Uniprocessor Feasibility of Sporadic Tasks Remains coNP-complete Under Bounded Utilization

Pontus Ekberg & Wang Yi

Uppsala University

RTSS, December 2015

The General Setting

Instances

Task set T of sporadic (or synchronous periodic) tasks with constrained deadlines.

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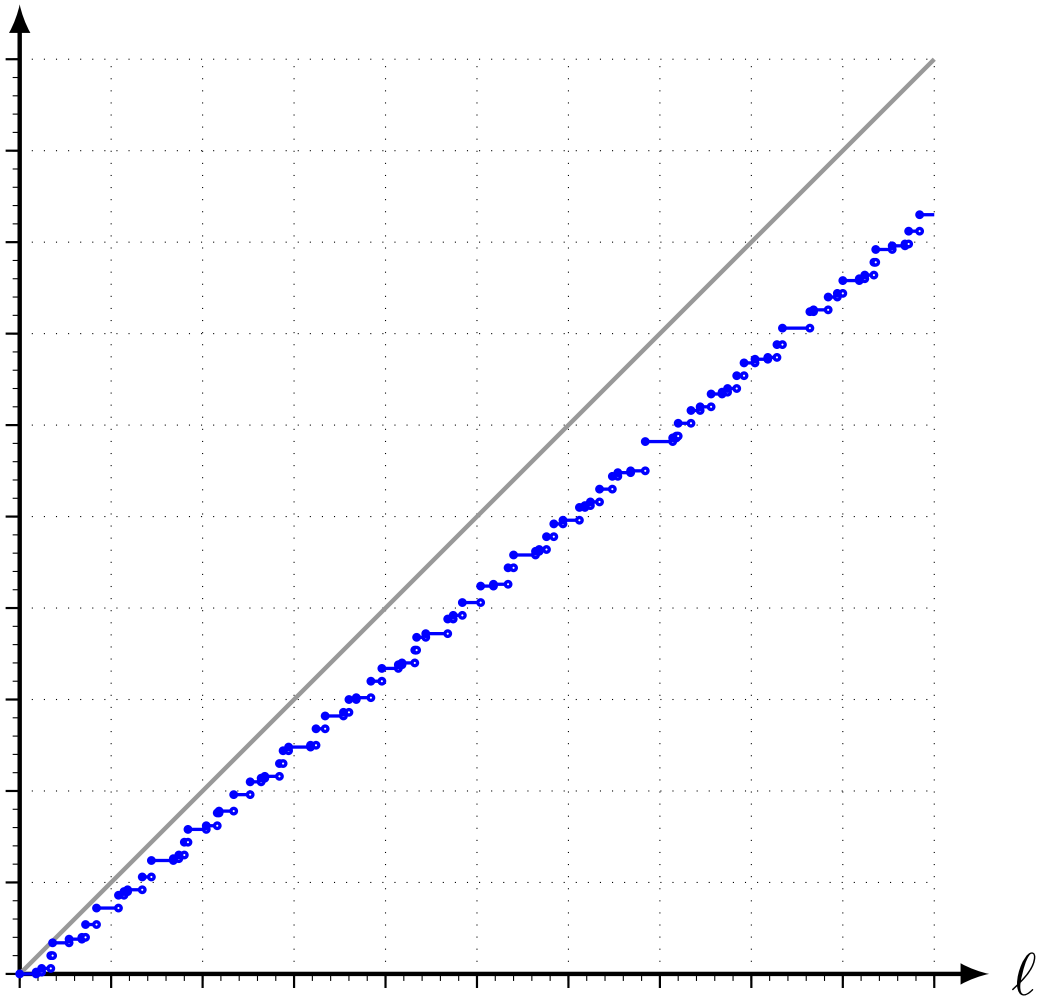
Task set T of sporadic (or synchronous periodic) tasks with constrained deadlines.

Question

Is T feasible on a preemptive uniprocessor?

An Algorithm for Feasibility [Baruah et al., 1990]



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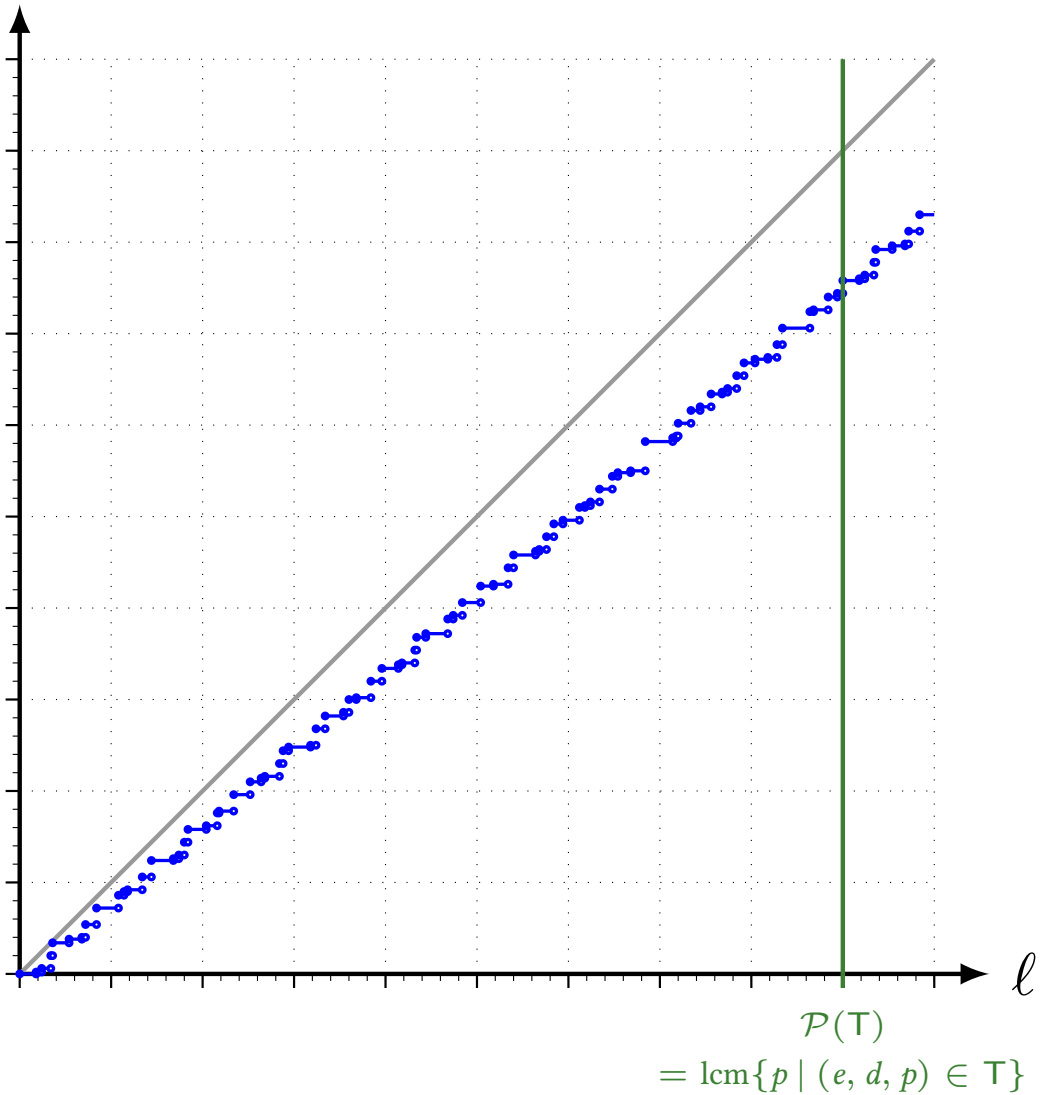

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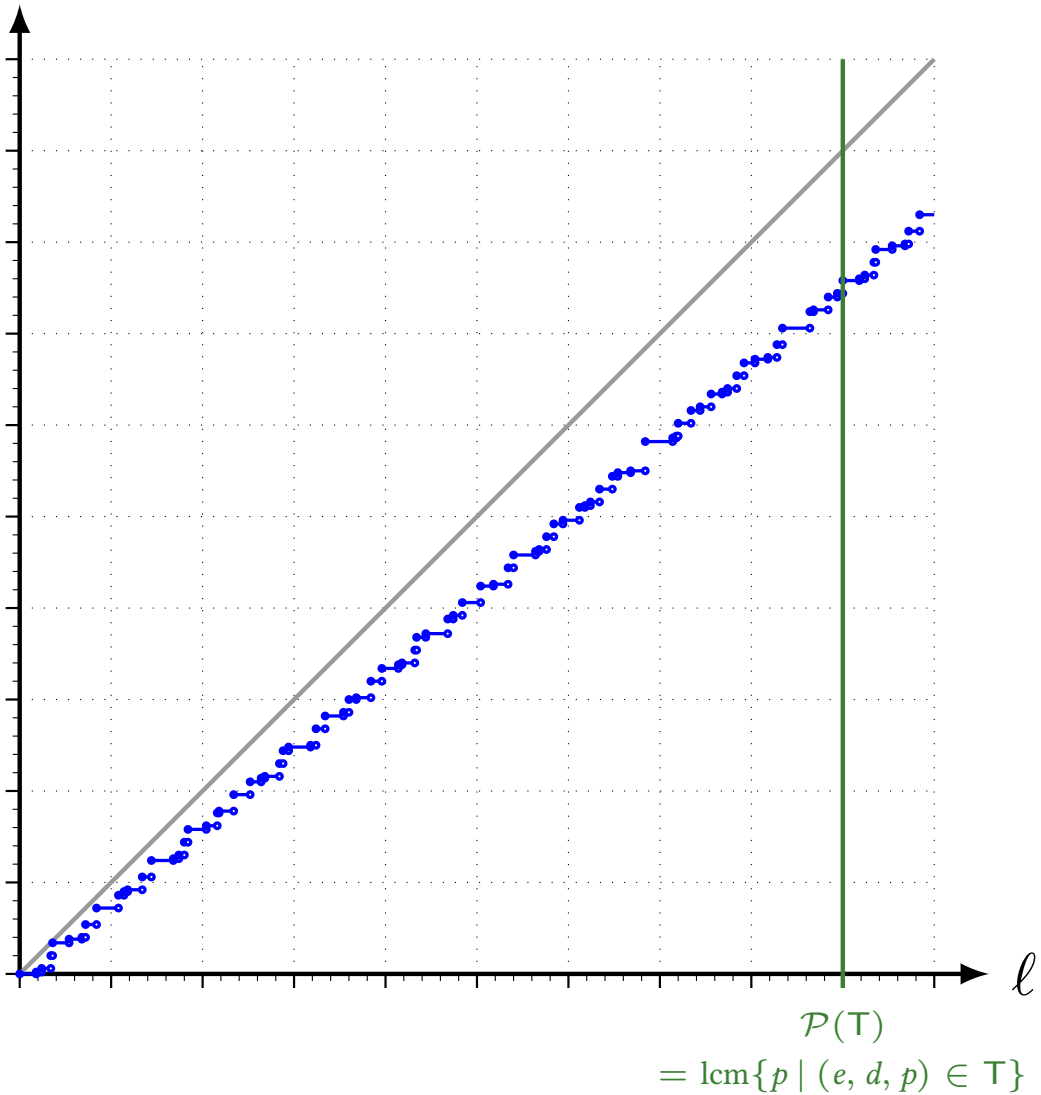

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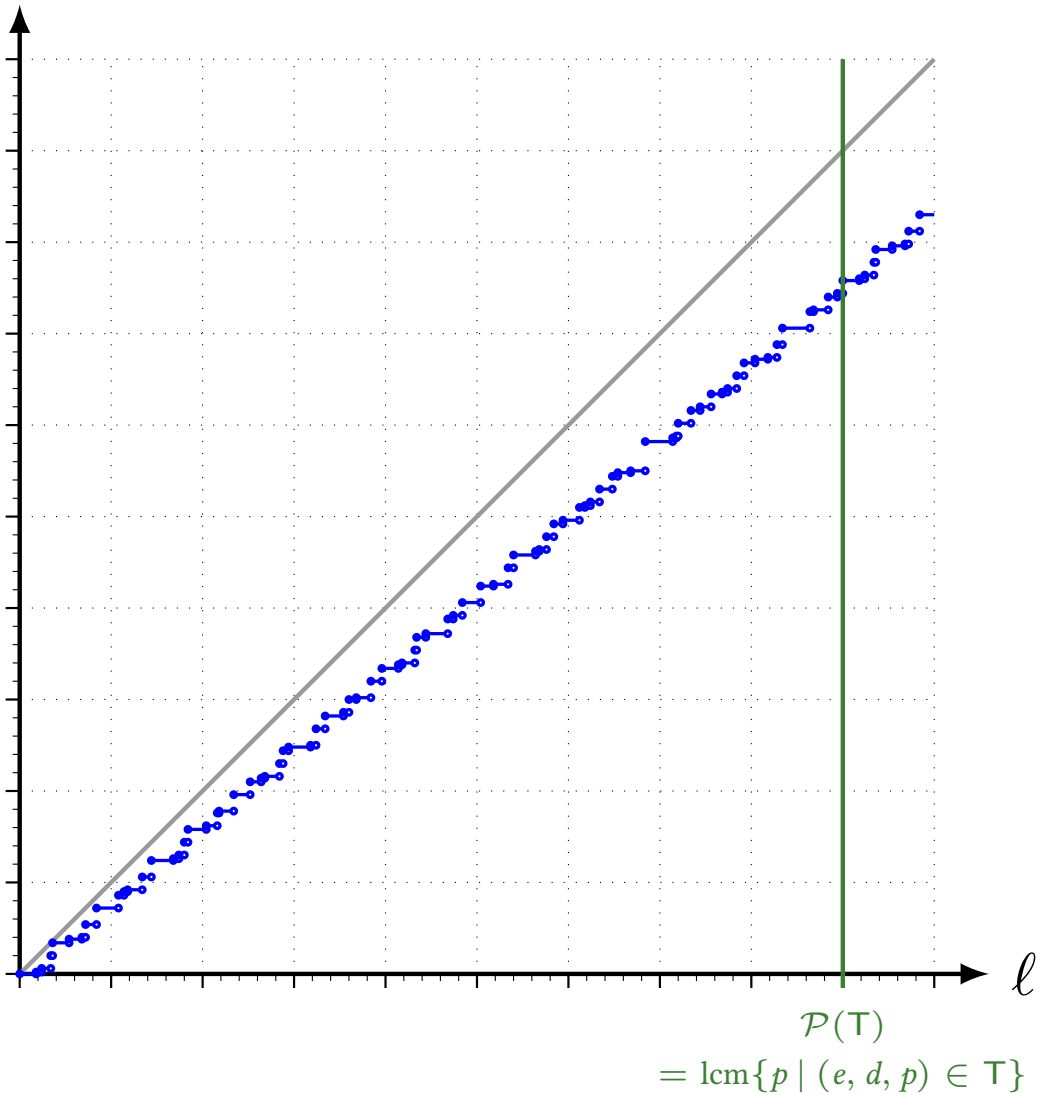

