
RFB	refined filter bank (p. 222)
QRD	Q–R decomposition (p. 368)
RCM	robust Capon method (p. 312)
SNR	signal-to-noise ratio (p. 85)
SVD	singular value decomposition (p. 351)
TLS	total least squares (p. 369)
ULA	uniform linear array (p. 283)
YW	Yule–Walker (p. 94)