Rethinking Tractability for Schedulability Analysis

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RTSS 2023

Moving the goalposts for fun and profit

’70s and early ’80s

Polynomial-time schedulability tests (Liu and Layland’s utilization bounds for implicit-deadline EDF end FP, etc.)

Complexity of uniprocessor sporadic schedulability

		Implicit deadlines ($d = p$)	Constrained deadlines ($d \leqslant p$)	Arbitrary deadlines (d, p unrelated)
FP	Arbitrary utilization			
	Utilization bounded by a constant c	Polynomial time for $c \leqslant \ln 2$ and RM priorities		
EDF	Arbitrary utilization	Polynomial time		
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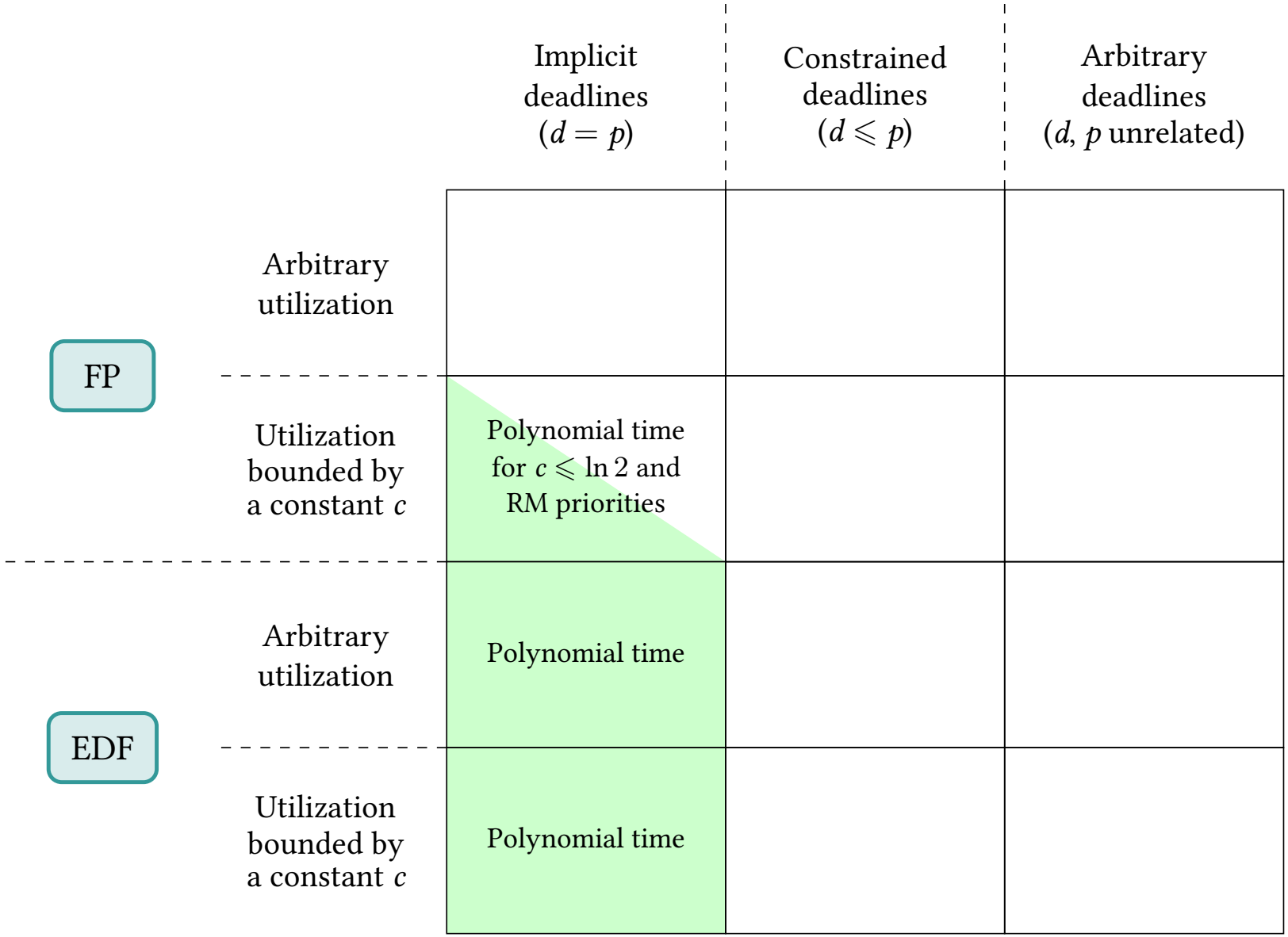

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Late ’80s and ’90s

Pseudo-polynomial time tests (Response-time analysis for FP, processor-demand analysis for EDF, etc.)

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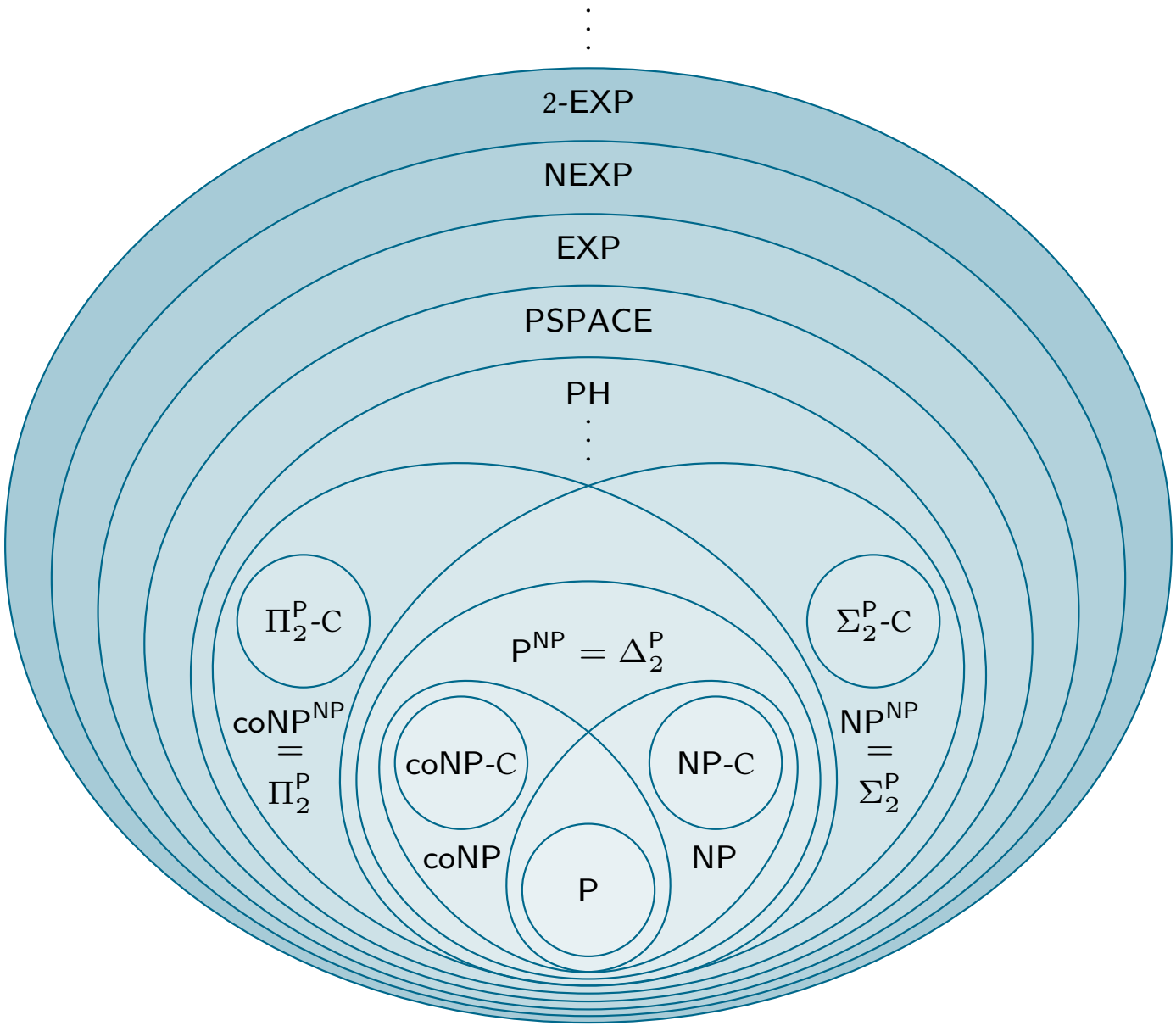
More recently

Integer Linear Programming (ILP) and similar optimized tools to implement non-pseudo-polynomial time tests.

What can be solved with ILPs?



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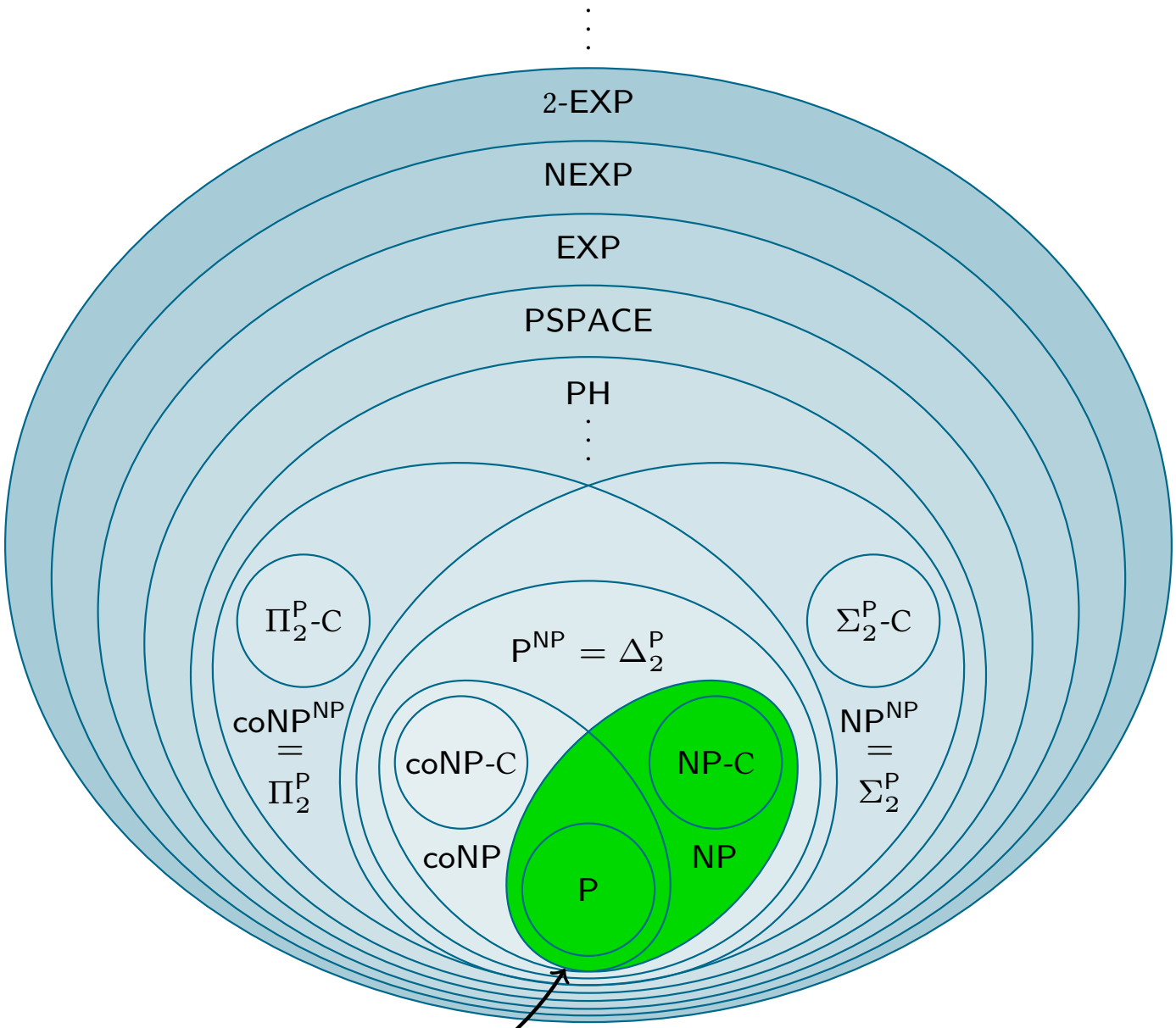