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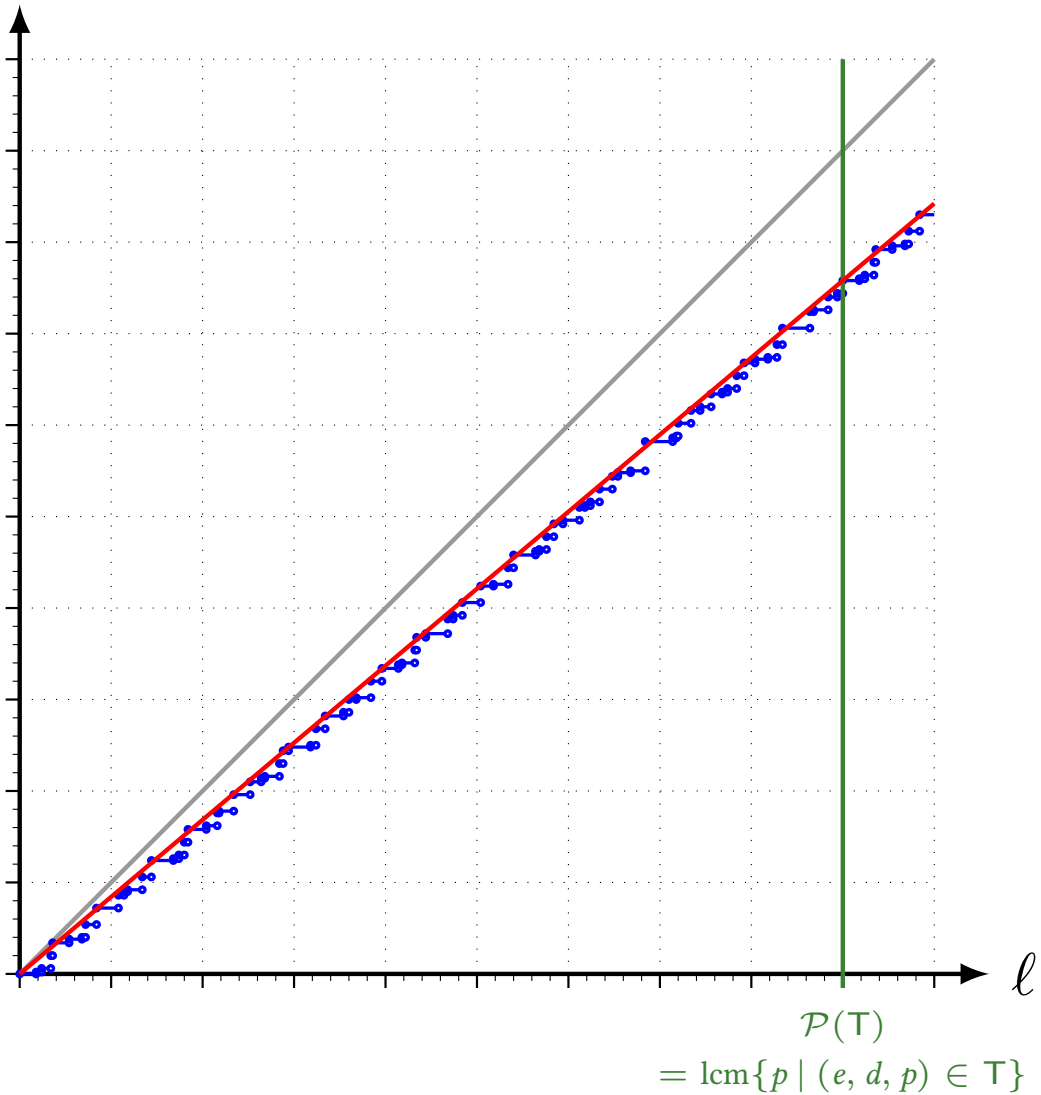

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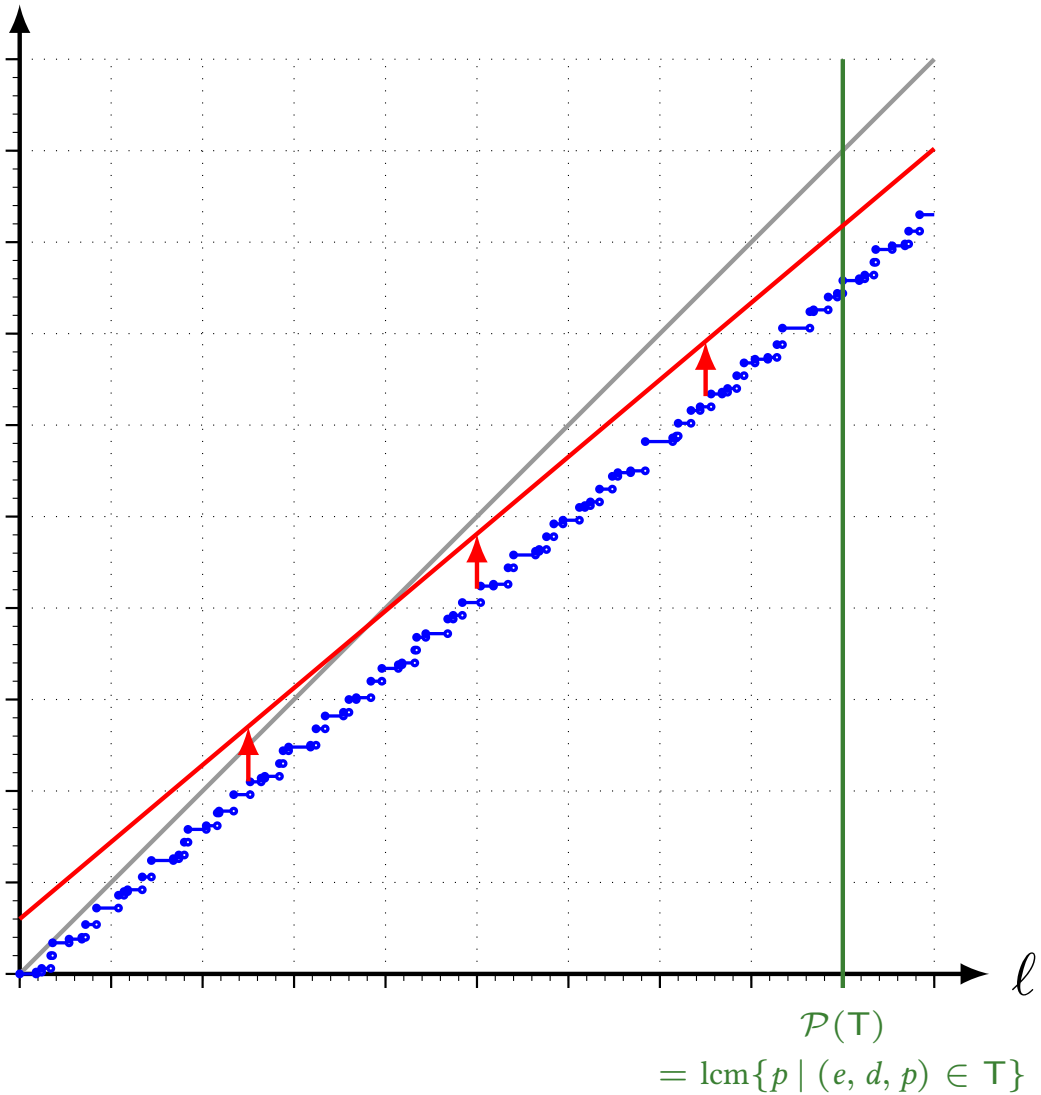

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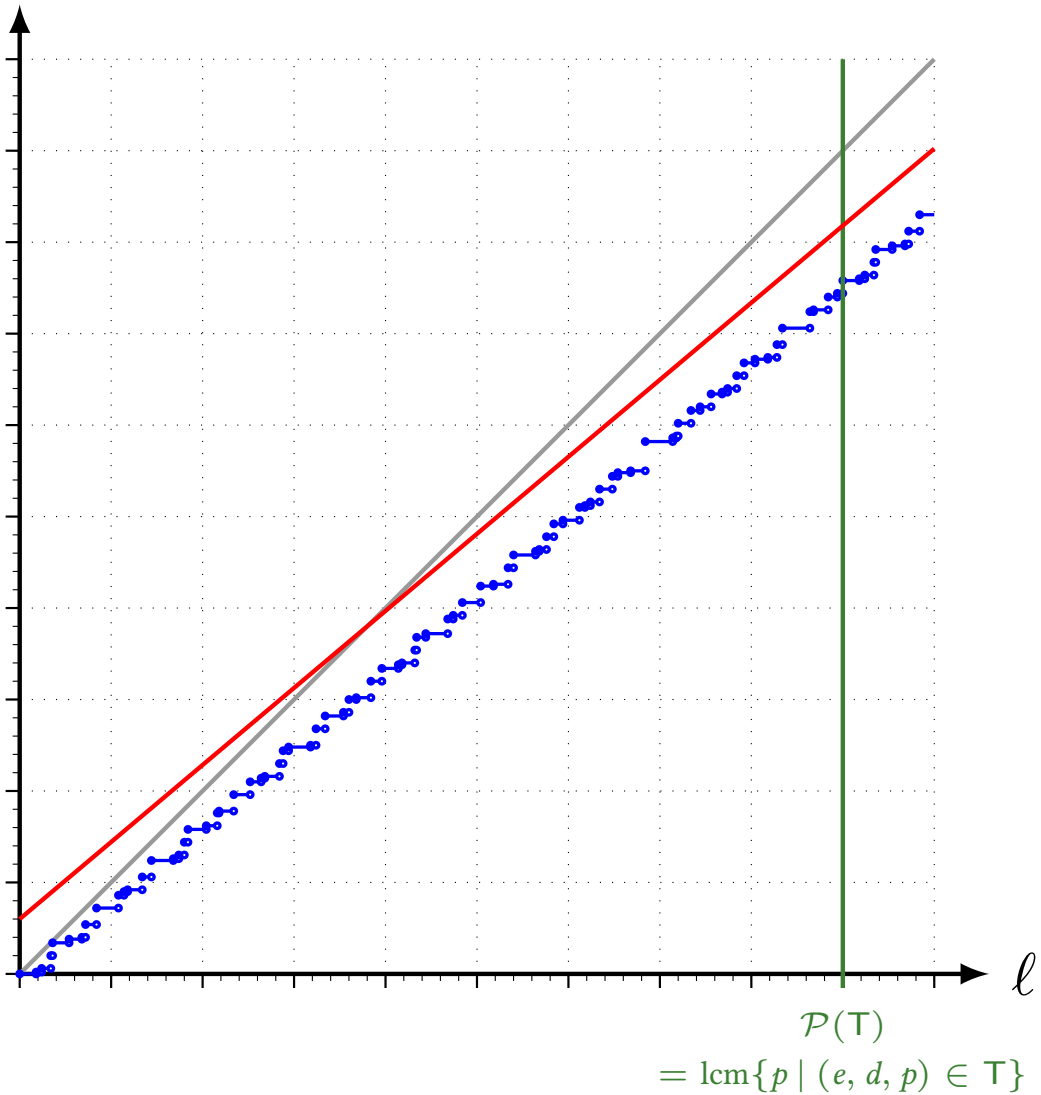

