Rate-Monotonic Schedulability of Implicit-Deadline Tasks is NP-hard Beyond Liu and Layland's Bound

Pontus Ekberg

Uppsala University

RTSS 2020

Background

	Implicit deadlines ($d = p$)	Constrained deadlines ($d \leqslant p$)	Arbitrary deadlines (d, p unrelated)
Arbitrary utilization	Weakly NP-complete Pseudo-polynomial time algorithm exists	Weakly NP-complete Pseudo-polynomial time algorithm exists	Weakly NP-hard Exponential time algorithm exists
Utilization bounded by a constant $c < 1$	Polynomial time with RM priorities and $c \leqslant \ln(2)$	Weakly NP-complete for $c > 0$ Pseudo-polynomial time algorithm exists	Weakly NP-hard for $c > 0$ Pseudo-polynomial time algorithm exists

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What if $c > \ln(2)$ or if the priorities are non-RM?

Complexity of bounded FP-schedulability

Conjecture by Rothvoß, 2009

RM-schedulability of task sets with implicit deadlines and utilization bounded by constant c is in P for all $c < 1$.

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Approach

Show that for any $c > \ln(2)$ there *exists* a reduction from some NP-complete problem.

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An RM-schedulability class

- Utilization bounded by *some* constant $c > \ln(2)$
- Implicit deadlines

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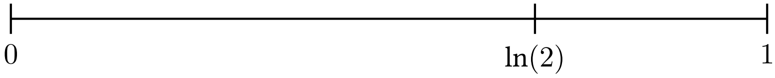
The existence of a reduction for a concrete c



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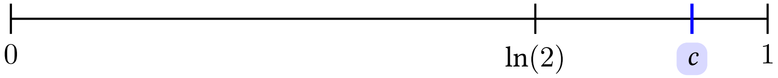

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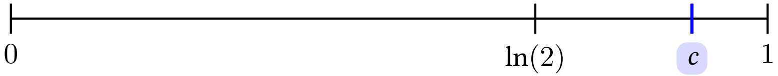

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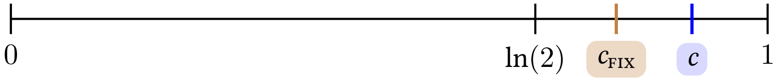

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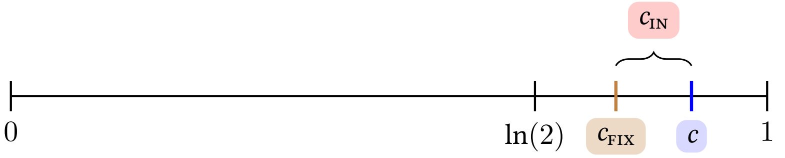

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Corollary from Liu & Layland, 1973

For every $c > \ln(2)$, there exists an implicit-deadline task set with utilization $< c$ that *fully utilizes*✽ the processor.

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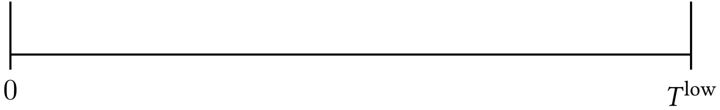

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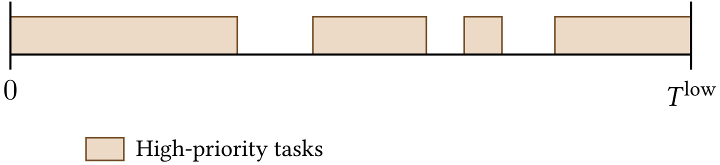

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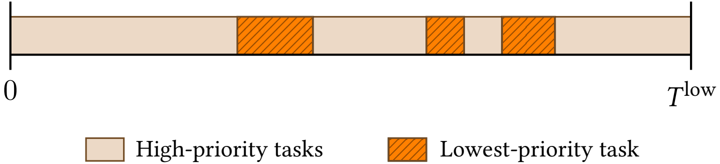