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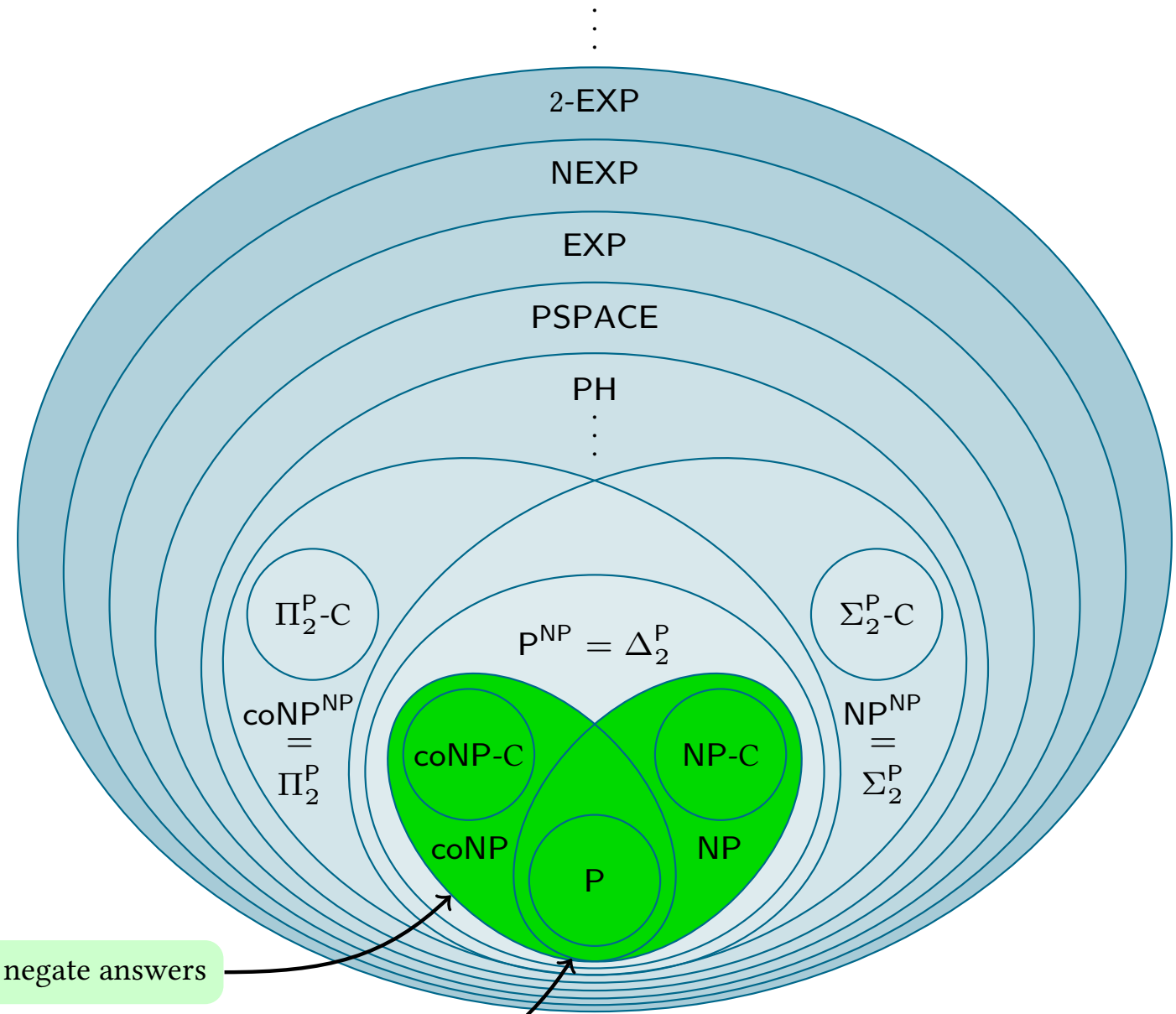

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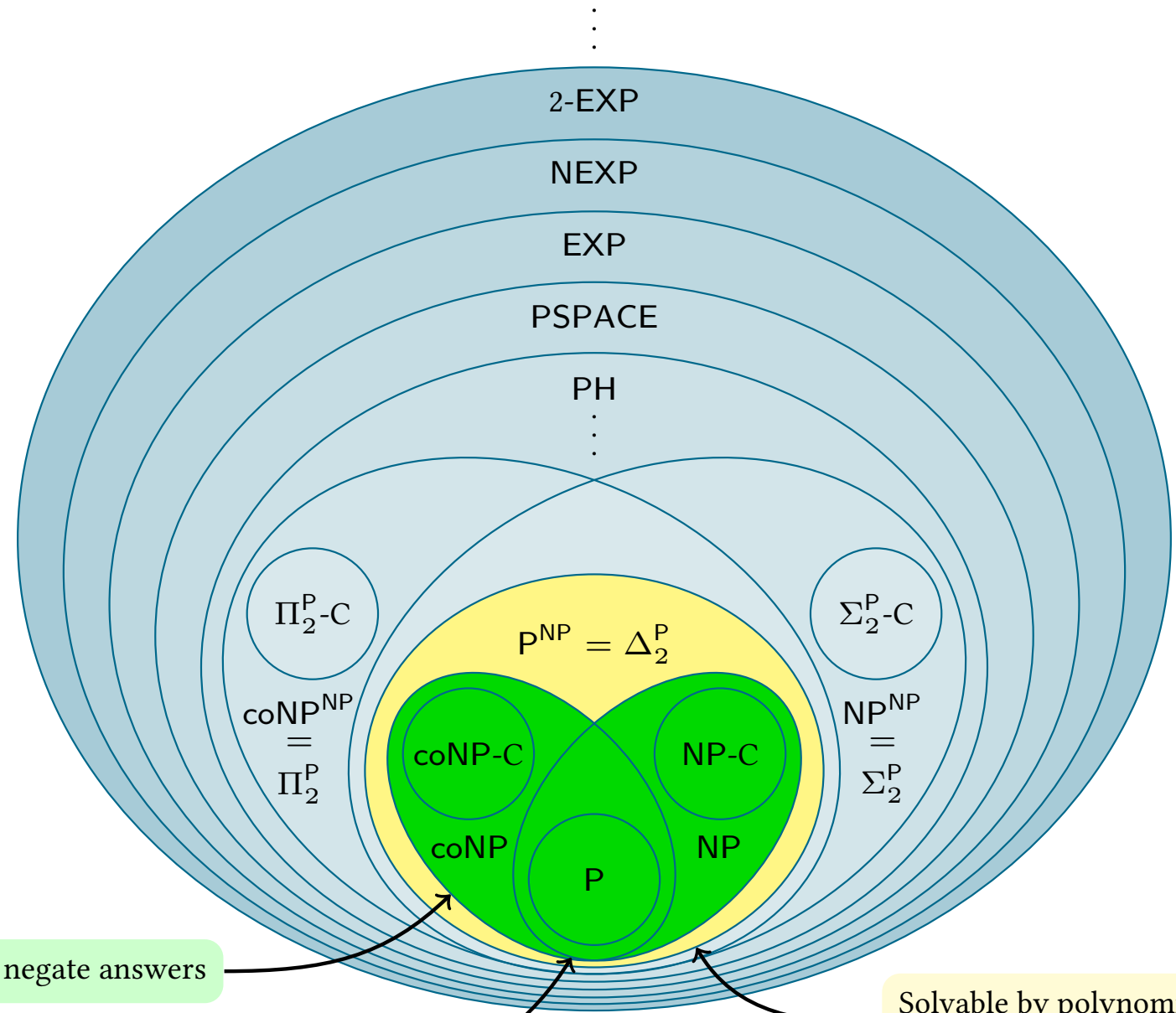

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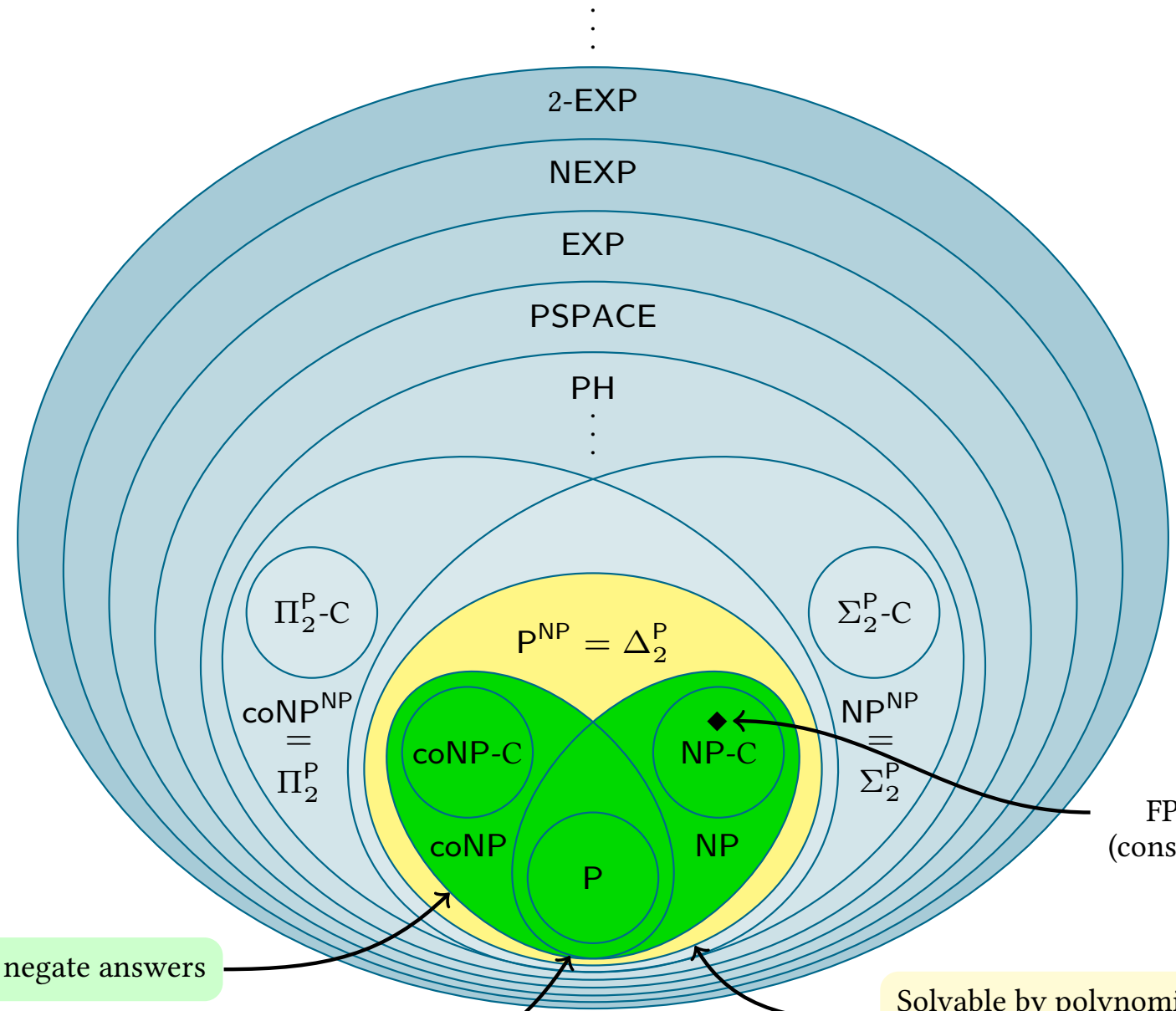