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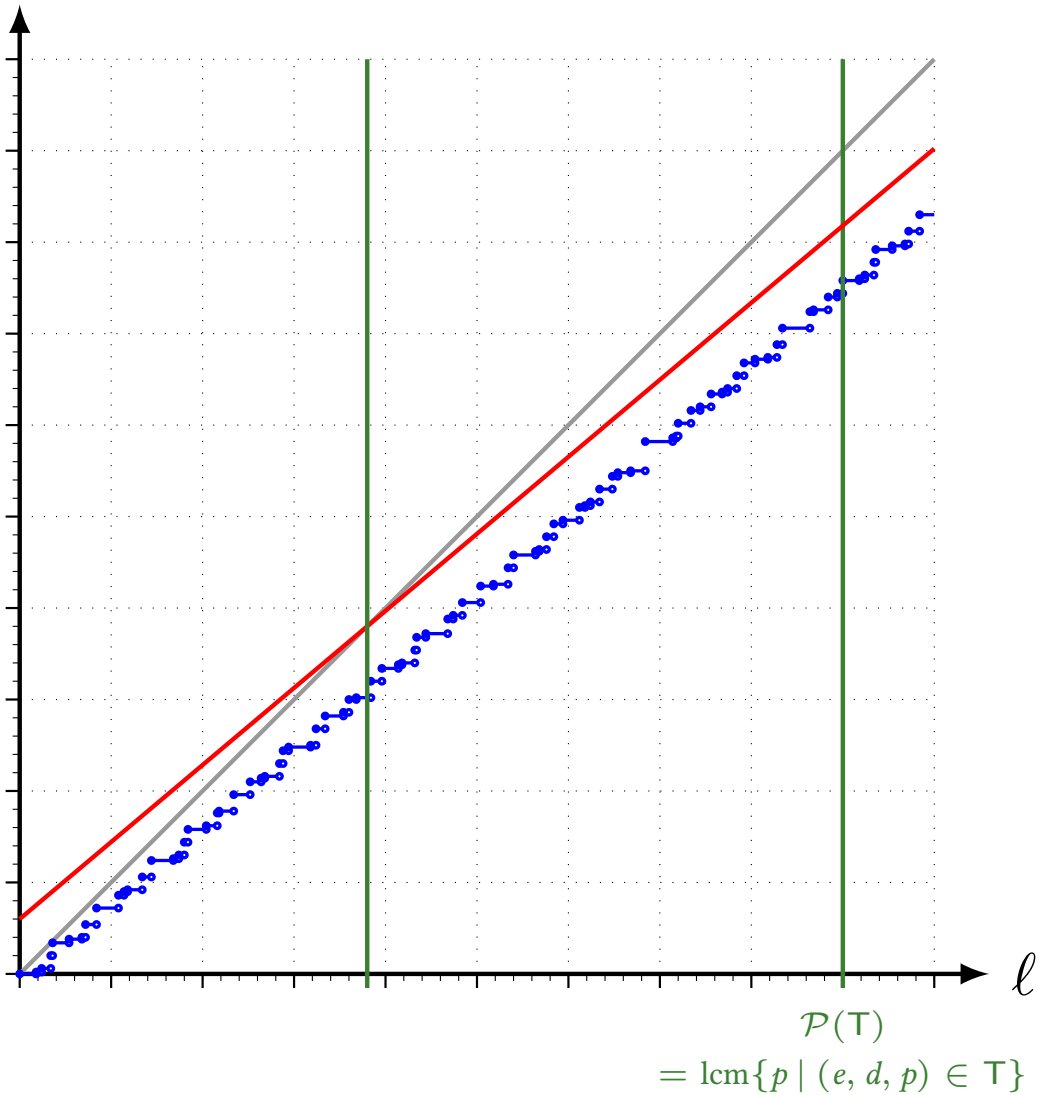

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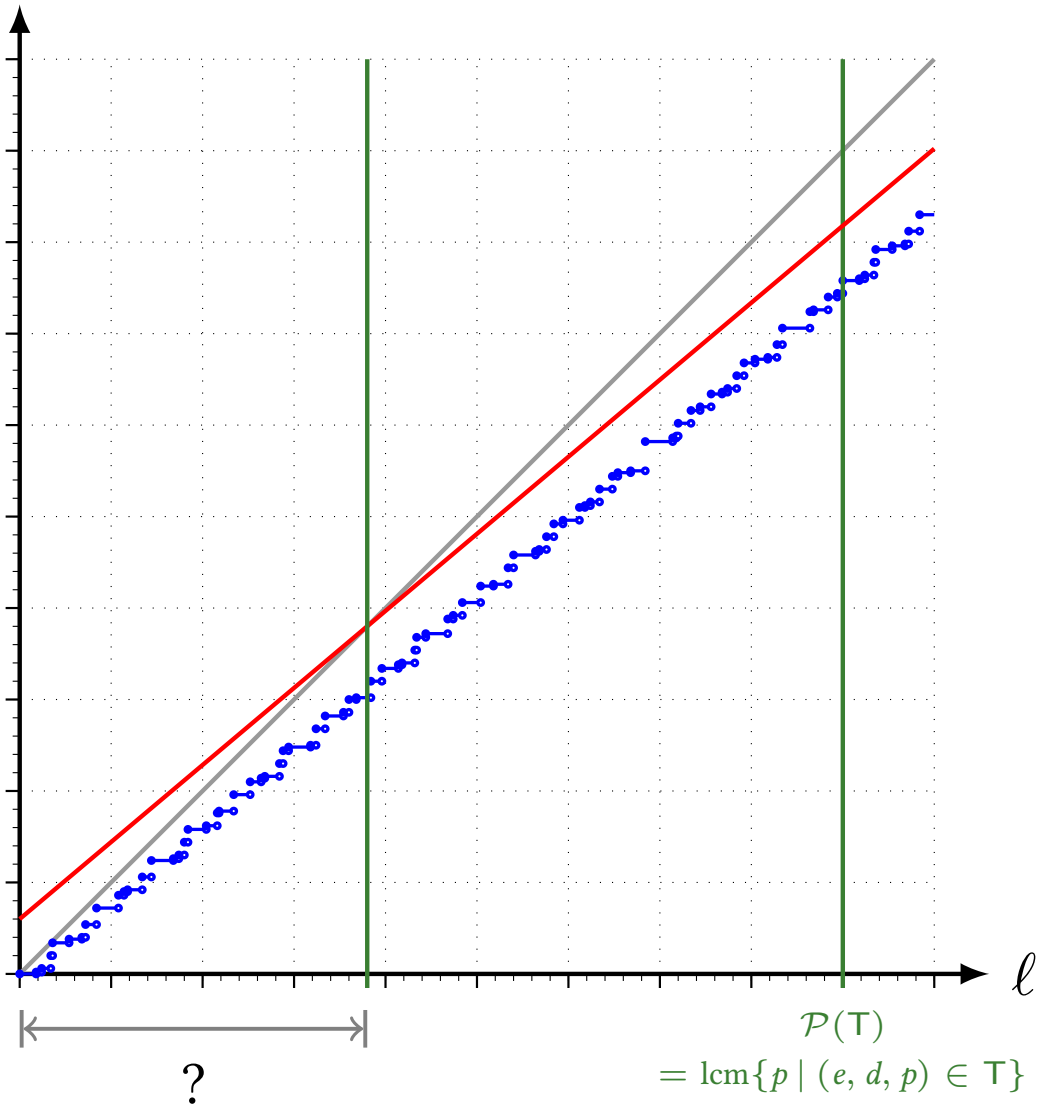

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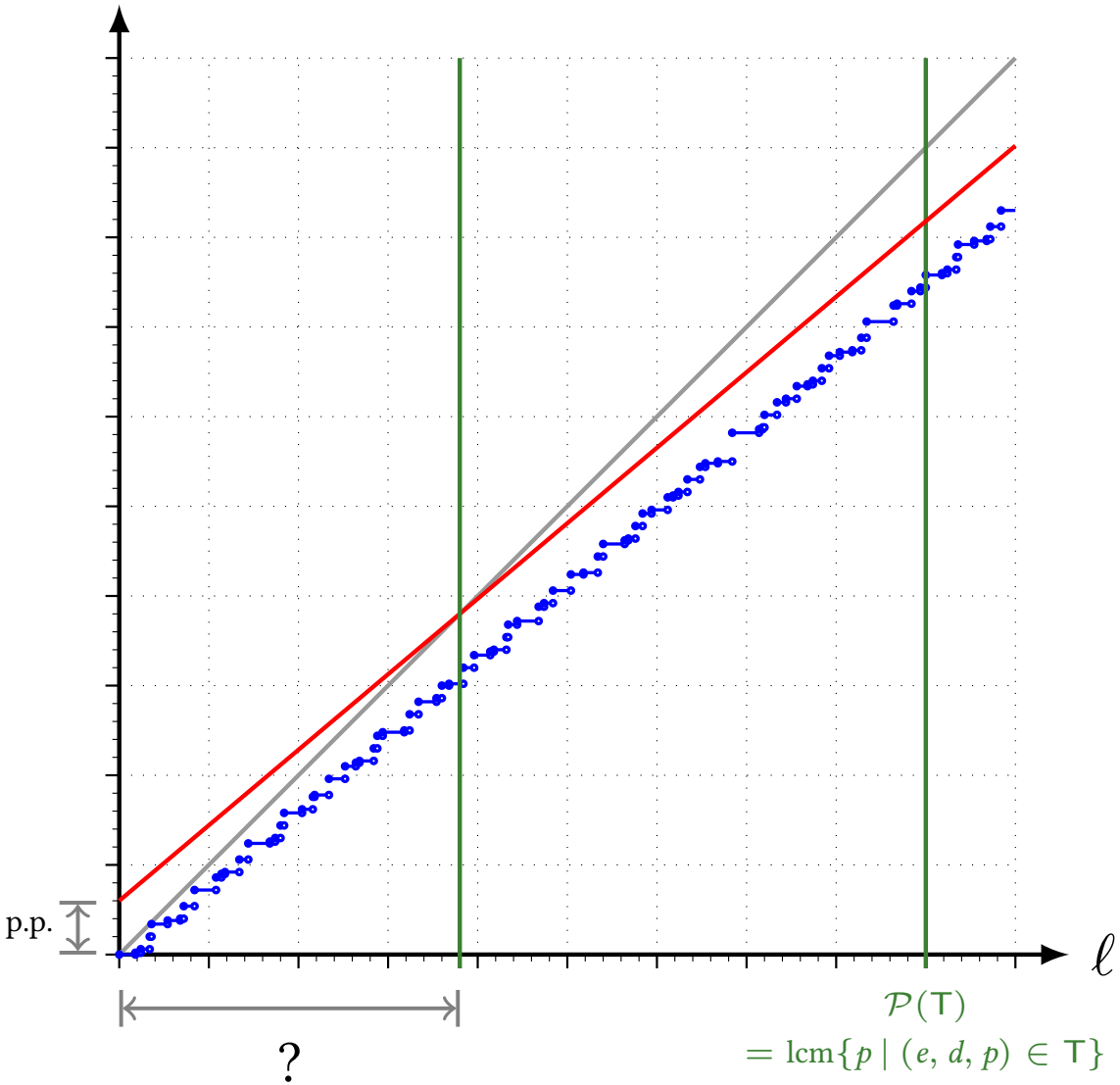

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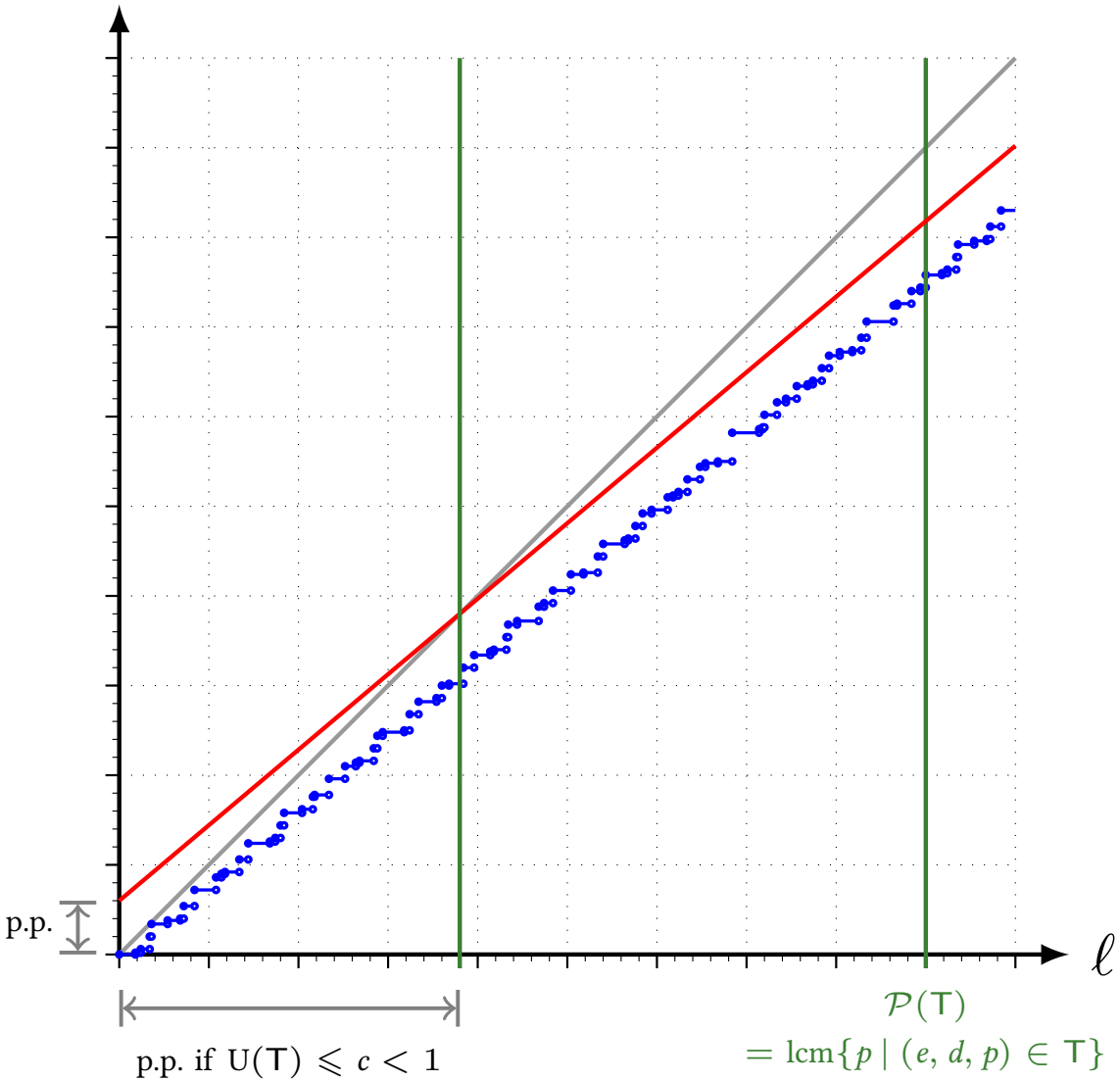