The existence of a reduction for a concrete c

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$\rightsquigarrow$

RM-schedulability

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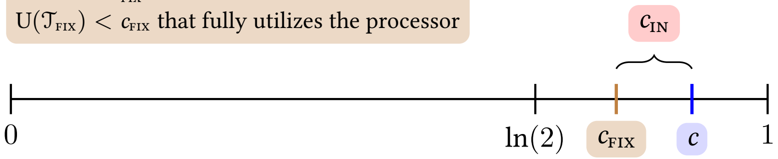
THE EXISTENCE OF A REDUCTION FOR A CONCRETE c

Let $\mathcal{T}_{\text{FIX}}$ be a *constant* task set with $U(\mathcal{T}_{\text{FIX}}) < c_{\text{FIX}}$ that fully utilizes the processor



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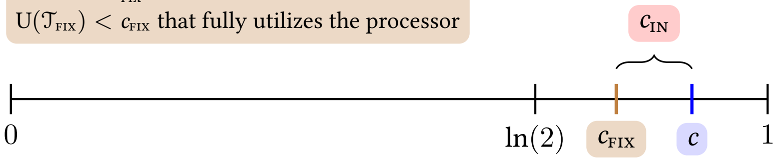

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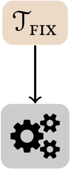

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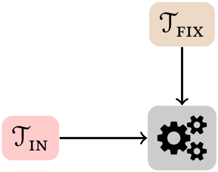

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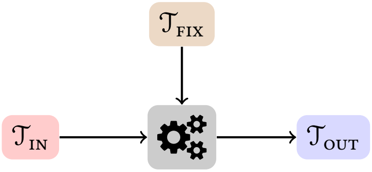

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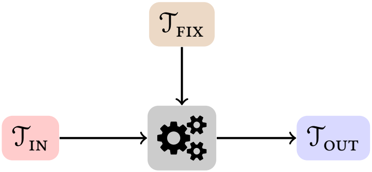

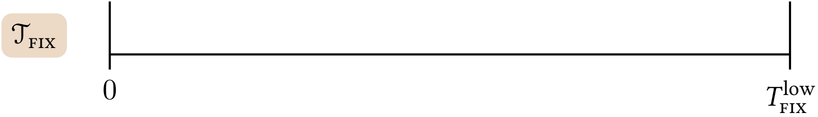

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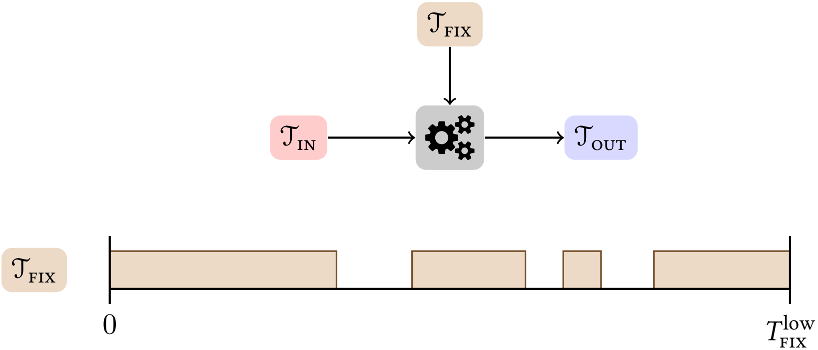

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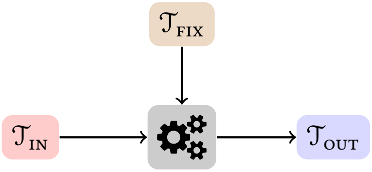