Partitioned FP-schedulability (constrained deadlines)

Ditto if we negate answers

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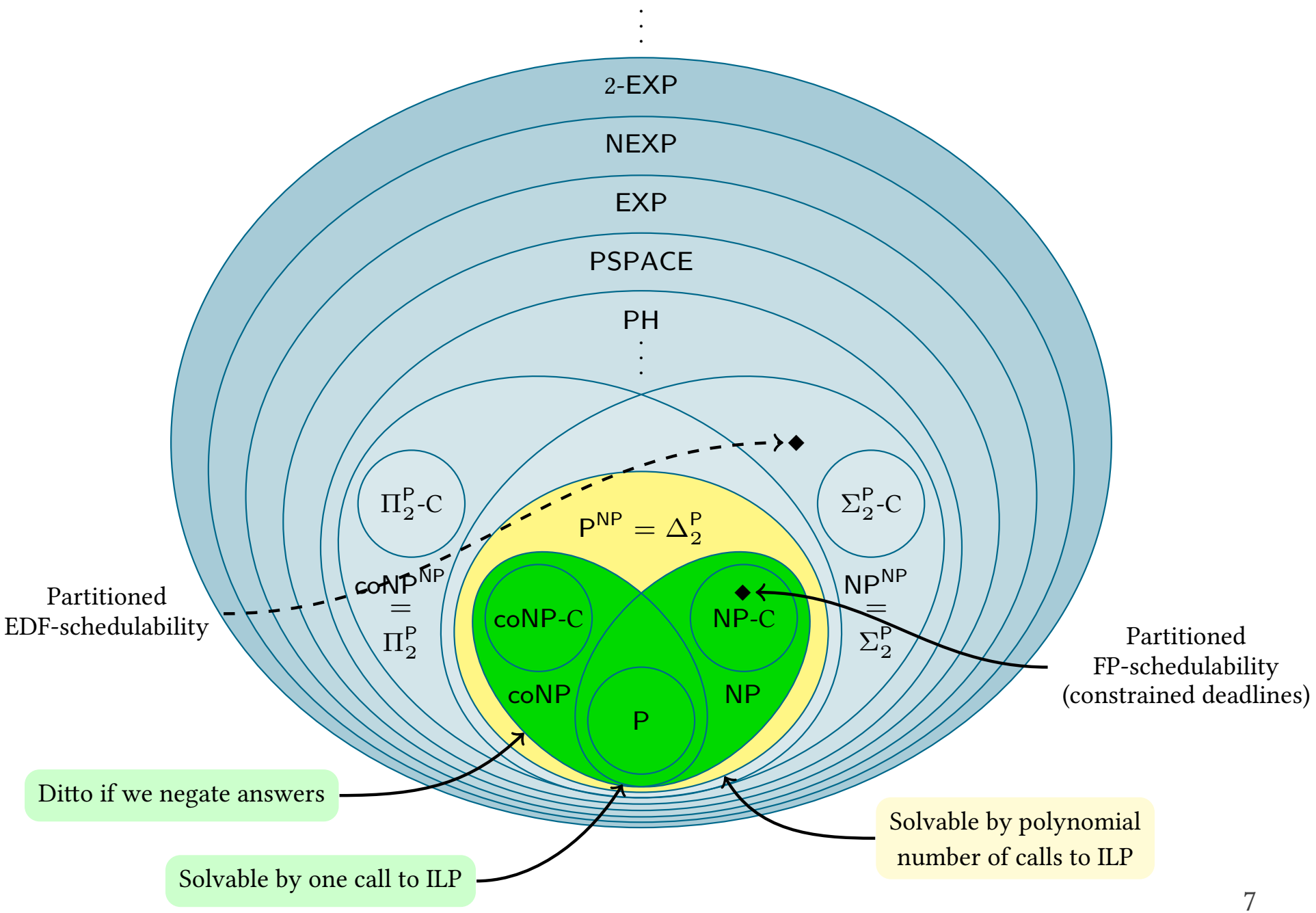

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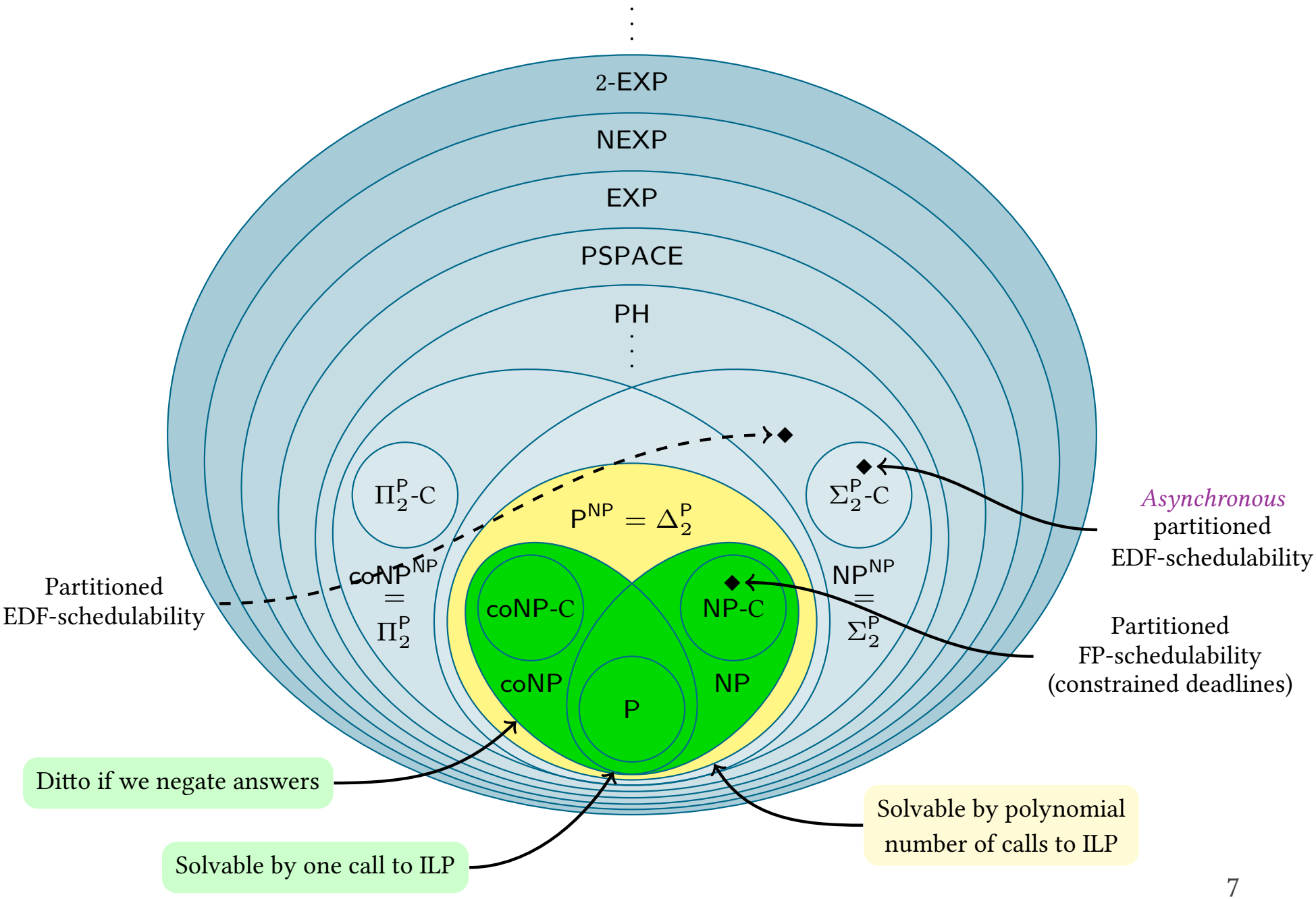

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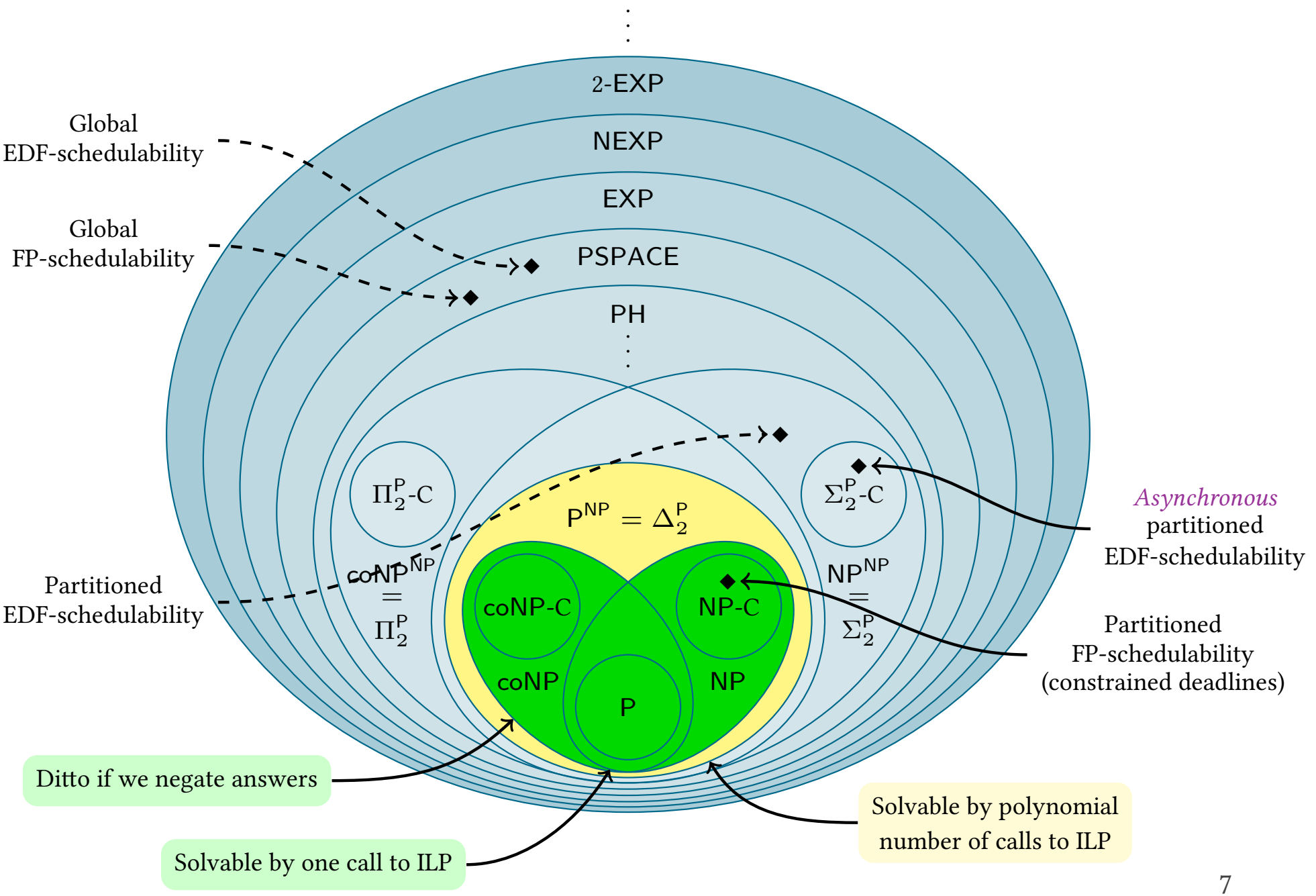