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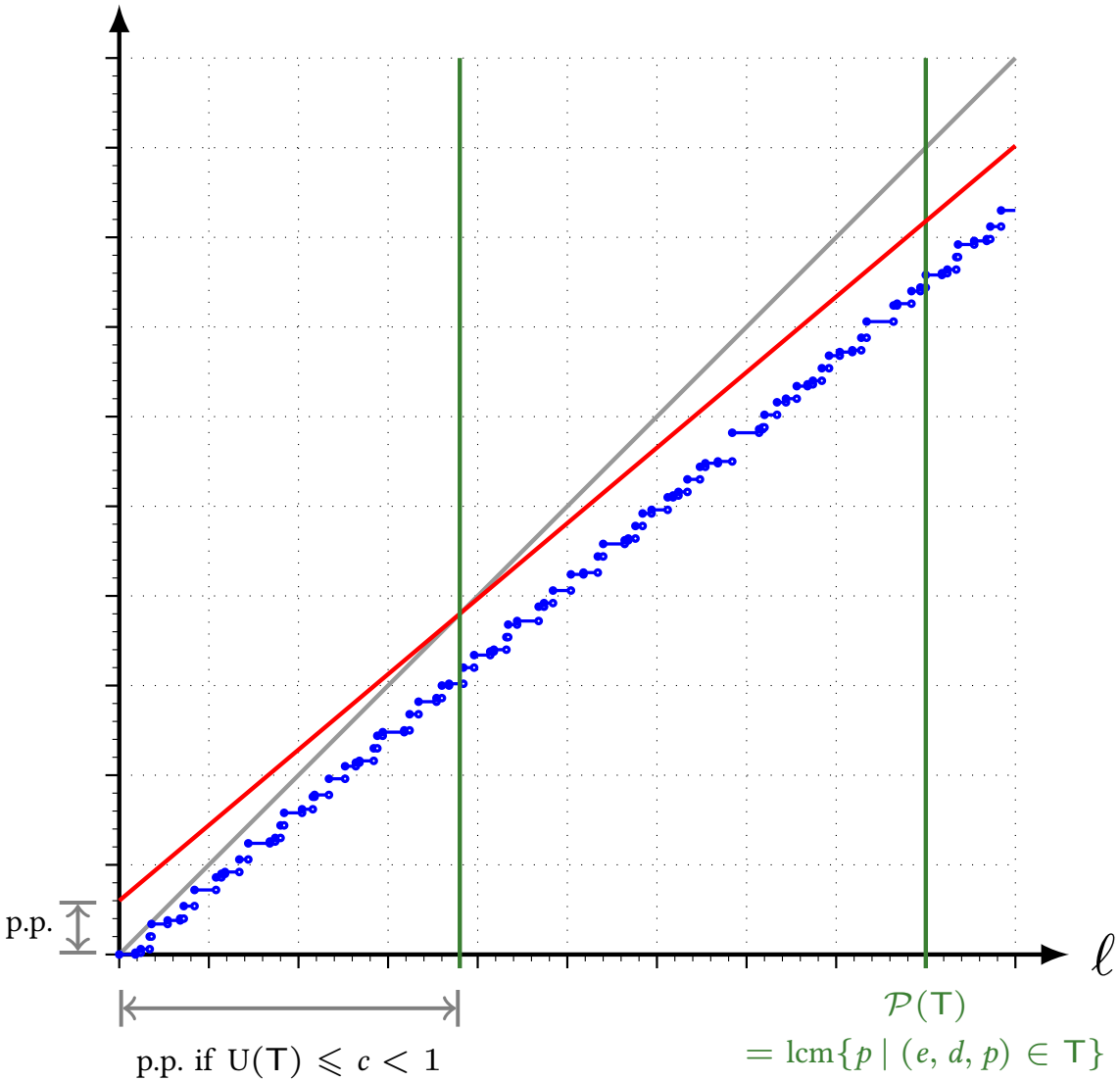

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c-Feasibility $(\mathrm{U}(\mathsf{T}) \leqslant c)$

- Pseudo-poly. time algorithm if $c < 1$
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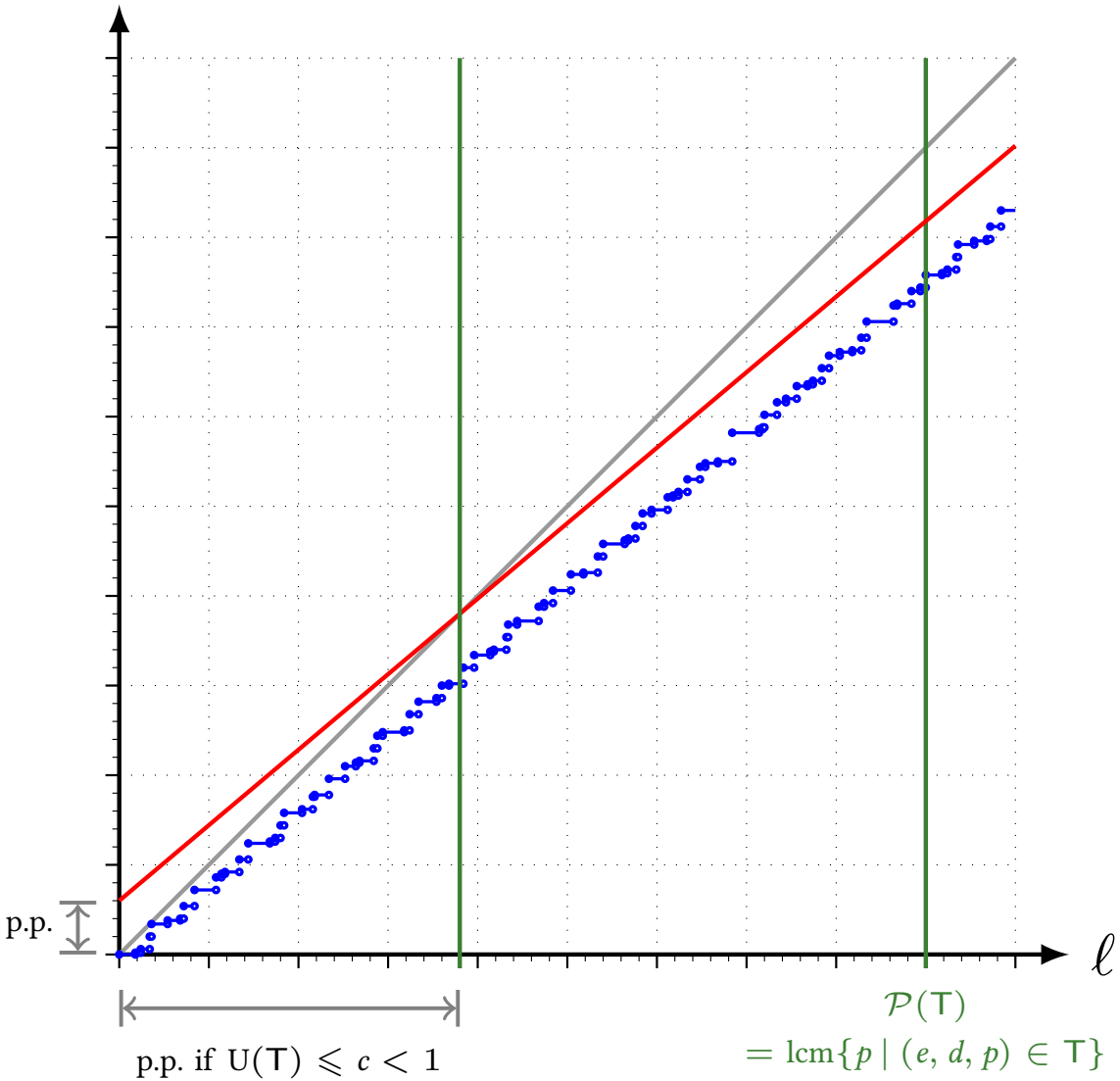

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  [Eisenbrand & Rothvoß, SODA'10]

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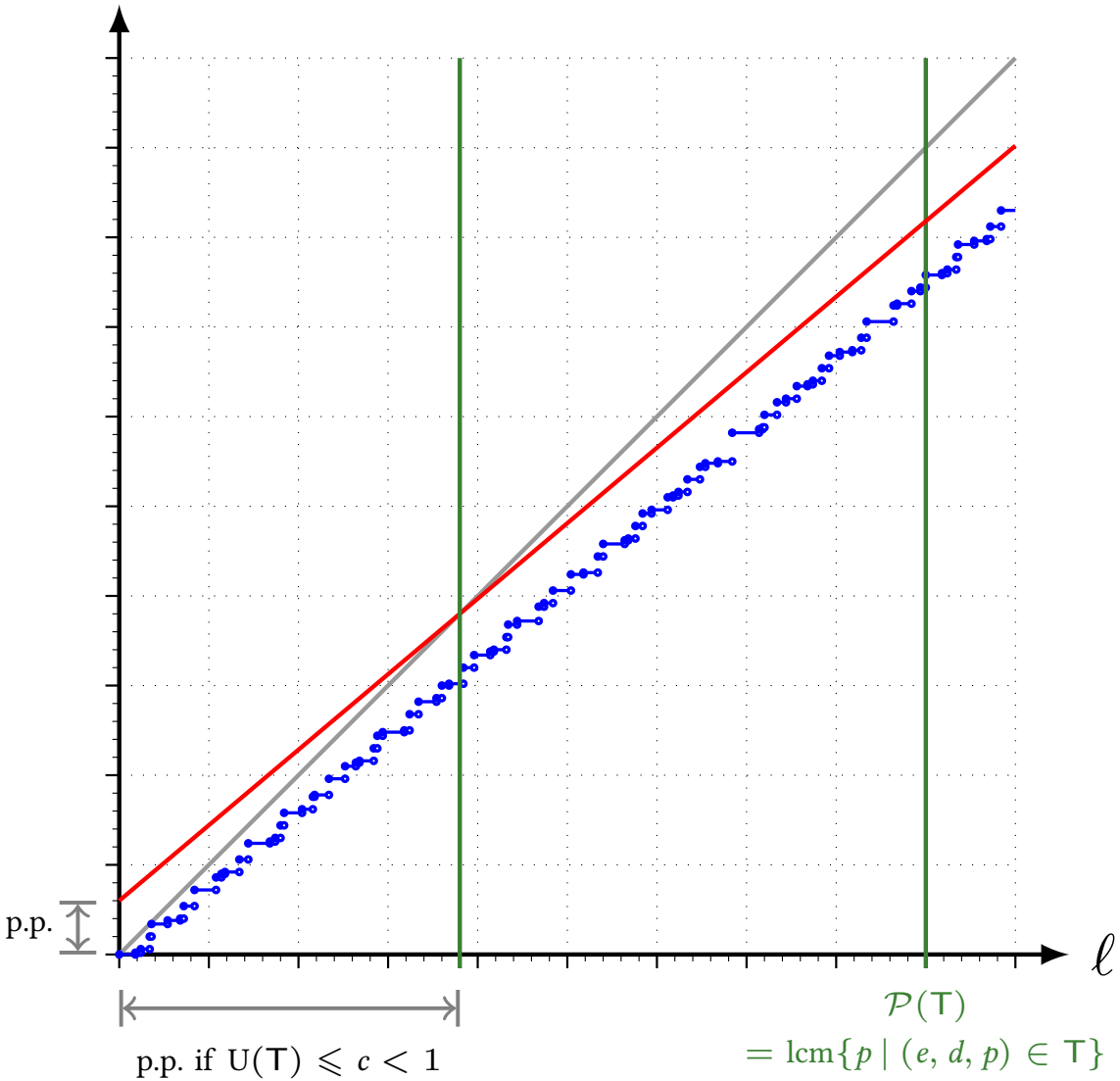