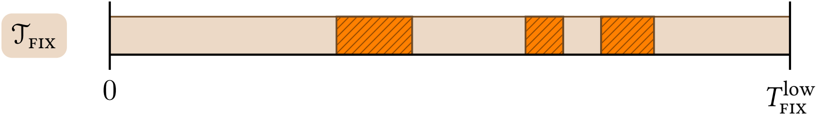

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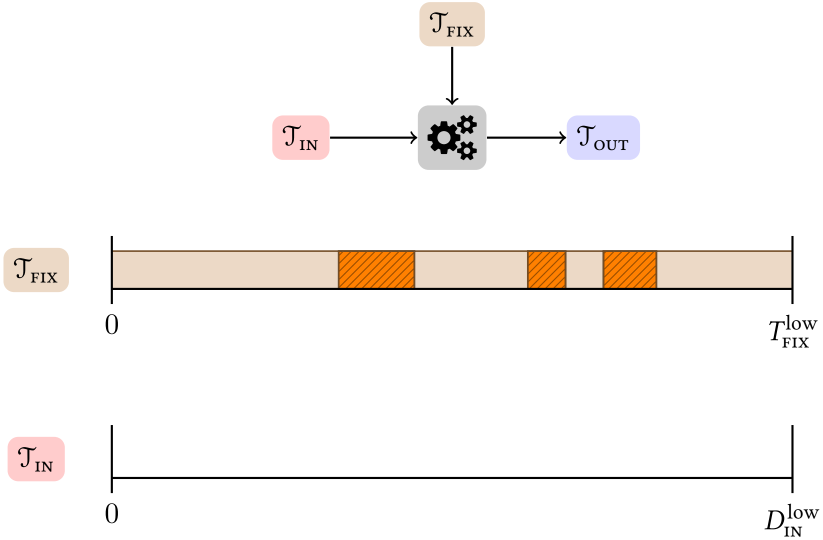

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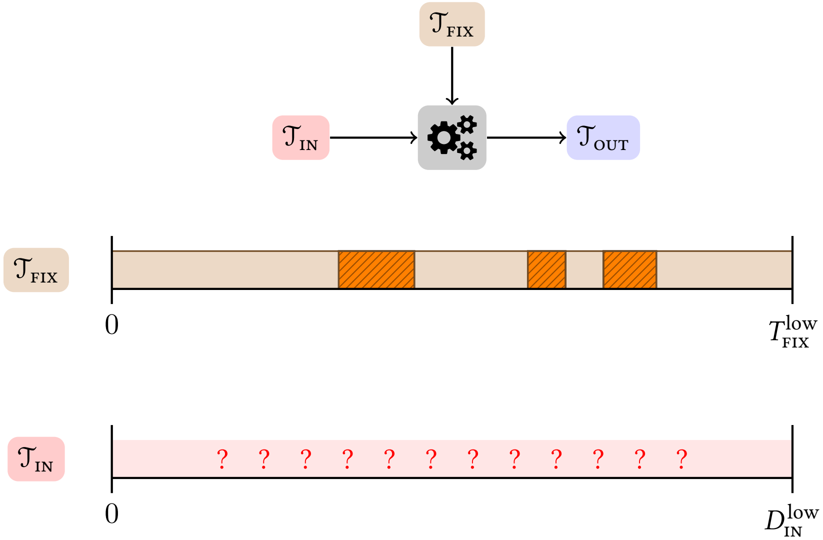

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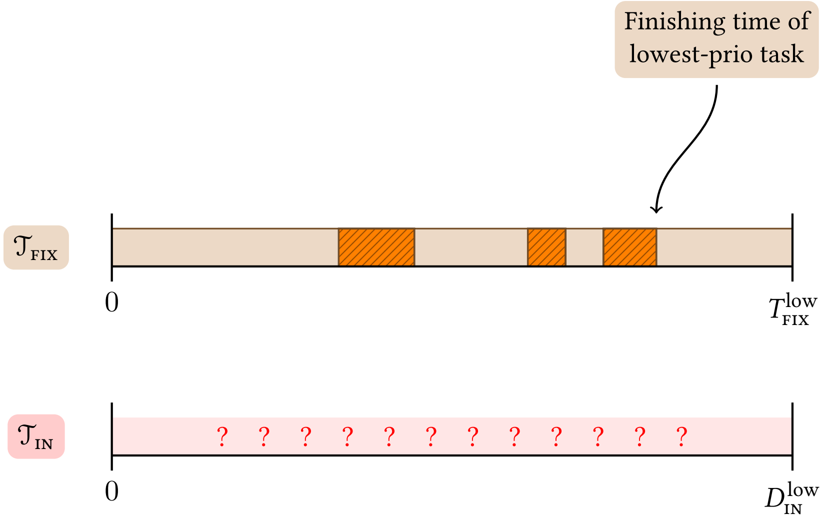