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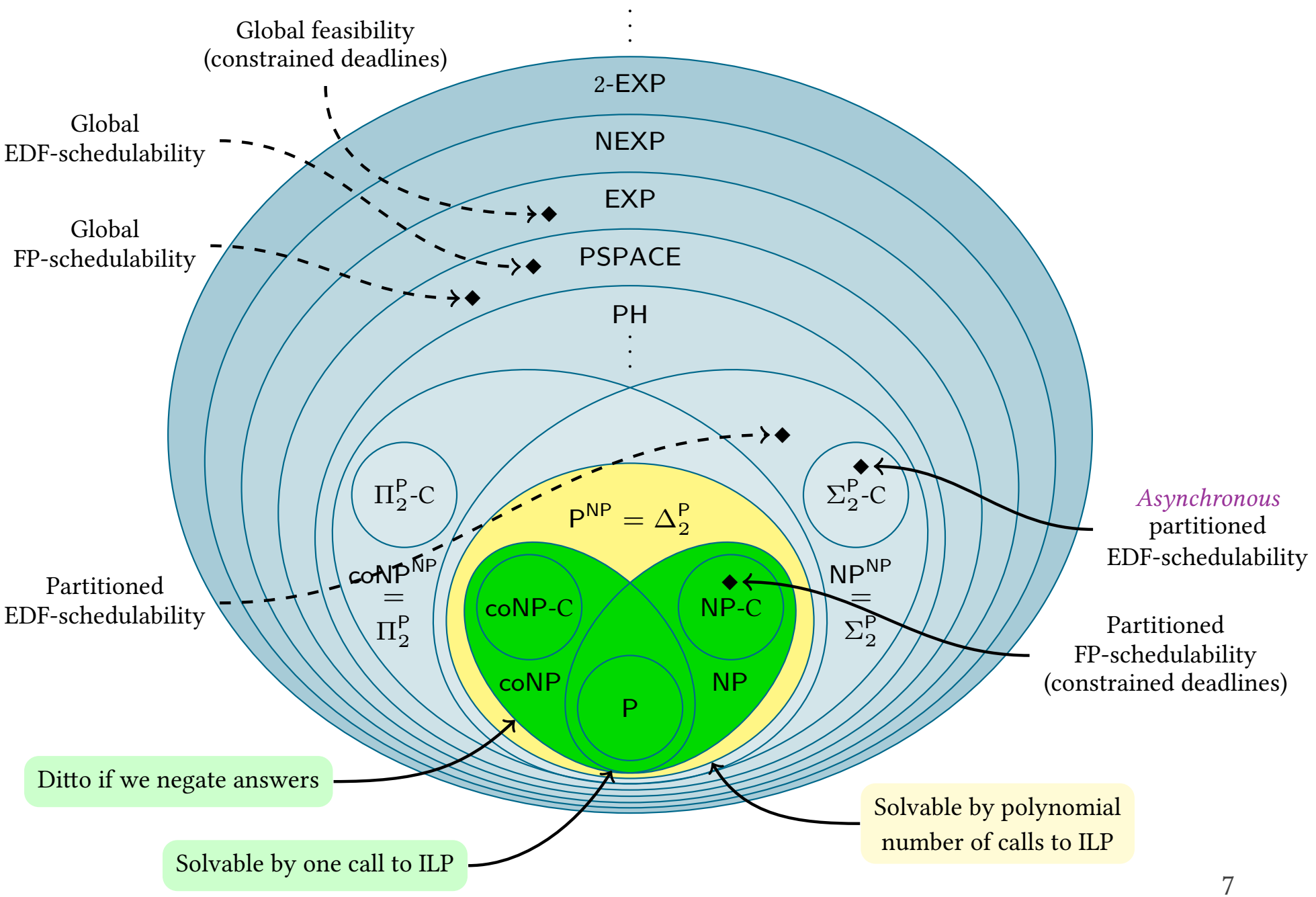

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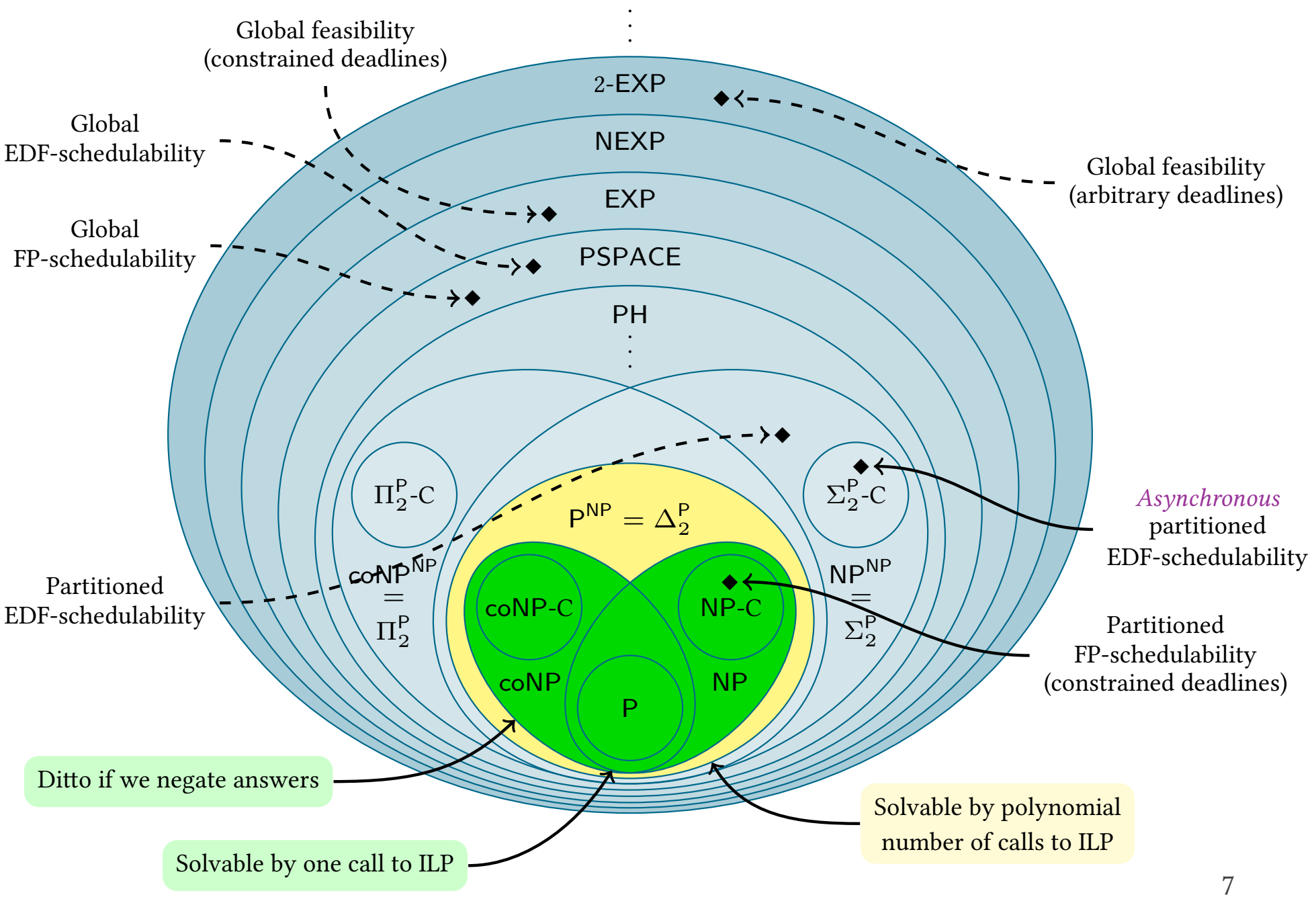

How hard is Σ_2^{P}-hard?

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The trouble with the second quantifier

Gerhard J. Woeginger[1]

Received: 6 February 2021 / Revised: 3 April 2021 / Accepted: 16 April 2021 / Published online: 26 April 2021

Abstract
We survey optimization problems that allow natural simple formulations with one existential and one universal quantifier. We summarize the theoretical background from computational complexity theory, and we present a multitude of illustrating examples. We discuss the connections to robust optimization and to bilevel optimization, and we explain the reasons why the operational research community should be interested in the theoretical aspects of this area.

Keywords Combinatorial optimization · Complexity theory · Polynomial hierarchy · Bilevel optimization

1 Introduction

The United Nations Security Council consists of 15 members: there are five permanent members (China, France, Russia, the United Kingdom, and the USA) and there are ten non-permanent members (which respectively serve for two-year terms). In order to pass a decision (i) at least nine of the fifteen members must agree, and furthermore

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Gerhard J. Woeginger[1]

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So…

Polynomial time

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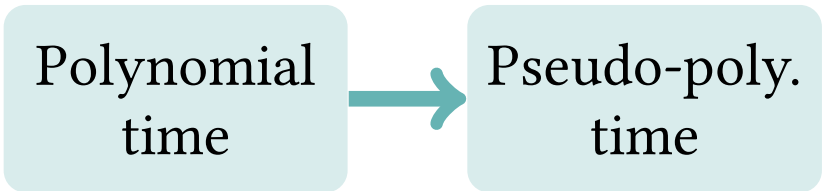

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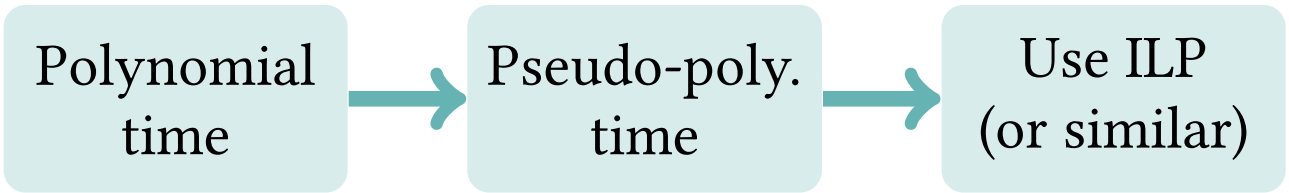

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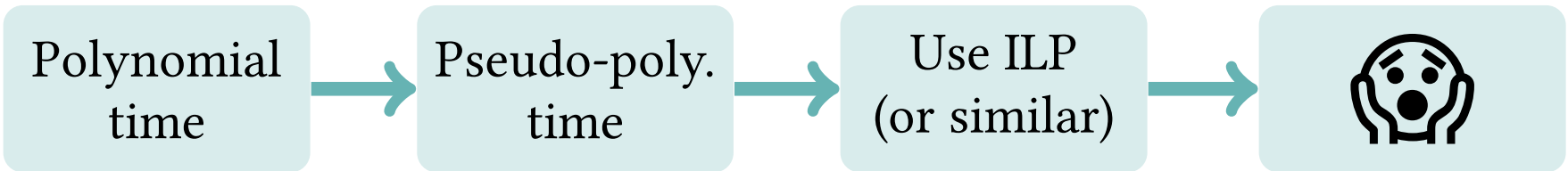

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Polynomial time → Pseudo-poly. time → Use ILP (or similar) → 