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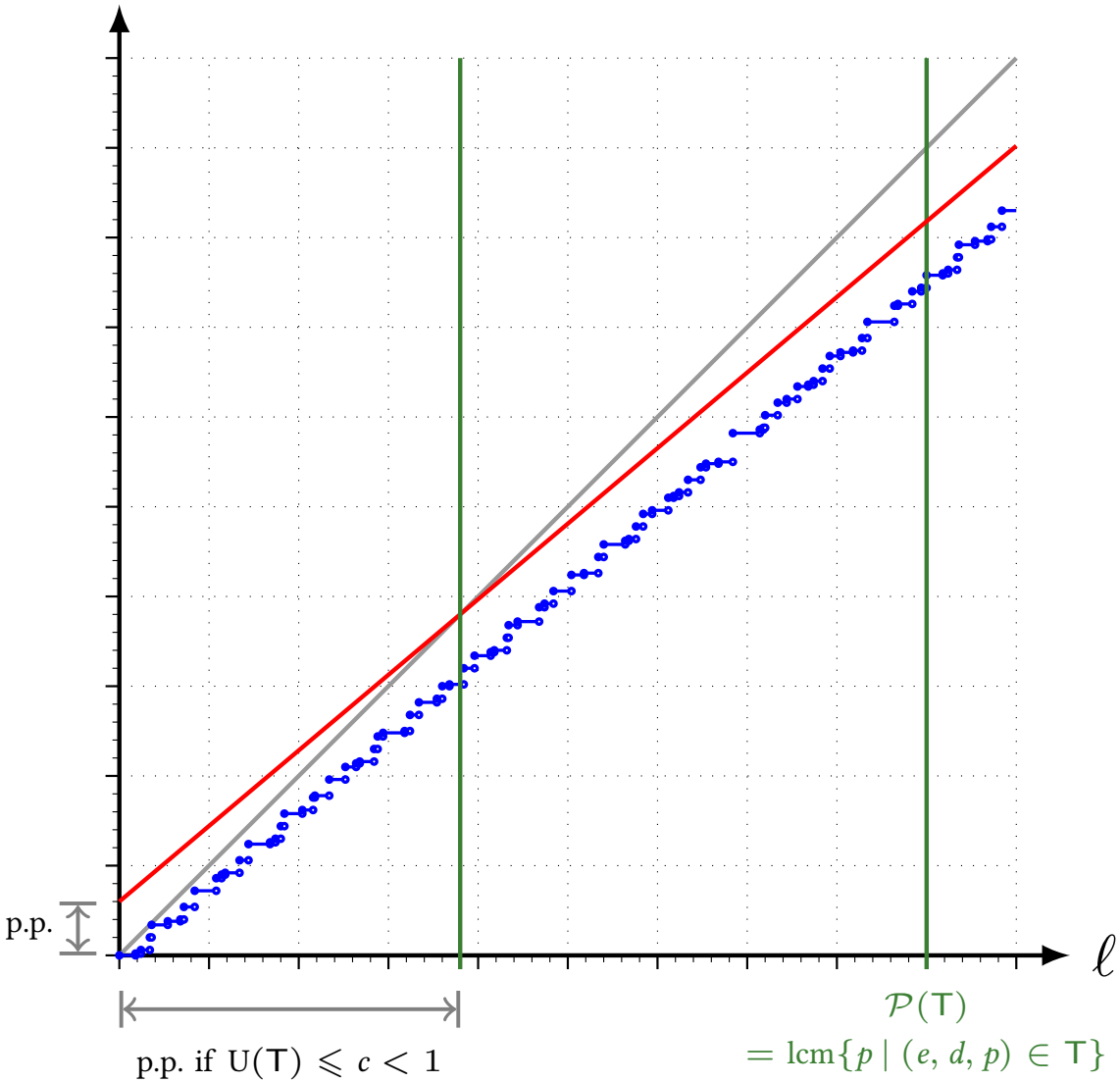

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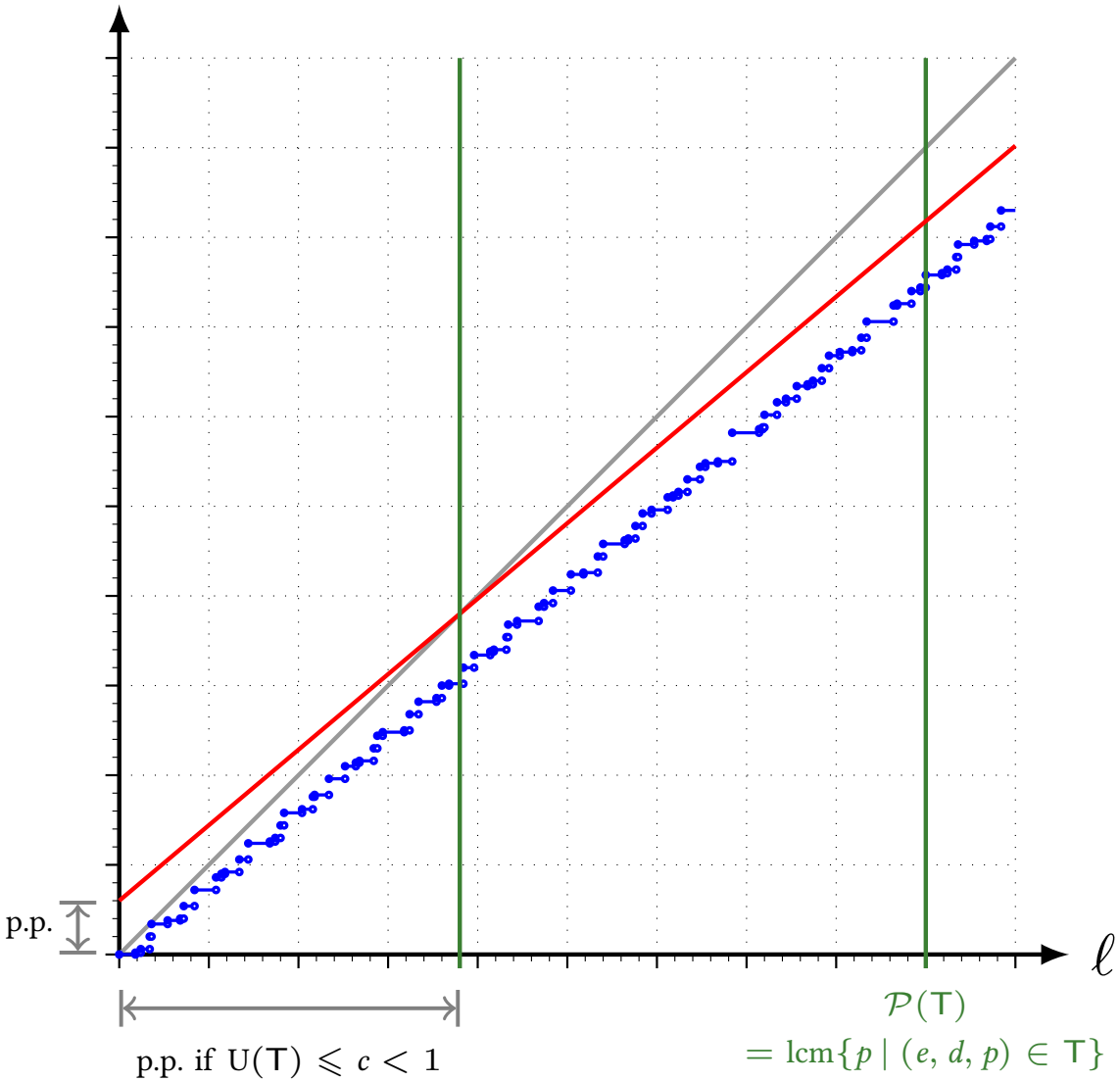

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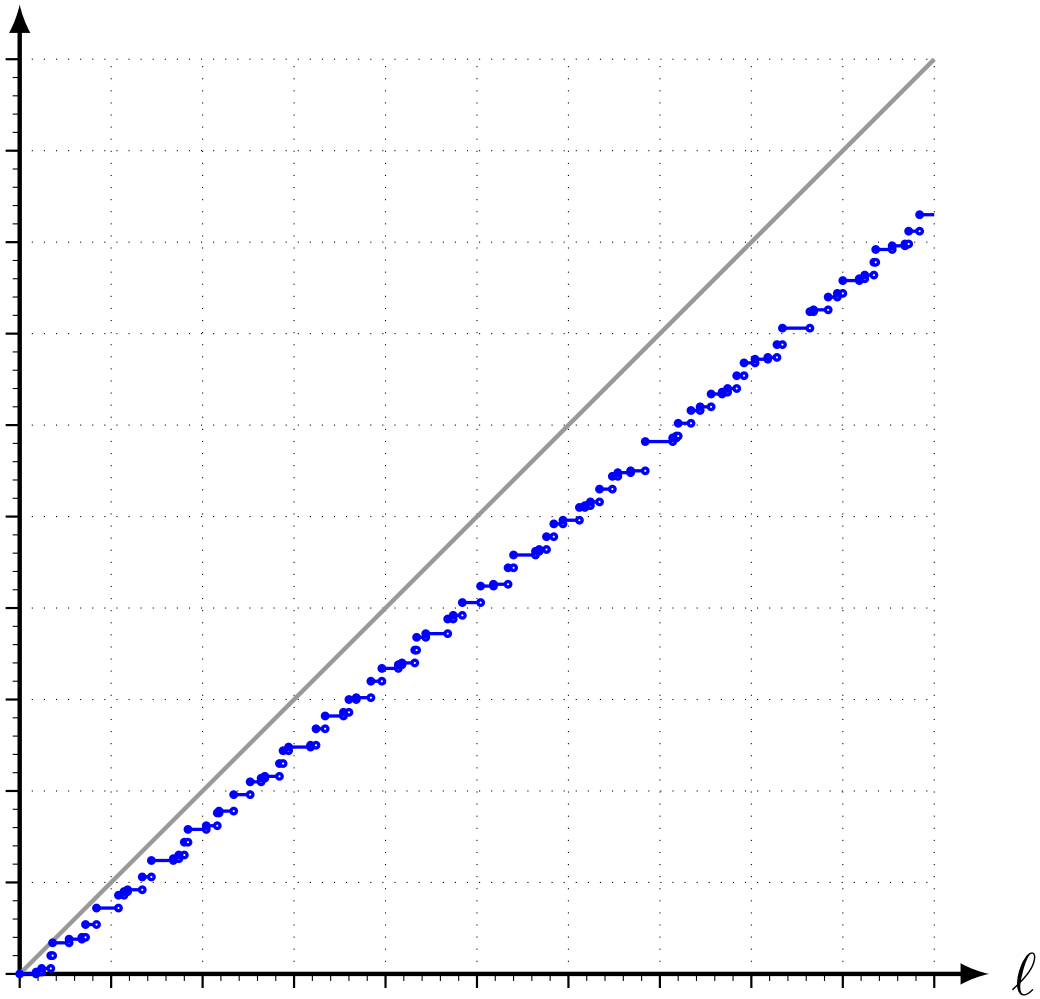

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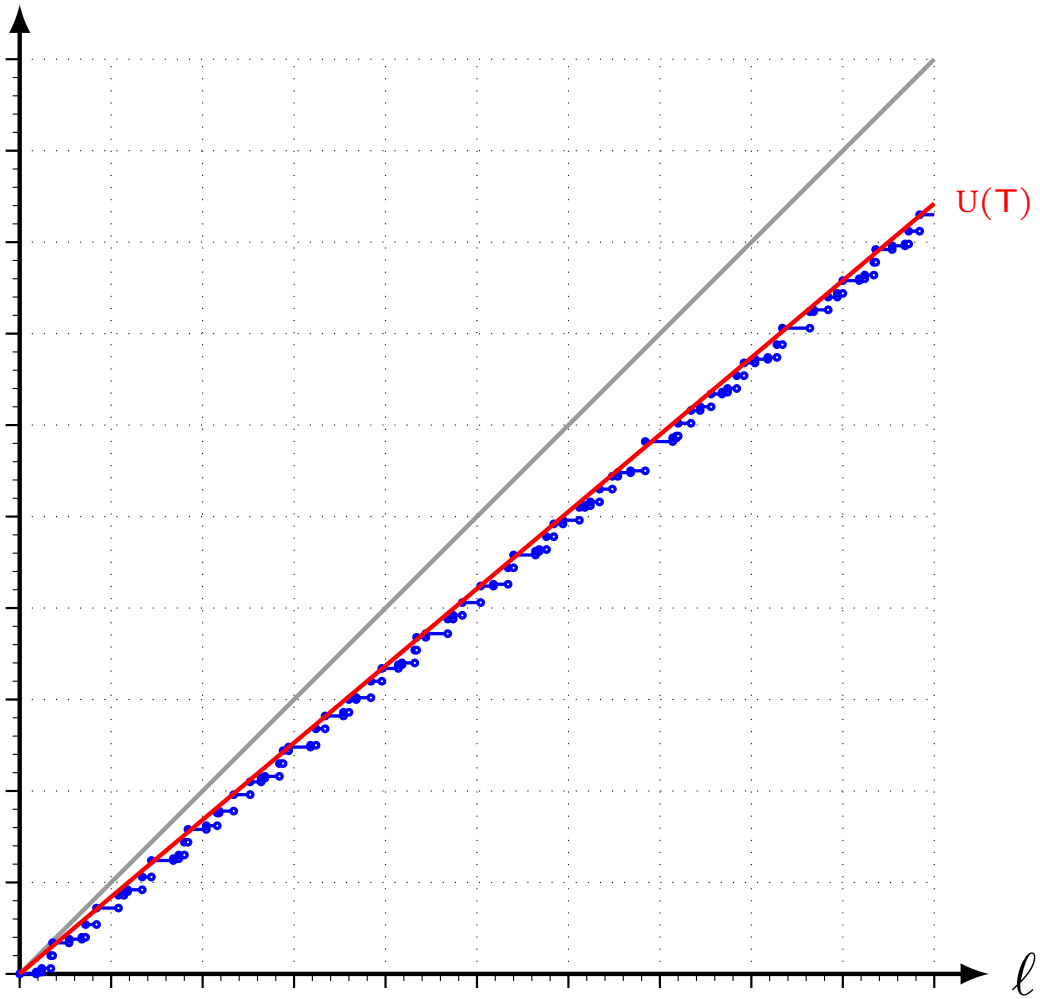