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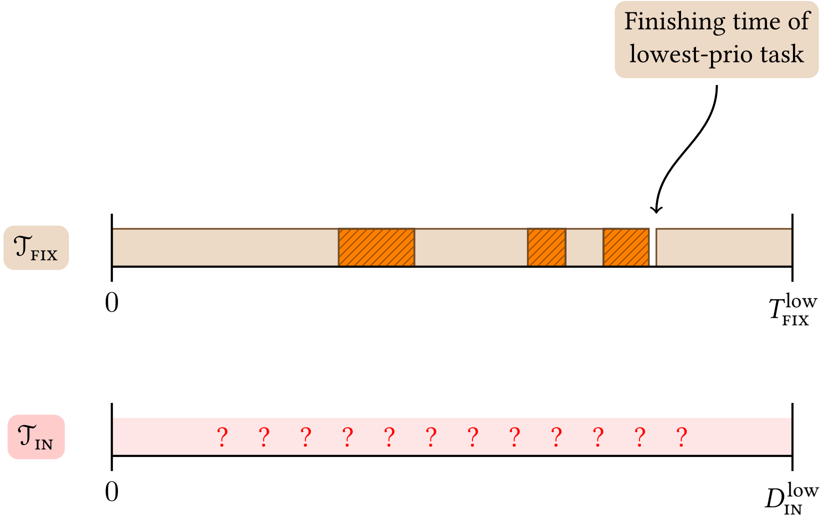

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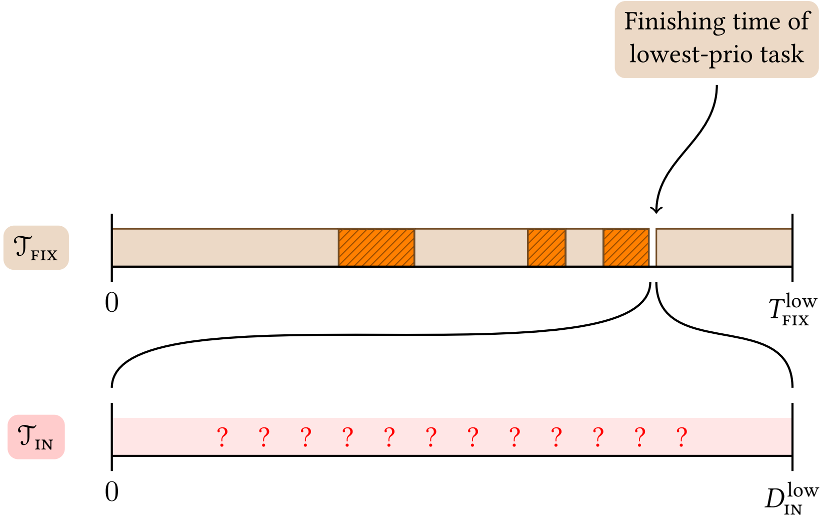

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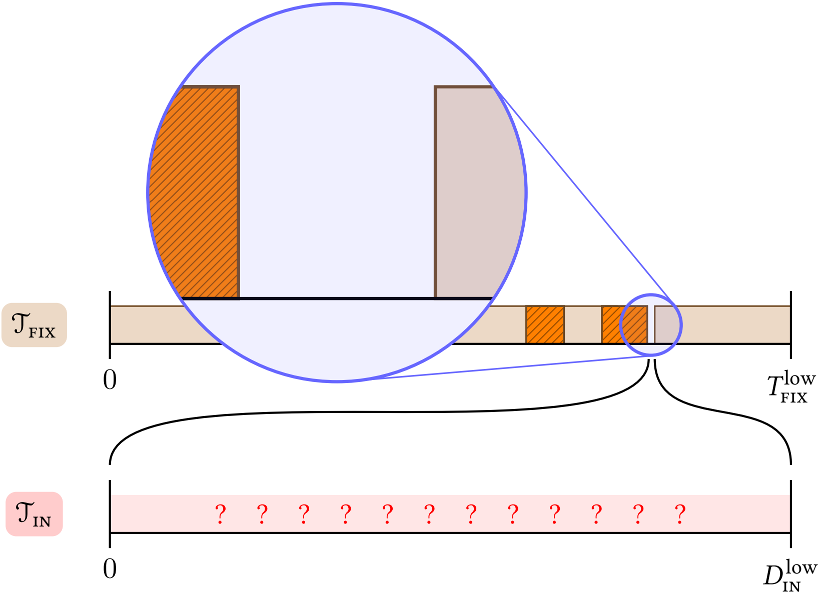