Weakly (co)NP-hard

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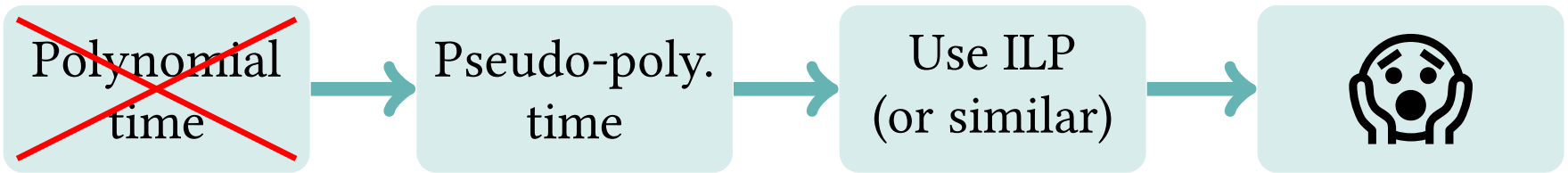
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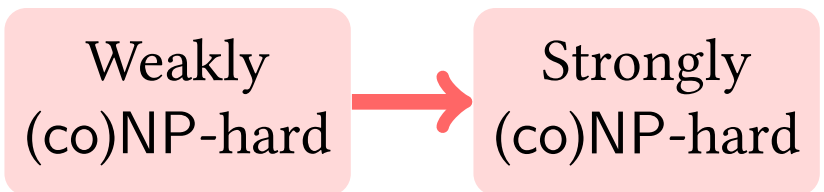
Weakly (co)NP-hard
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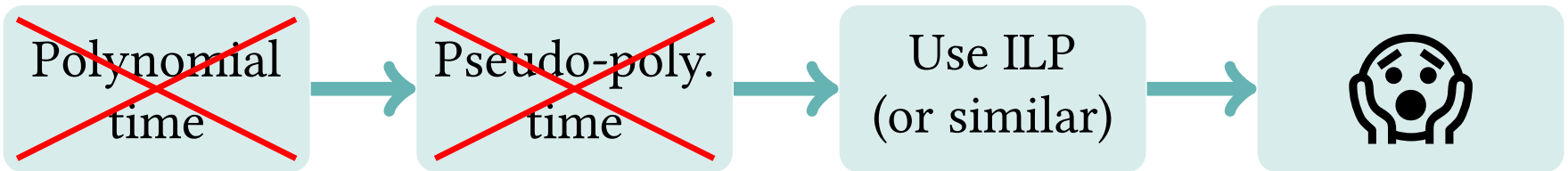
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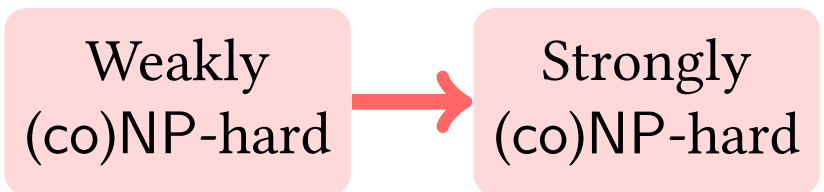
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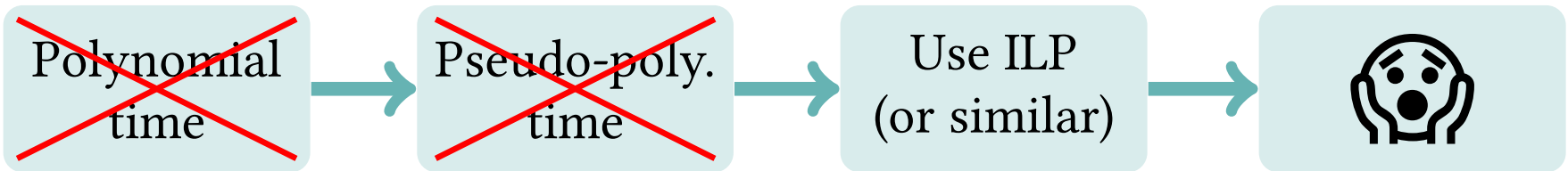
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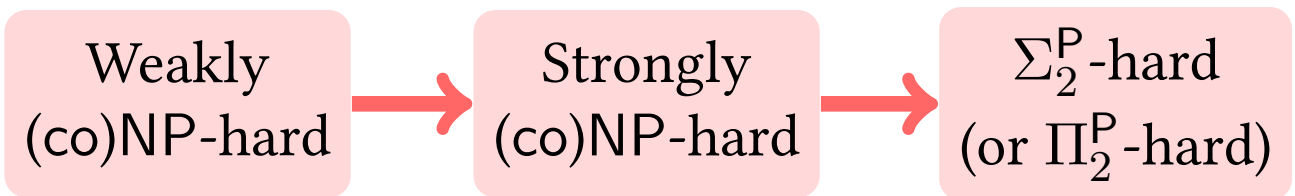
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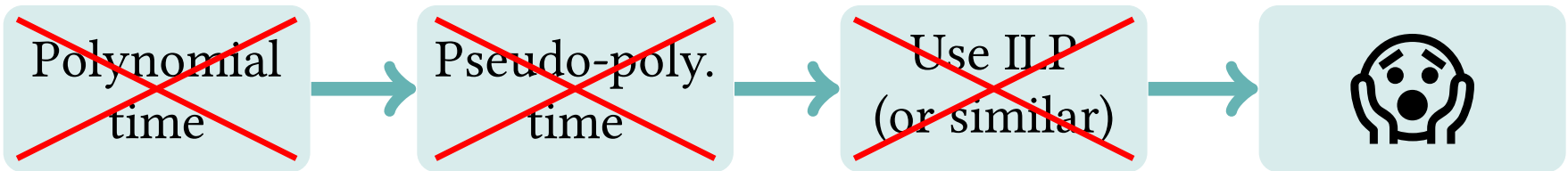
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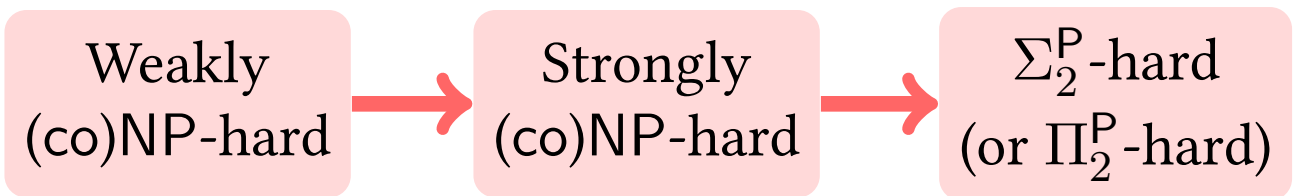
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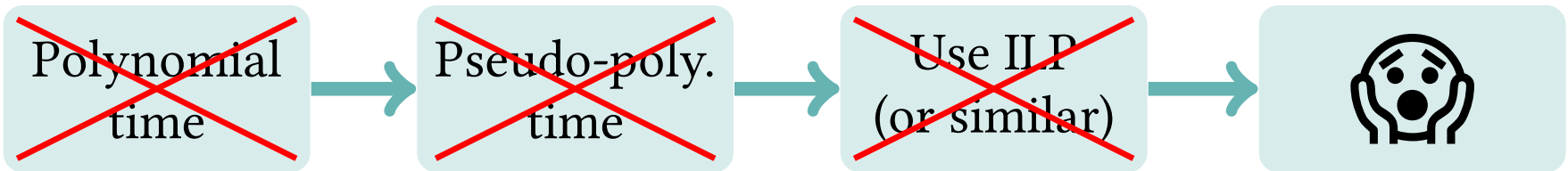
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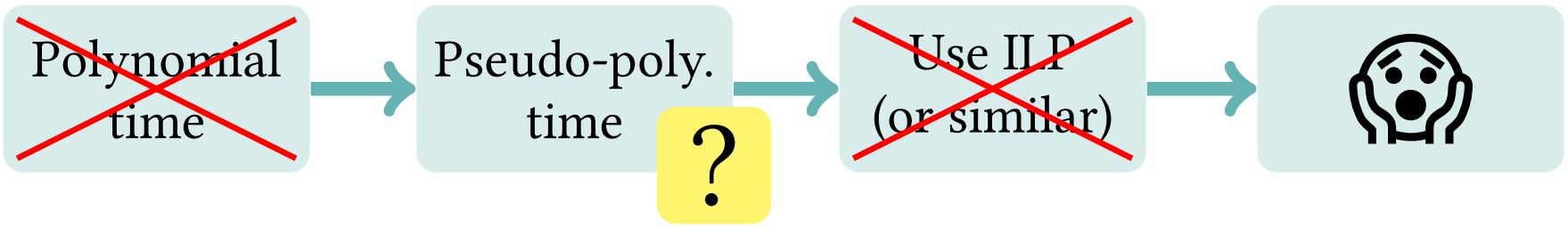
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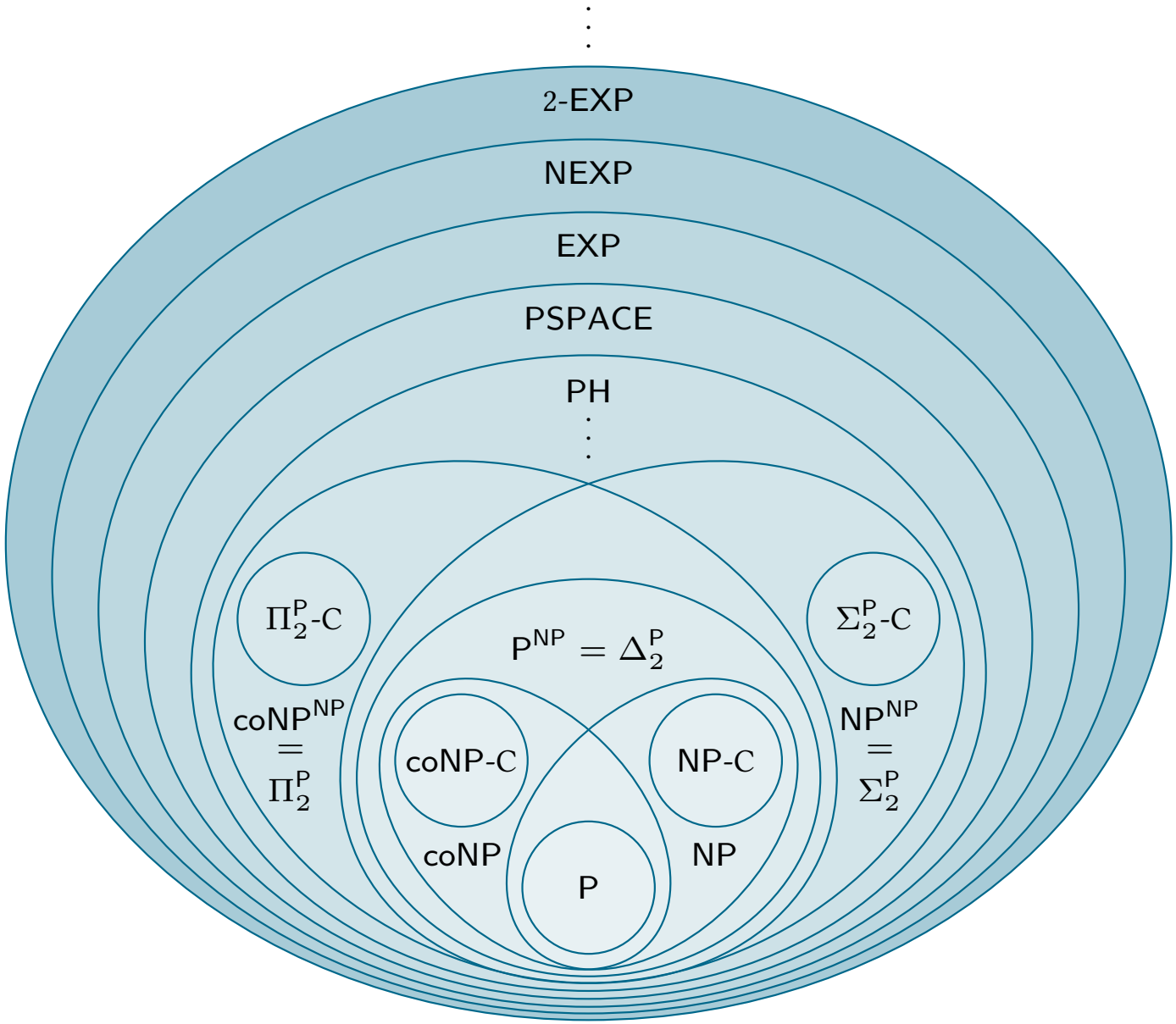