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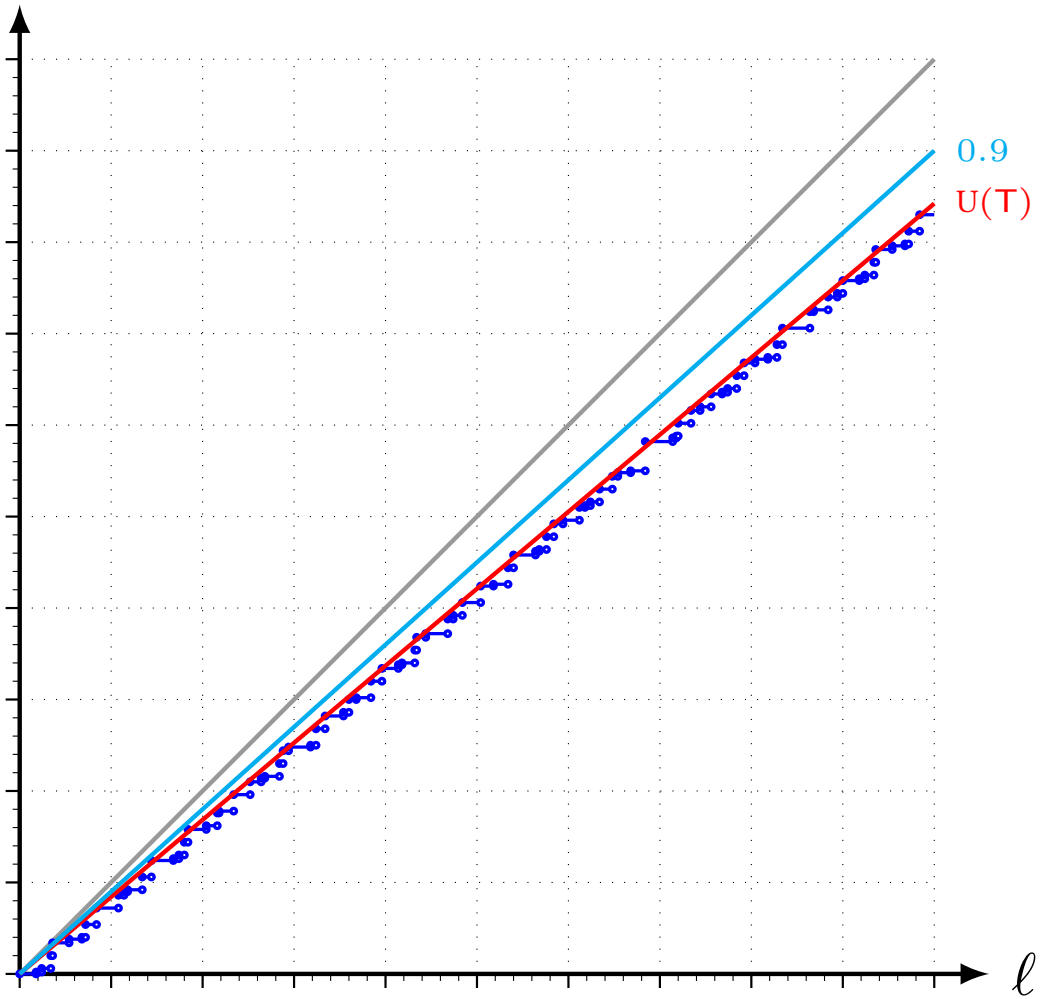

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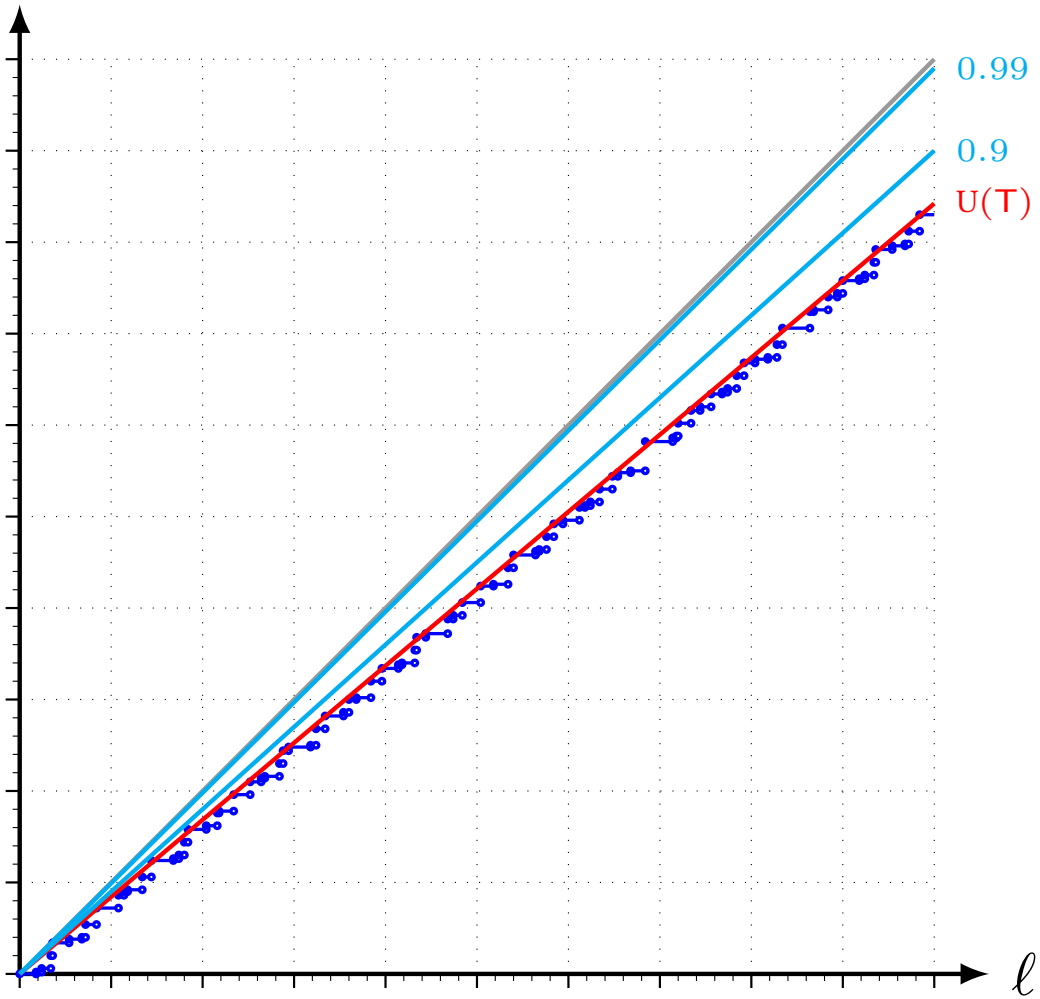

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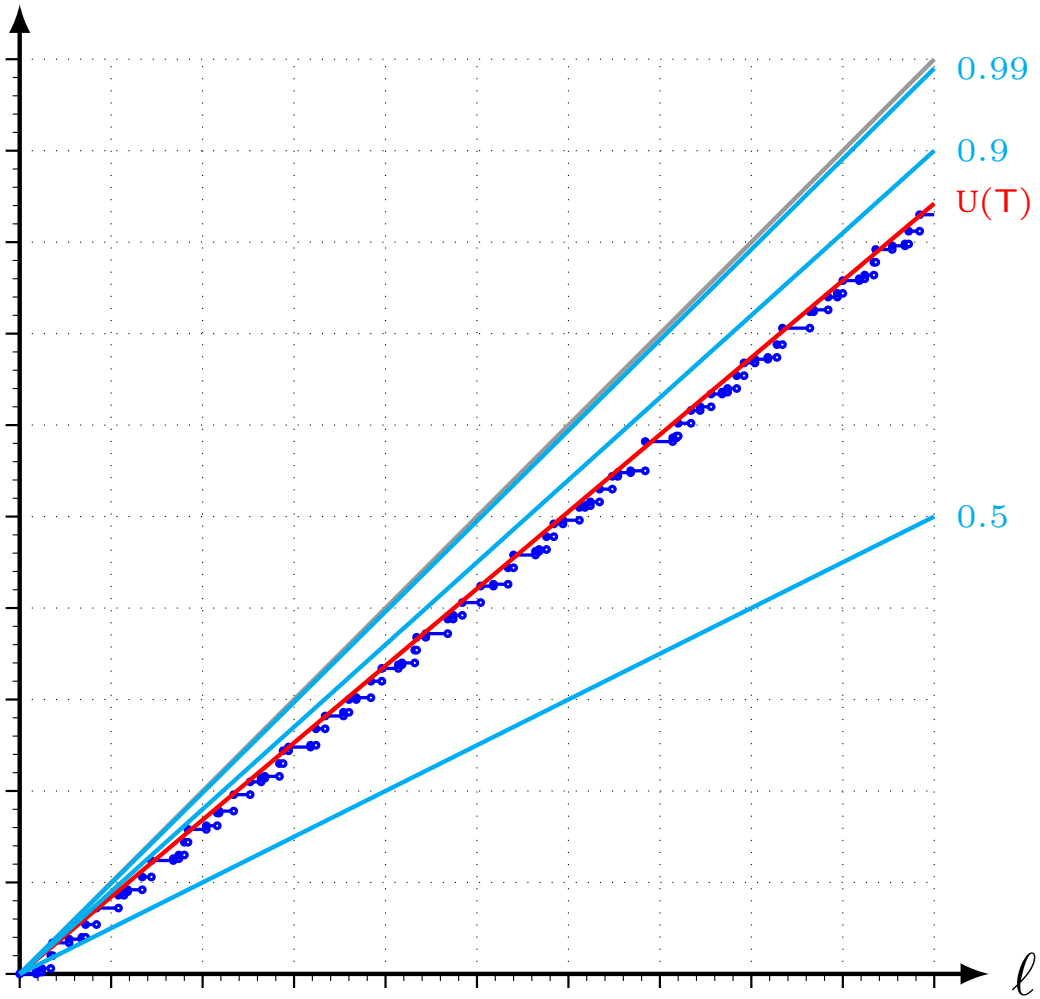

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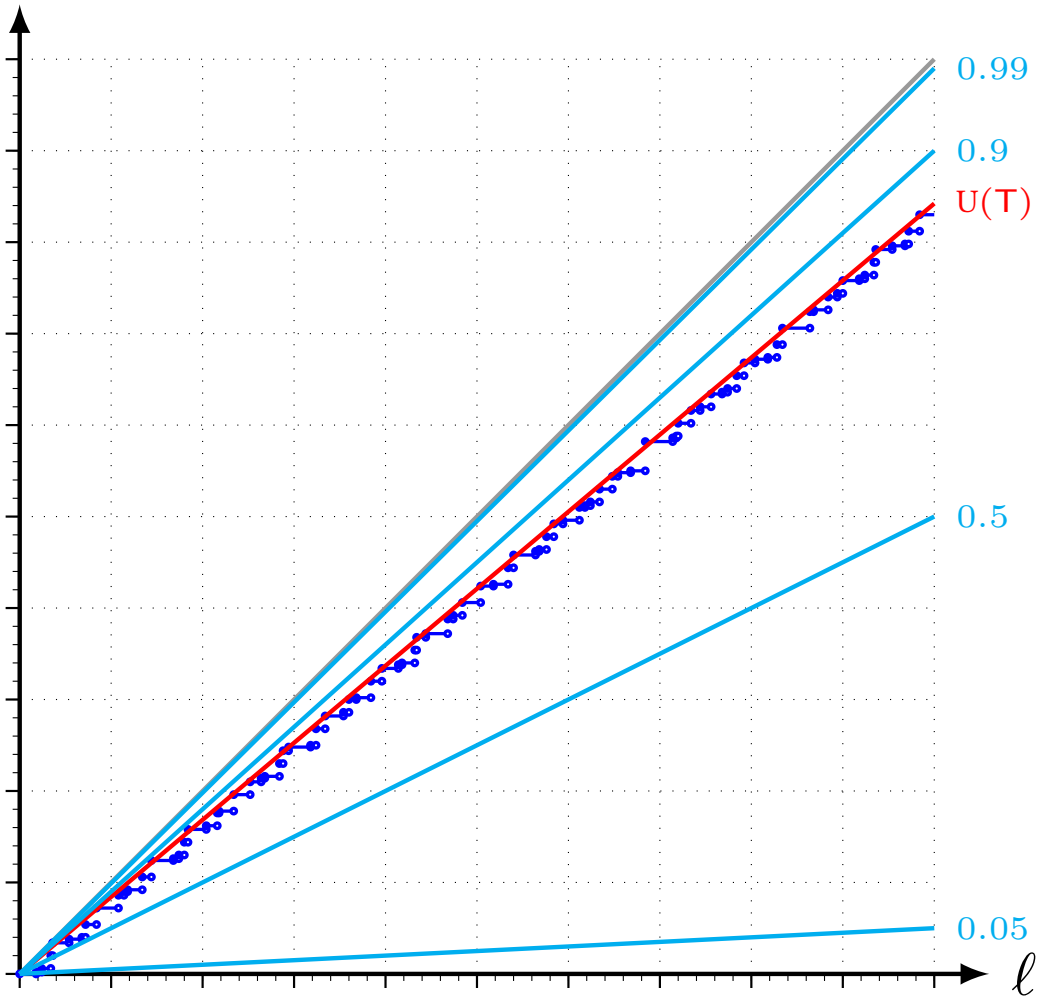