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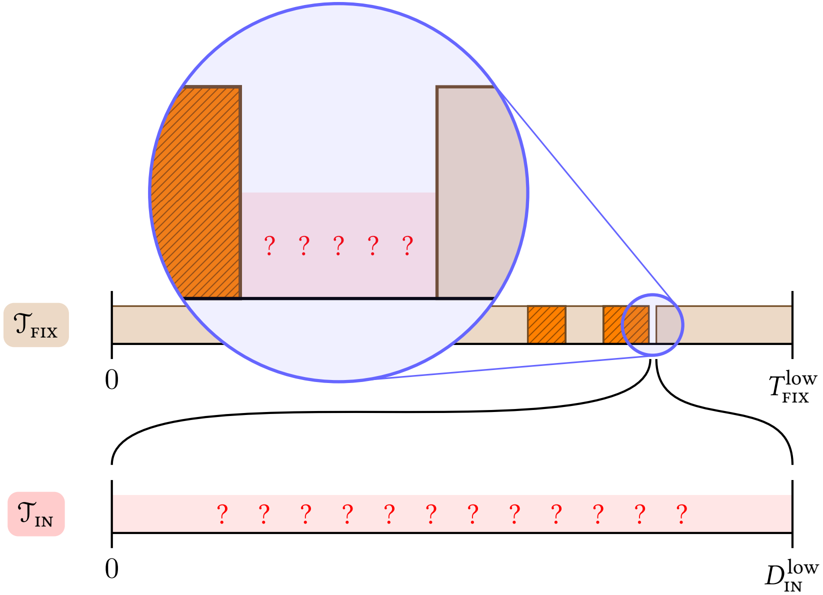

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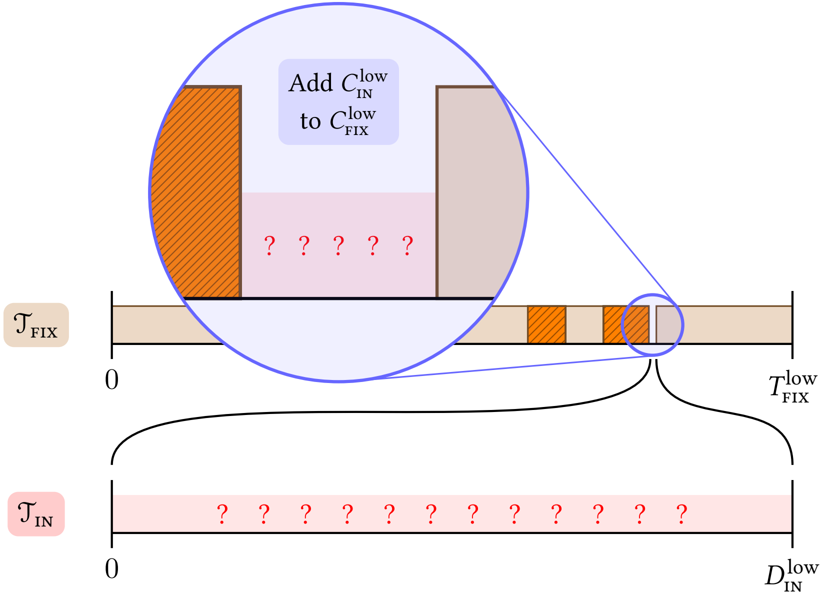