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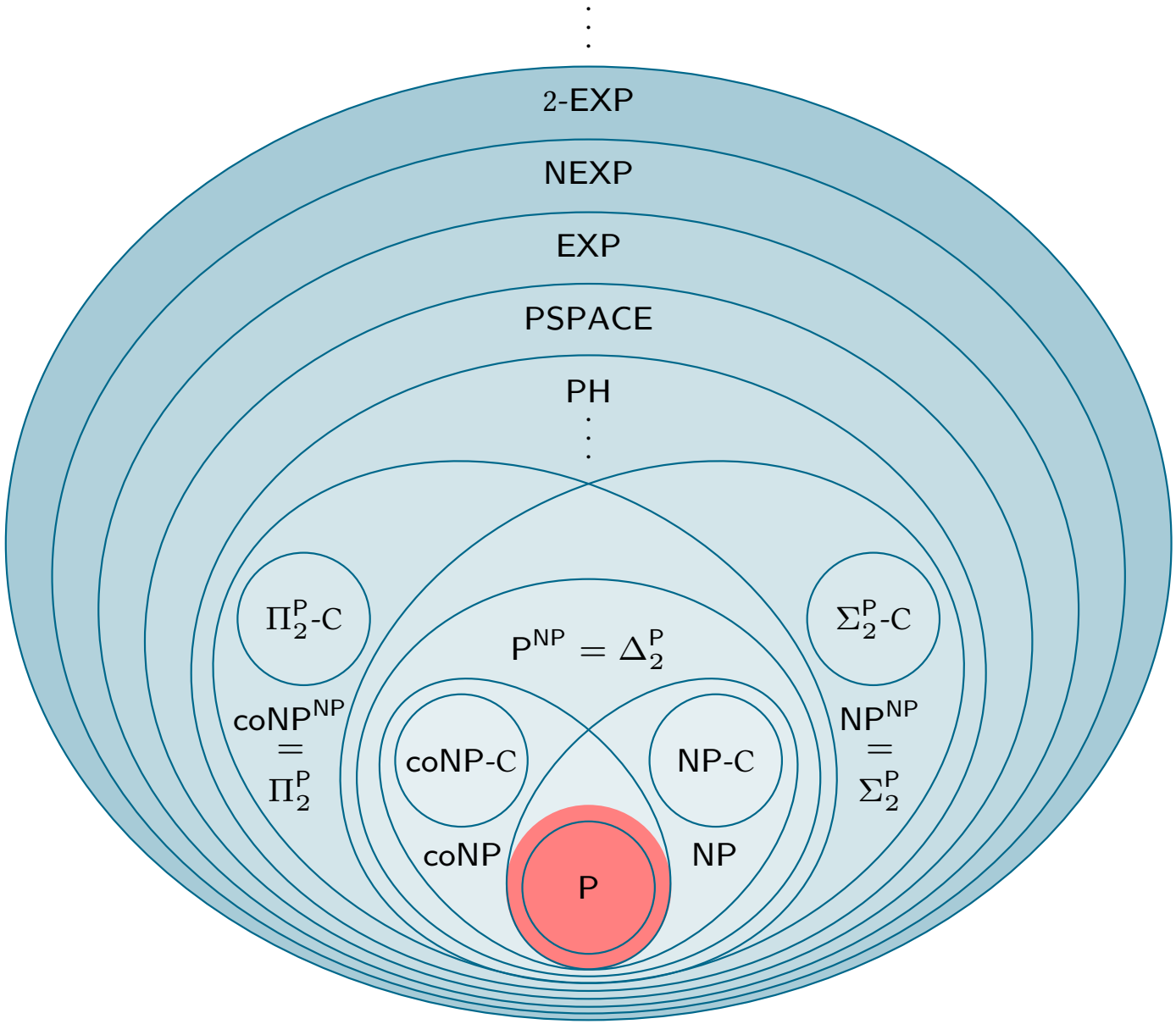

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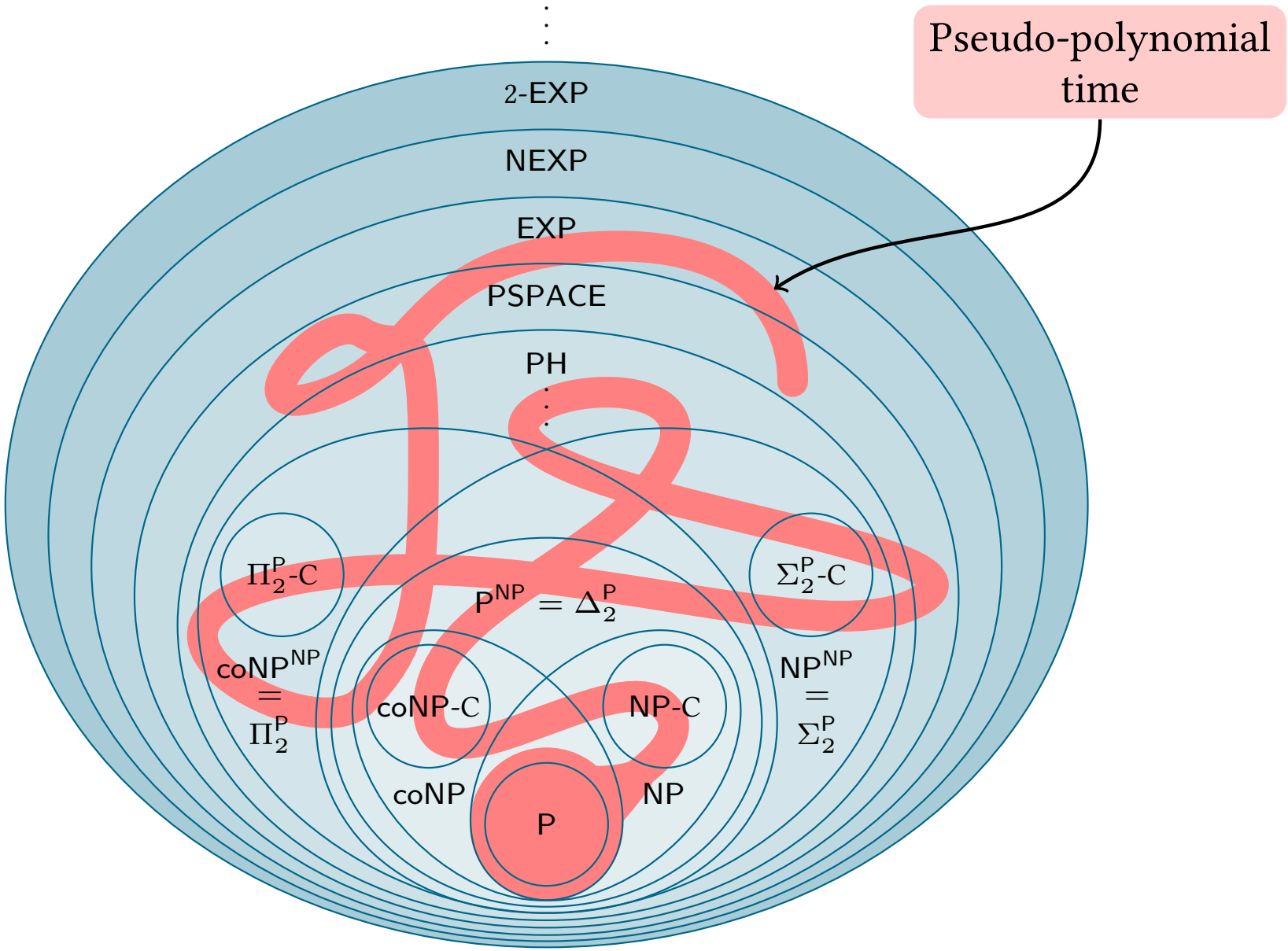

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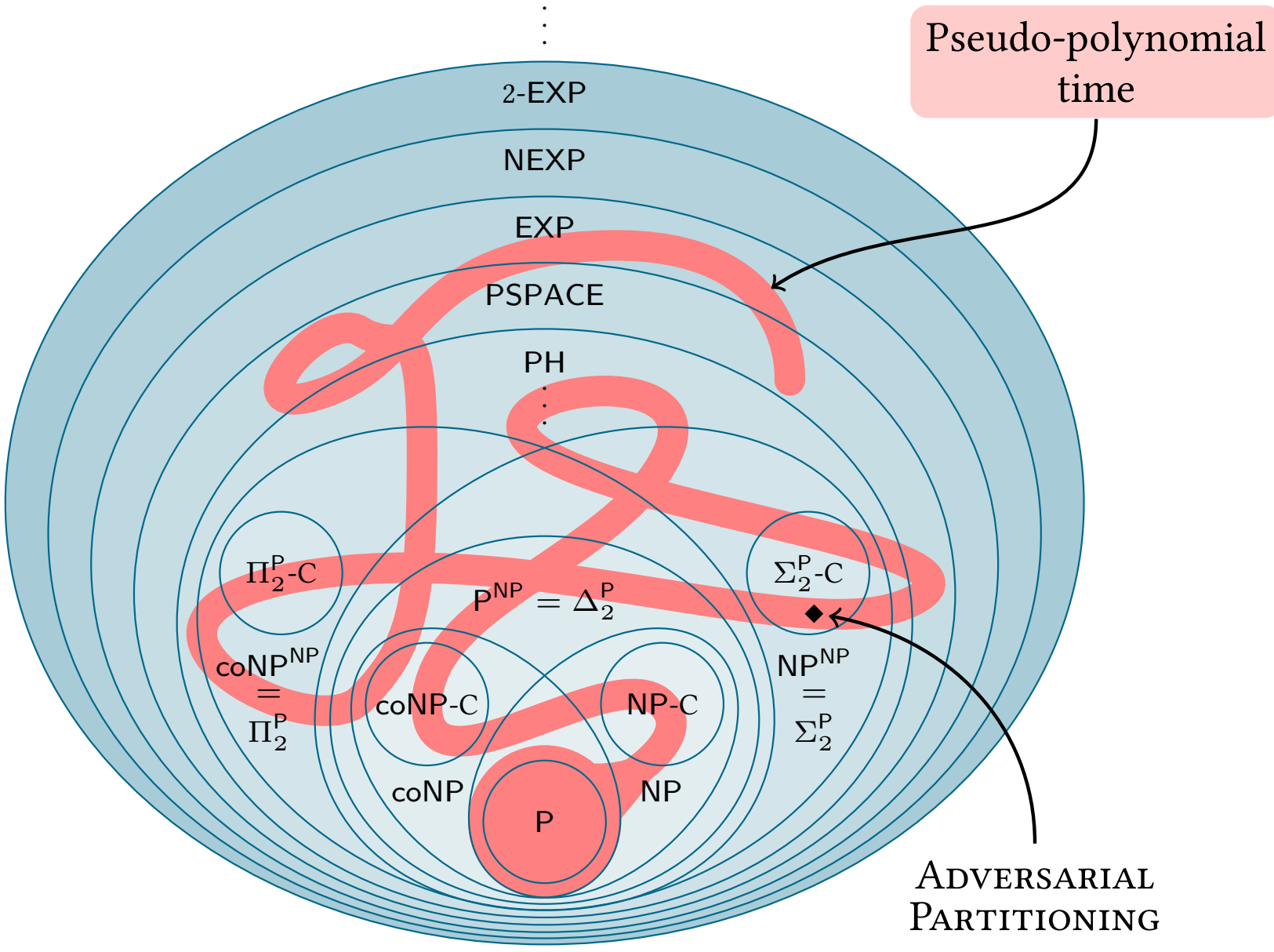

Adversarial Partitioning

Definition

Instance: Sets A and $B \subset \mathbb{N}$, and two bins of capacity S.

