# PARTITIONED SCHEDULING OF RECURRENT REAL-TIME TASKS

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## WHAT?

What is the complexity of partitioned schedulability?



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What is the complexity of partitioned schedulability?

- 1 Exact schedulability tests!
- 2 Schedulers:
  - FP
  - EDF / feasibility
- 3 Tasks:
  - Synchronous periodic / sporadic
  - Asynchronous periodic
- 4 Processors:
  - Identical
  - Unrelated



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#### NP-hard is just a lower bound on complexity.

What is the exact complexity?

















































# Simultaneous Congruences

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Example: 
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SCP is NP-complete (Leung and Whitehead, 1982)

#### PARTITIONED SCP:

Example:  $A = \{(2,4), (4,6), (3,8), (0,3), \ldots\}, m, k$ 

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 $\Sigma_2^{\mathsf{P}}$ -complete, Rutenburg, 1986











 $\begin{array}{c} \begin{array}{c} \text{GENERALIZED} \\ \text{GRAPH COLORING} \end{array} \longrightarrow \text{PARTITIONED SCP} \longrightarrow \begin{array}{c} \text{Some partitioned} \\ \text{schedulability} \\ \text{problems} \end{array} \end{array}$ 













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- Some are essentially the same as the uniprocessor case!
- Some can not be formulated as ILP in polynomial time.
- No problem is higher up than Σ<sup>P</sup><sub>3</sub>.

# ∀Thank you!↓∃Questions?