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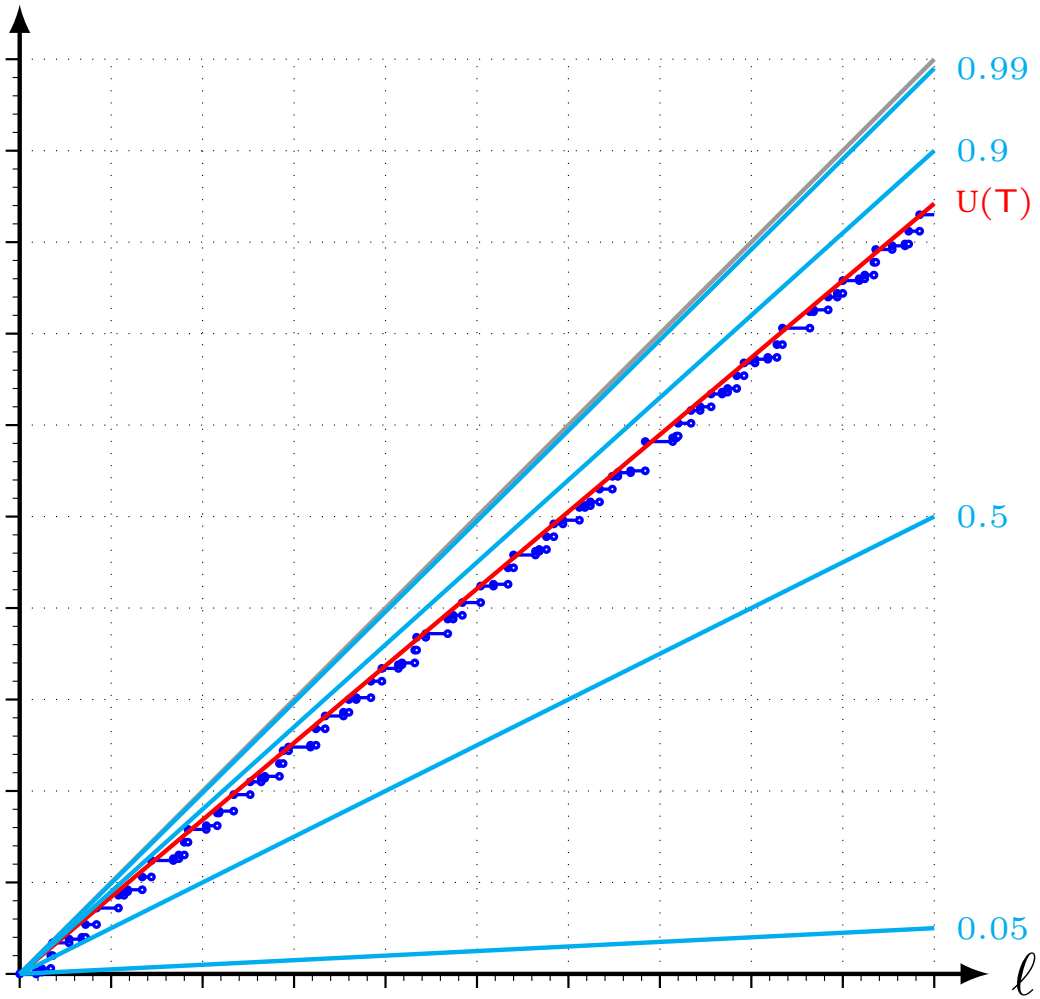

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- Pseudo-poly. time algorithm if $c < 1$
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- ~~Conjectured poly. time for all $c < 1$~~
- Weakly coNP-hard for all $c > 0$

Proving Hardness for c-Feasibility

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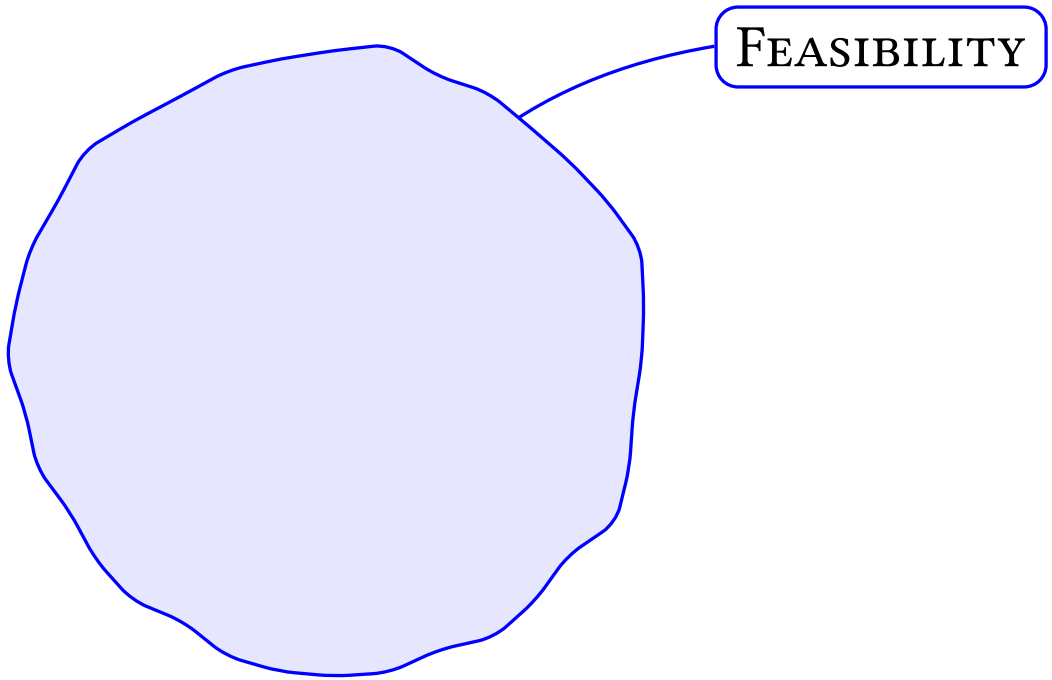

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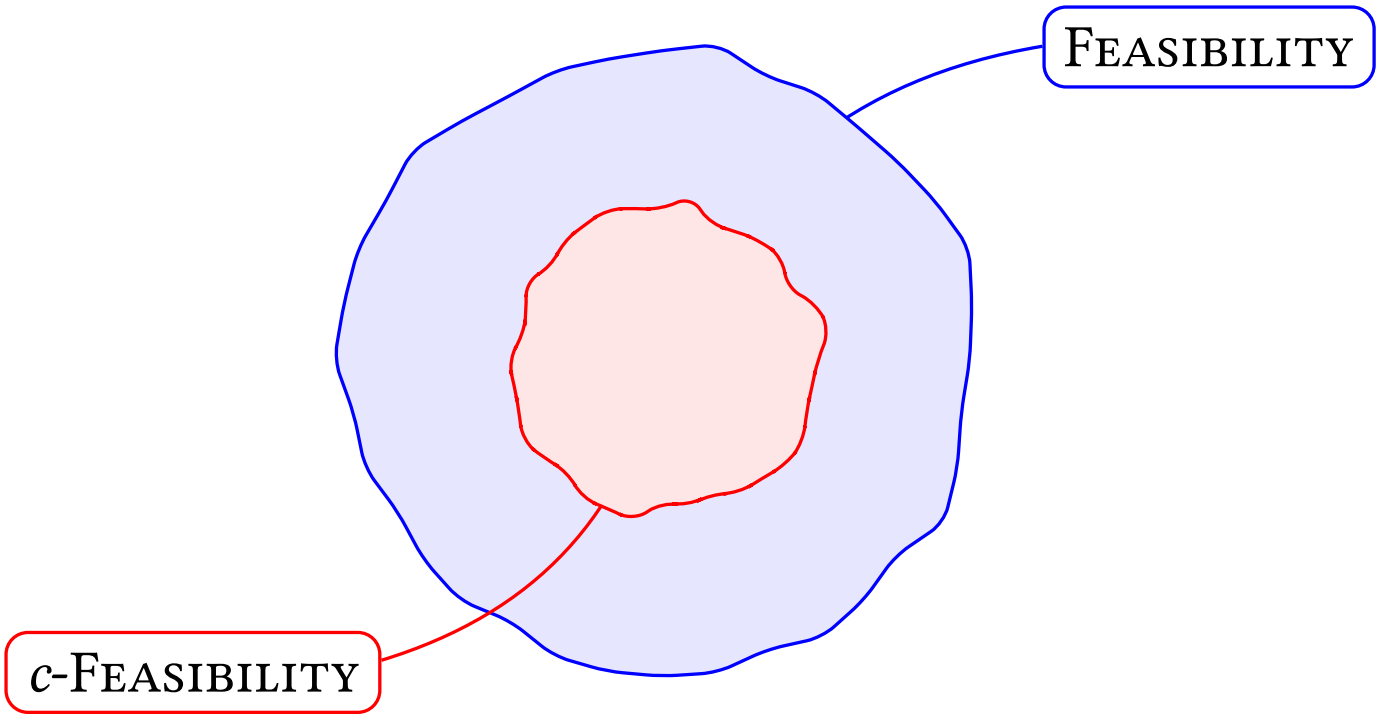

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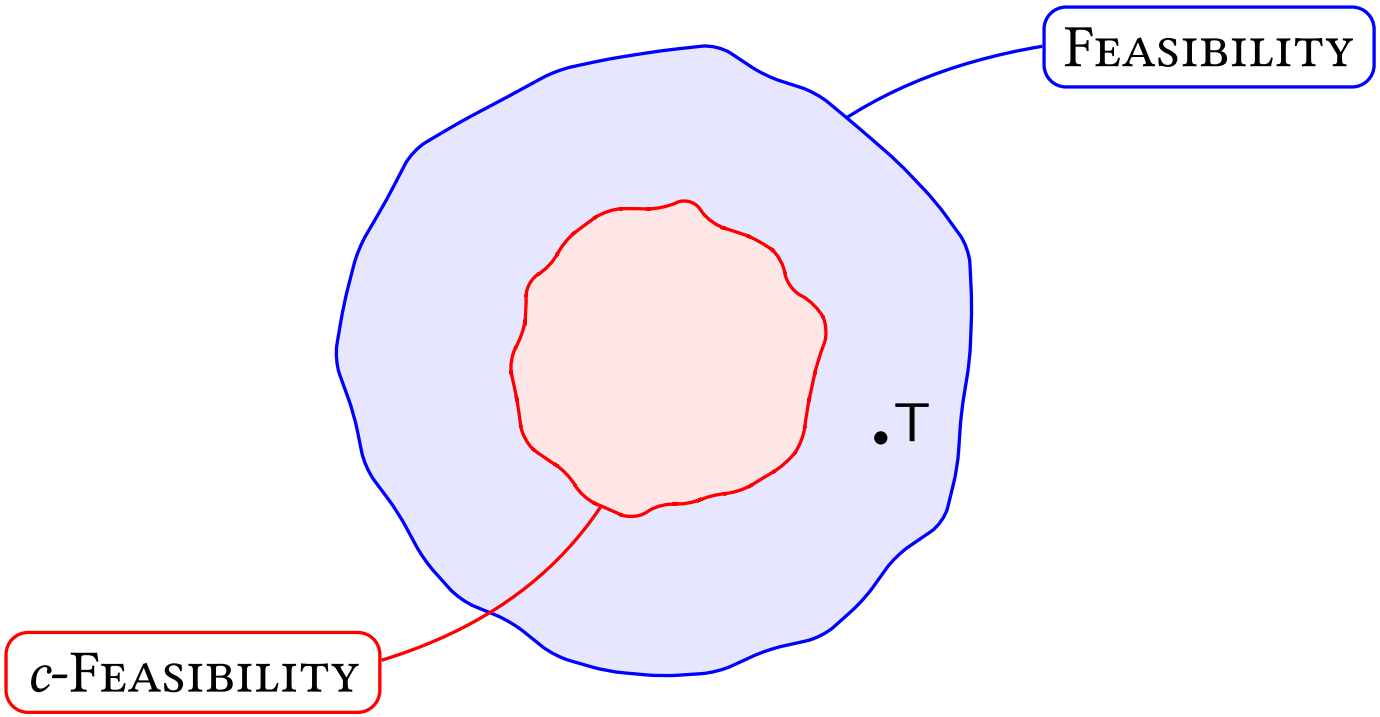