Question: Can the items in A be partitioned upon the bins such that the items in B cannot be partitioned upon the remaining capacities?

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Berit Johannes, PhD thesis, 2011

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Adversarial Partitioning is Σ_2^{P}-complete.

"Adversarial" problems in general are relevant for security.
Many are Σ_2^{P}-complete.

Adversarial Partitioning

Pseudo-polynomial time algorithm

1. Generate the possible sizes of a partitioning of A (using dynamic programming).
2. Ditto for B.
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Runtime is $\mathcal{O}((|A| + |B| + \log S)S)$

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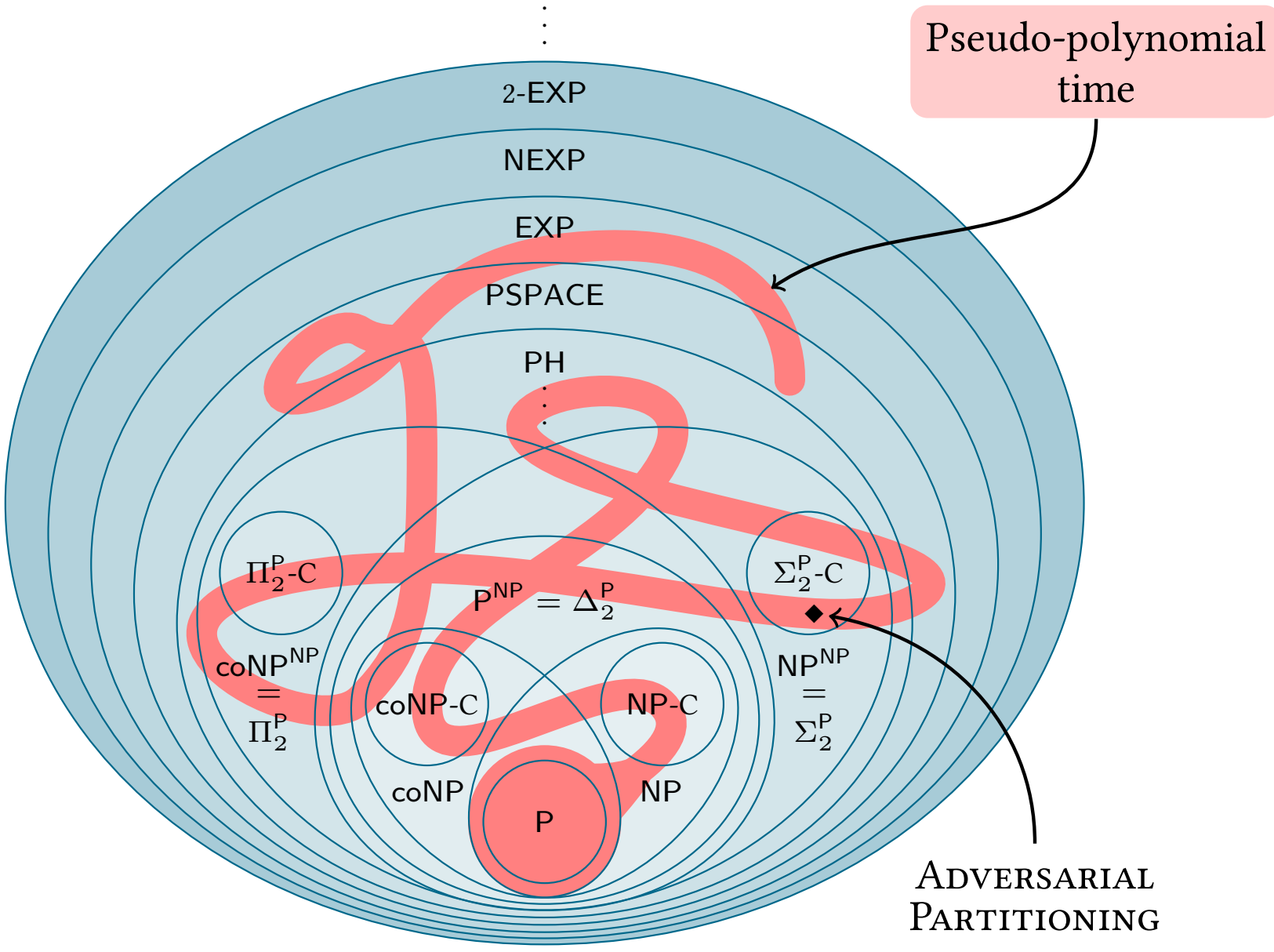

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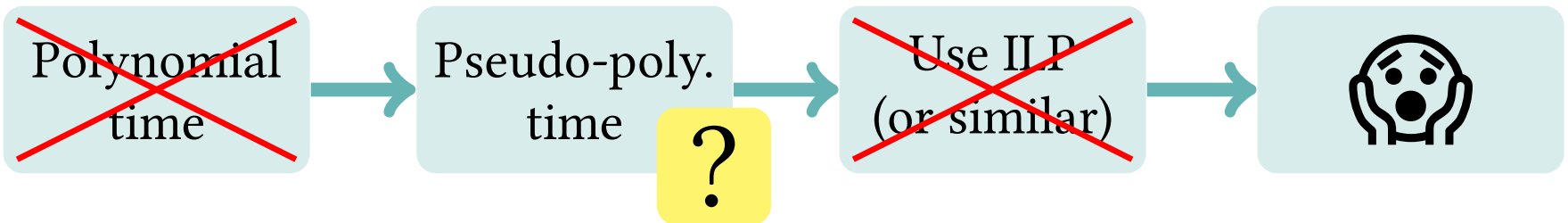
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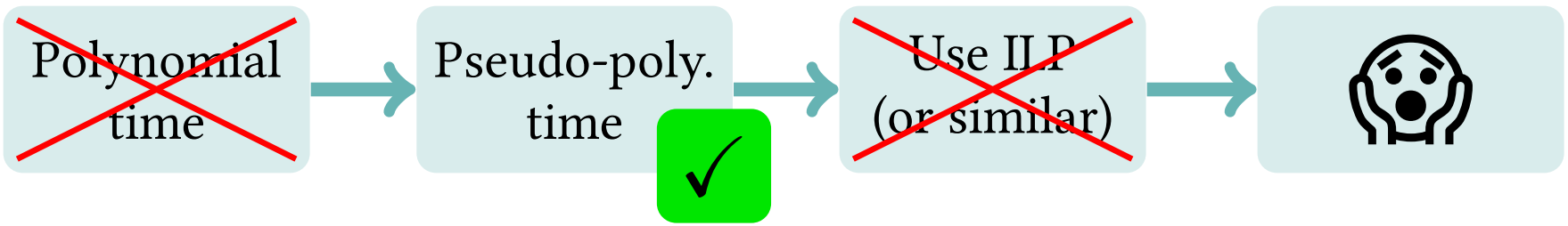

(Adversarial Partitioning)

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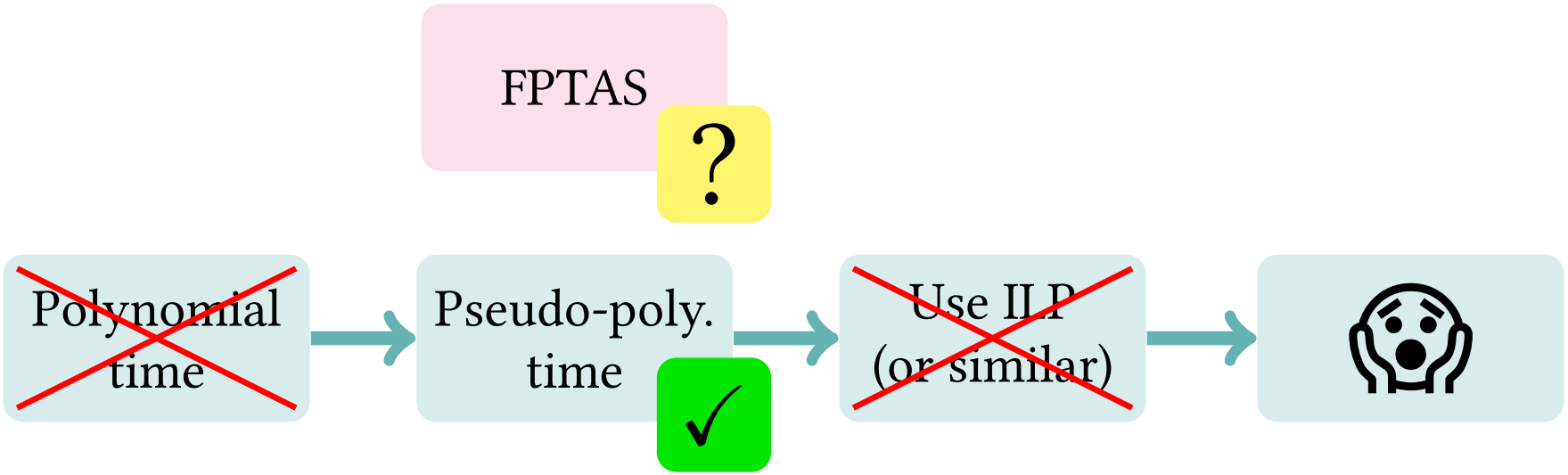

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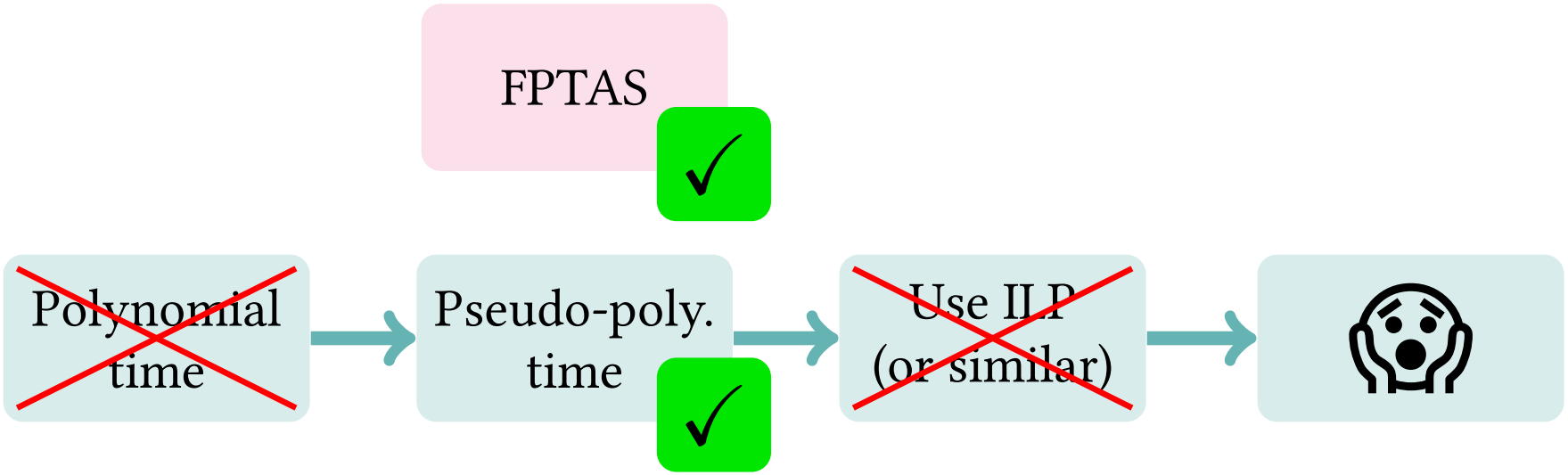