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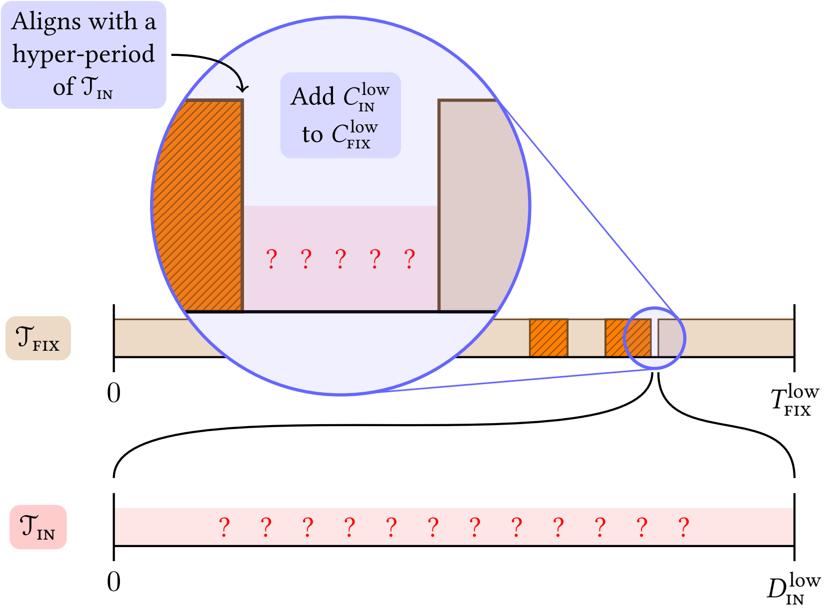

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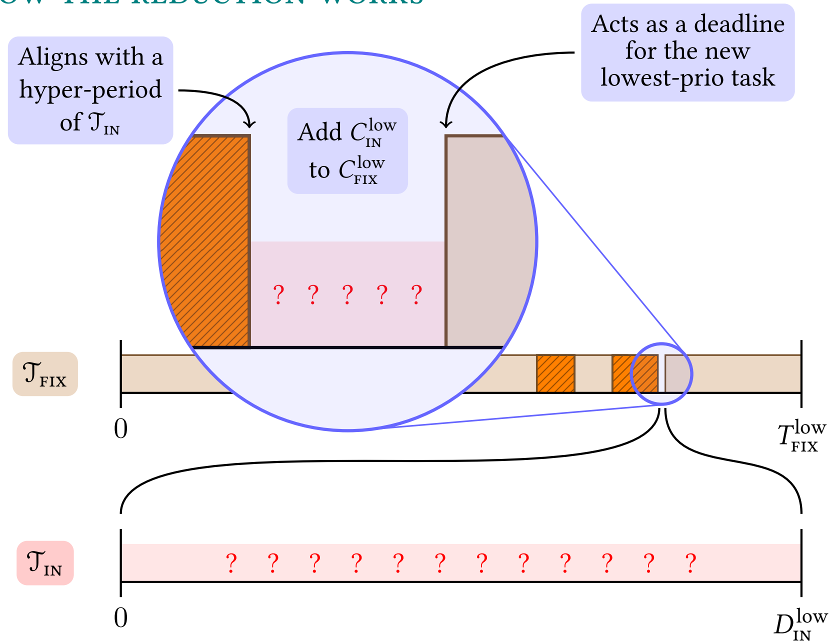

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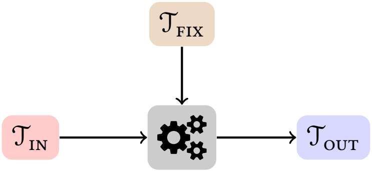

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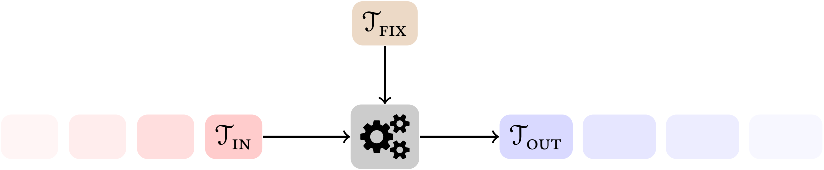