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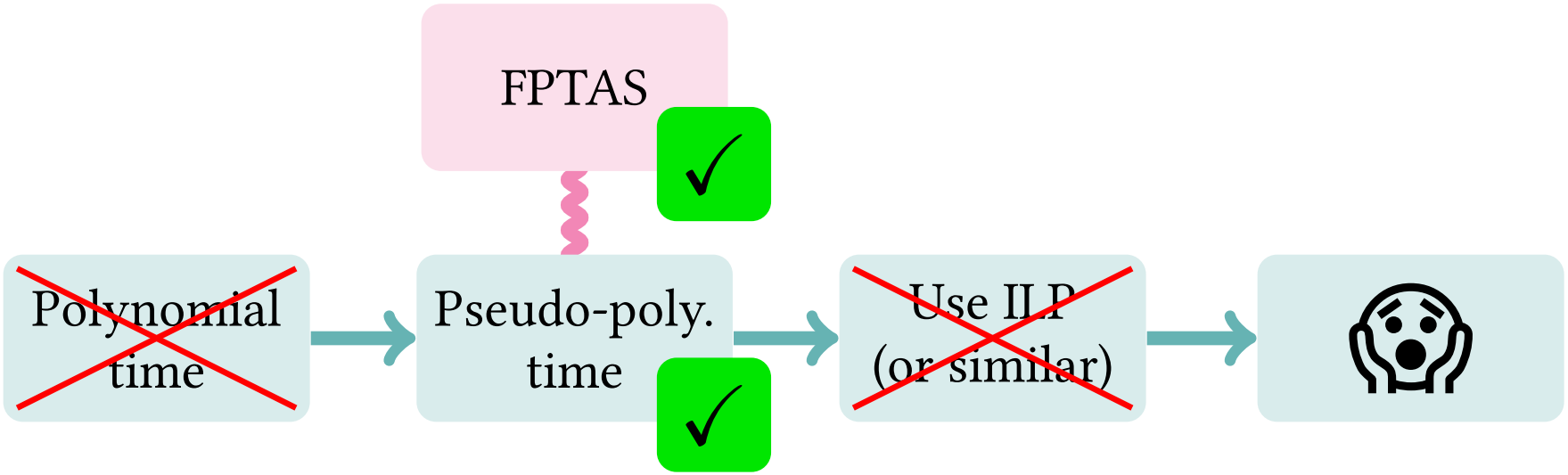

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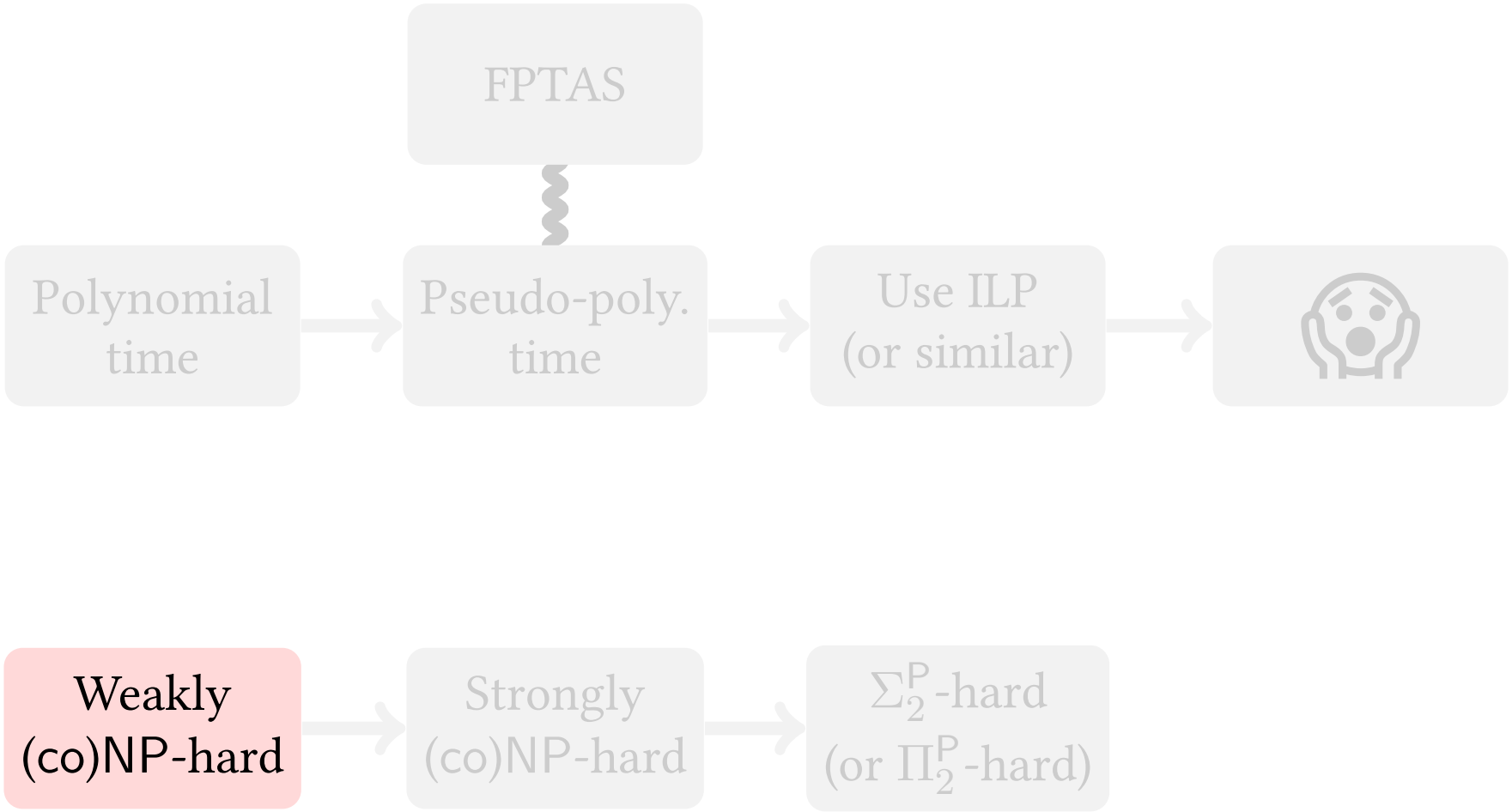
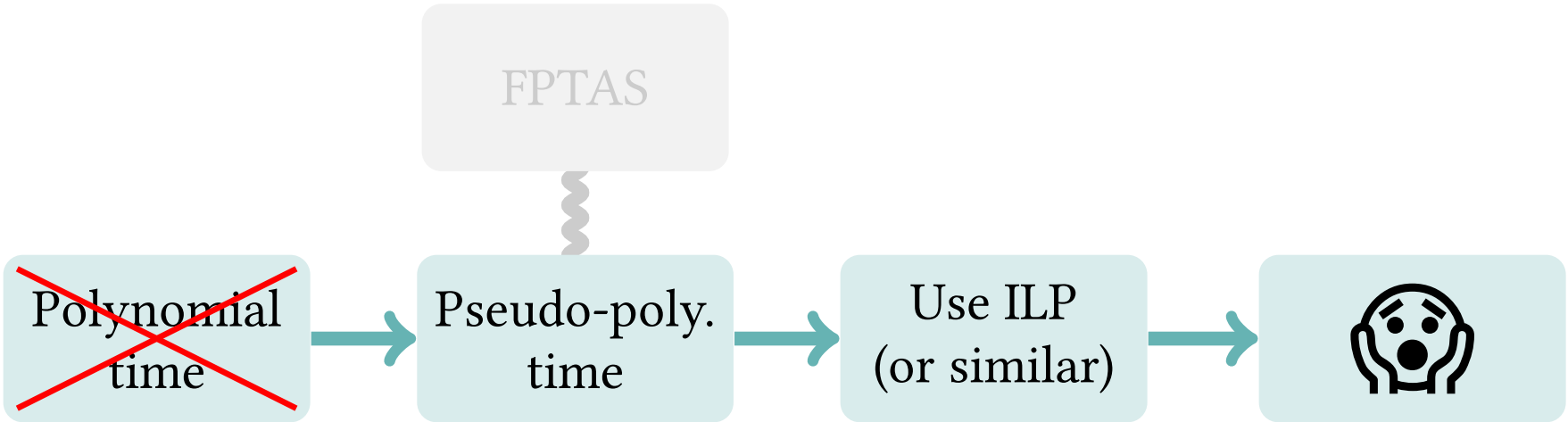


(Memory-Constrained Task Selection)

So...



(Memory-Constrained Task Selection)

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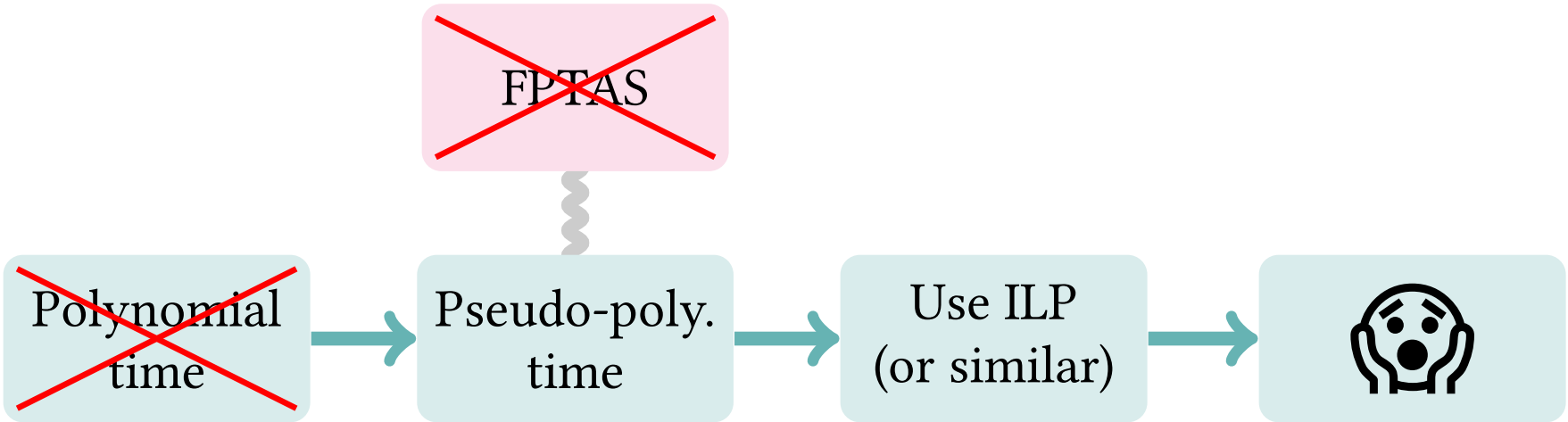

(Memory-Constrained Task Selection)

A more fine-grained take on pseudo-polynomial time

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$\forall$Thank you!

$\exists$Questions?