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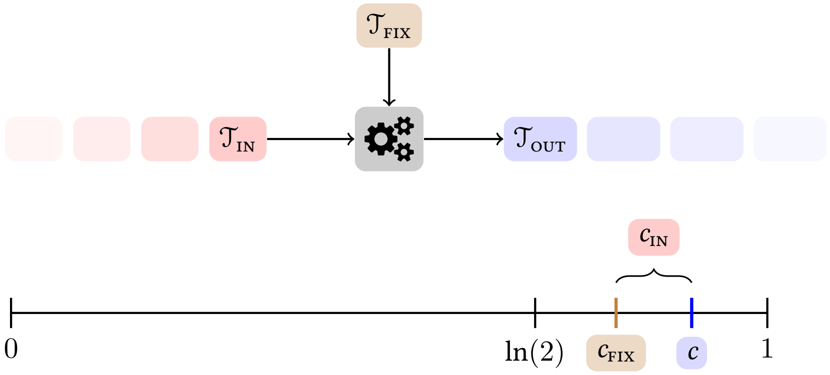

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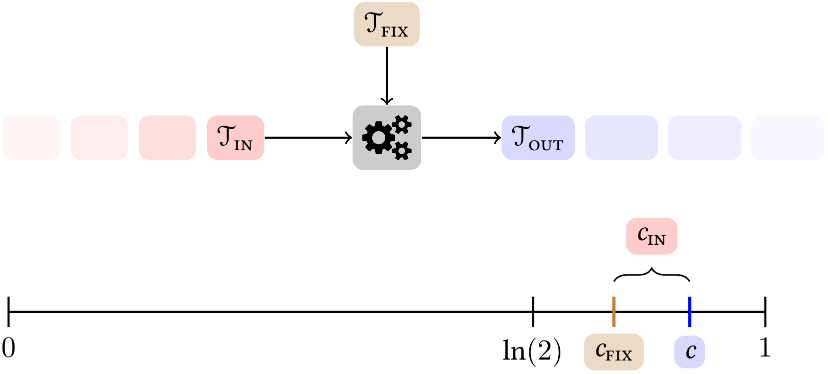

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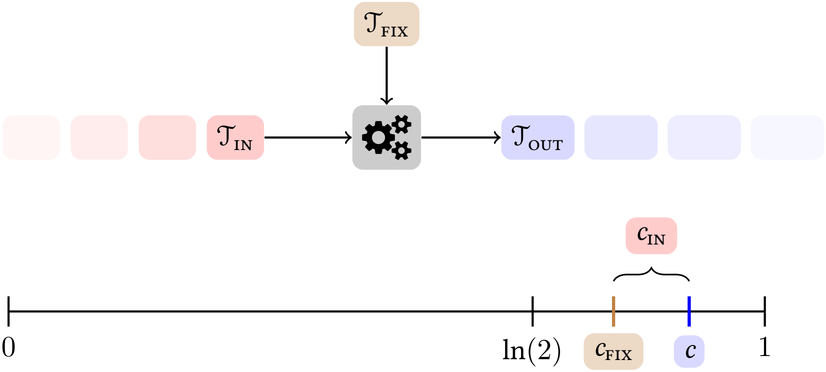

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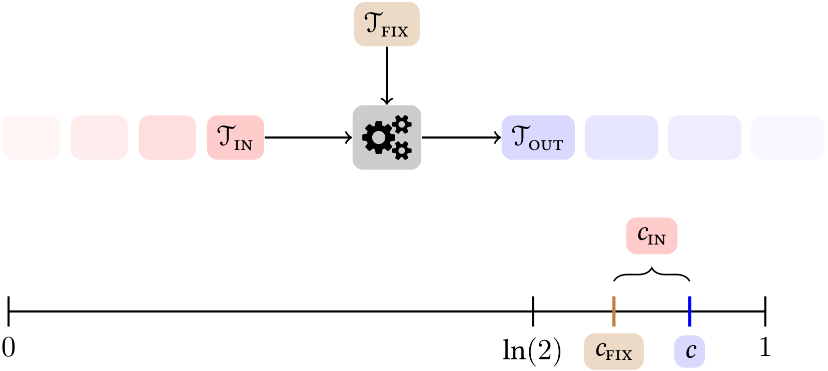

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$\exists$Questions?