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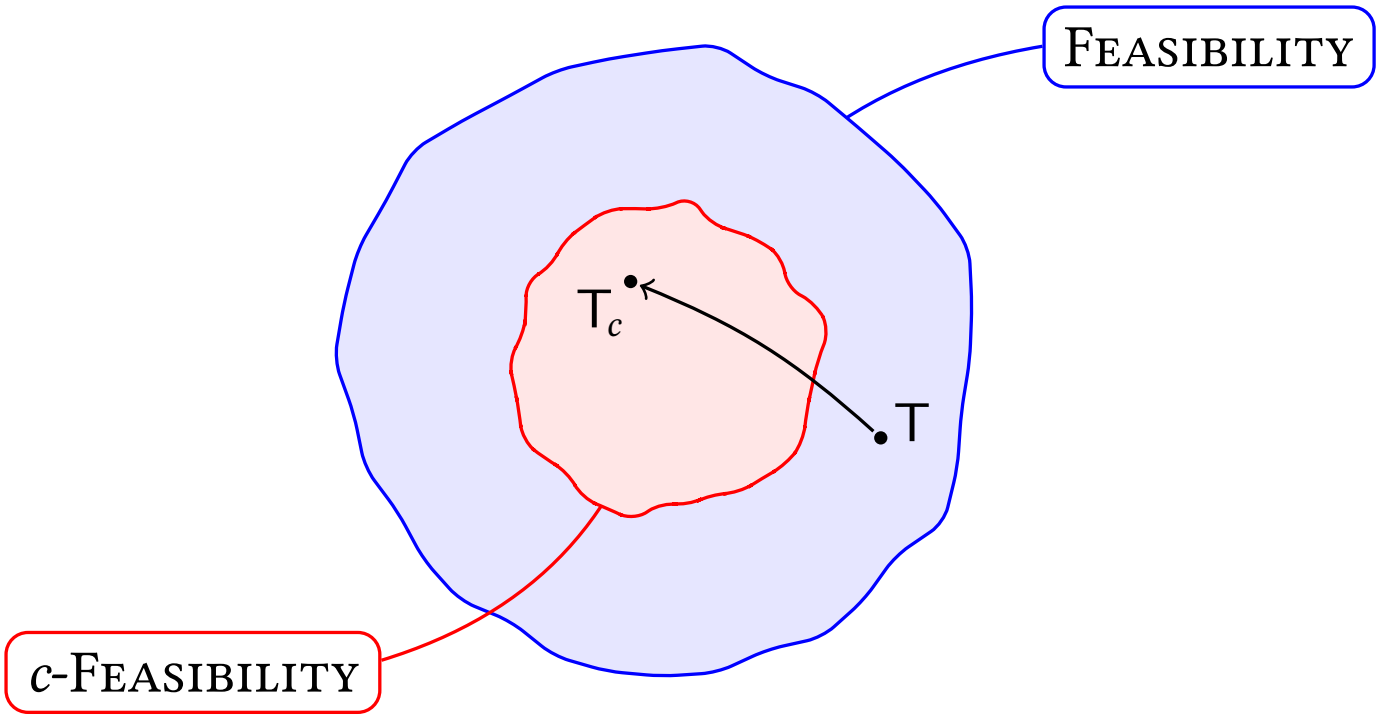

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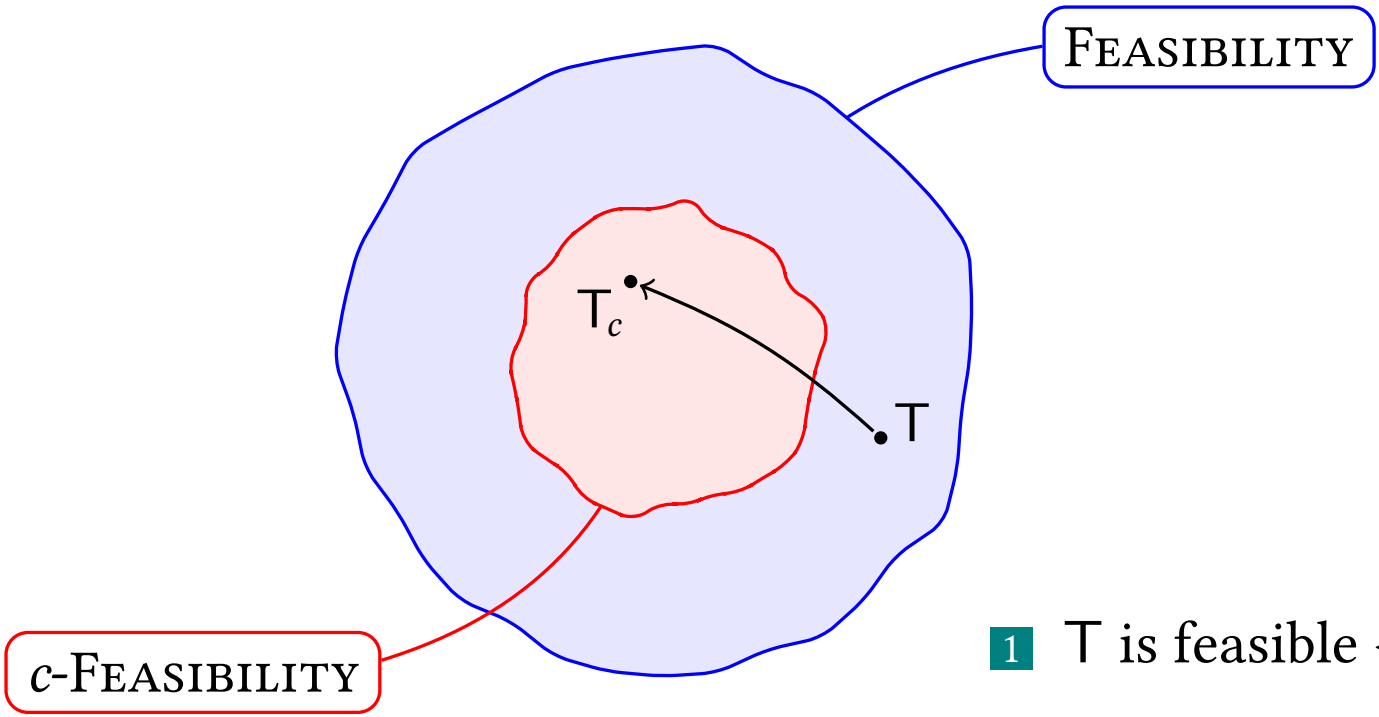
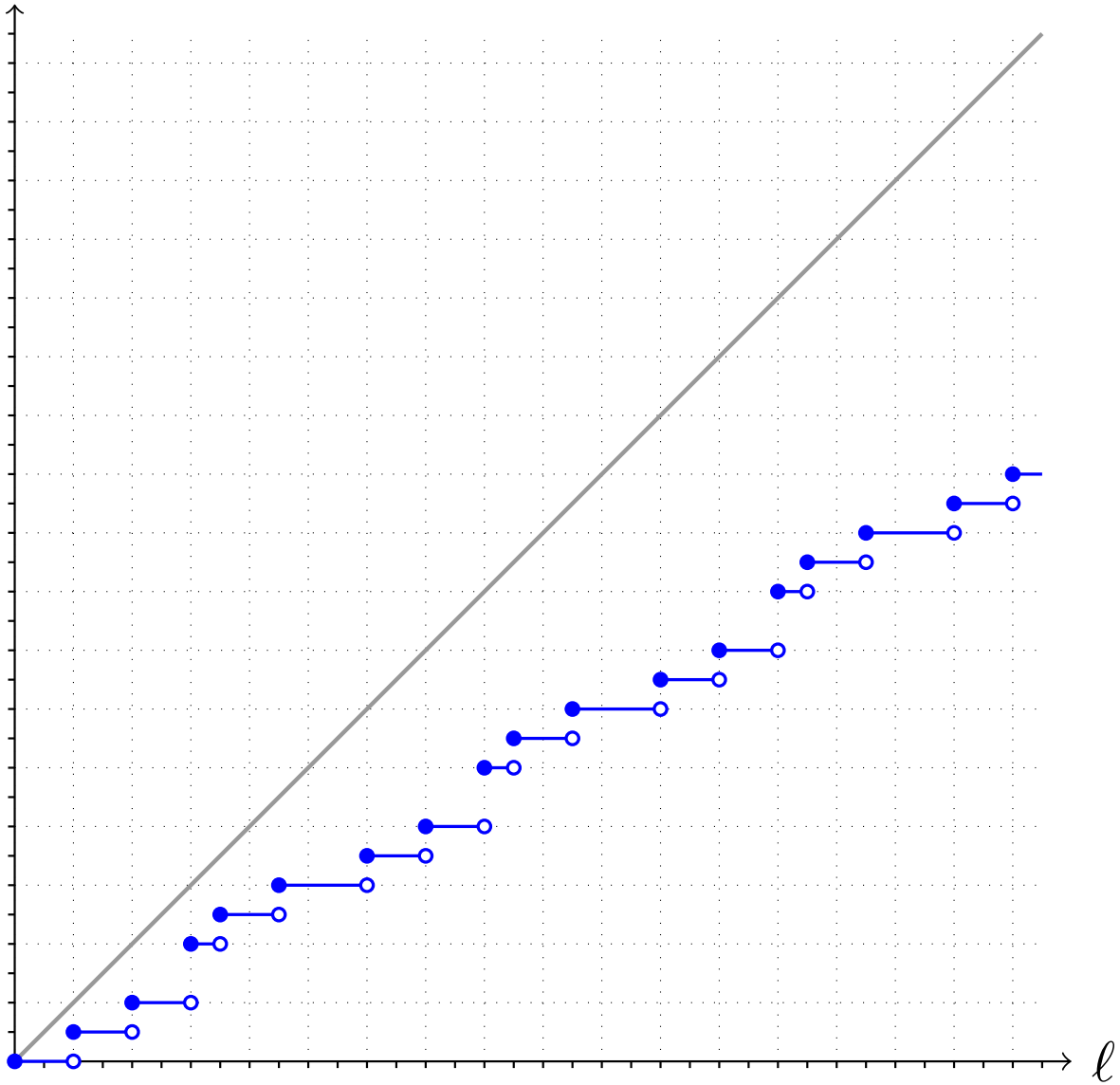


1. T is feasible $\Leftrightarrow$ T_c is feasible
2. $U(T_c) \leqslant c$
3. T_c is computed in poly. time

Feasibility $\propto$ c-Feasibility, Step 1



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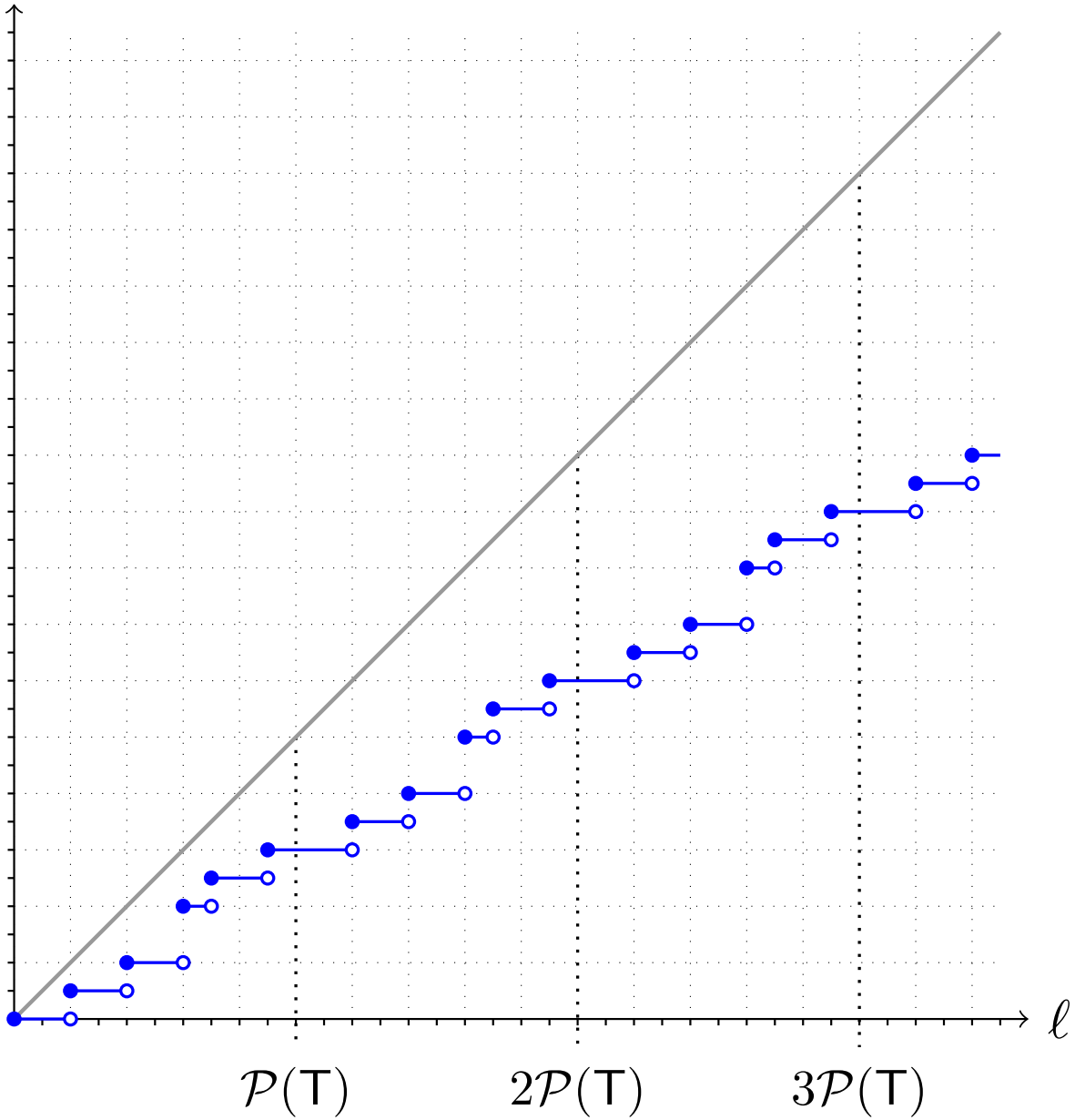

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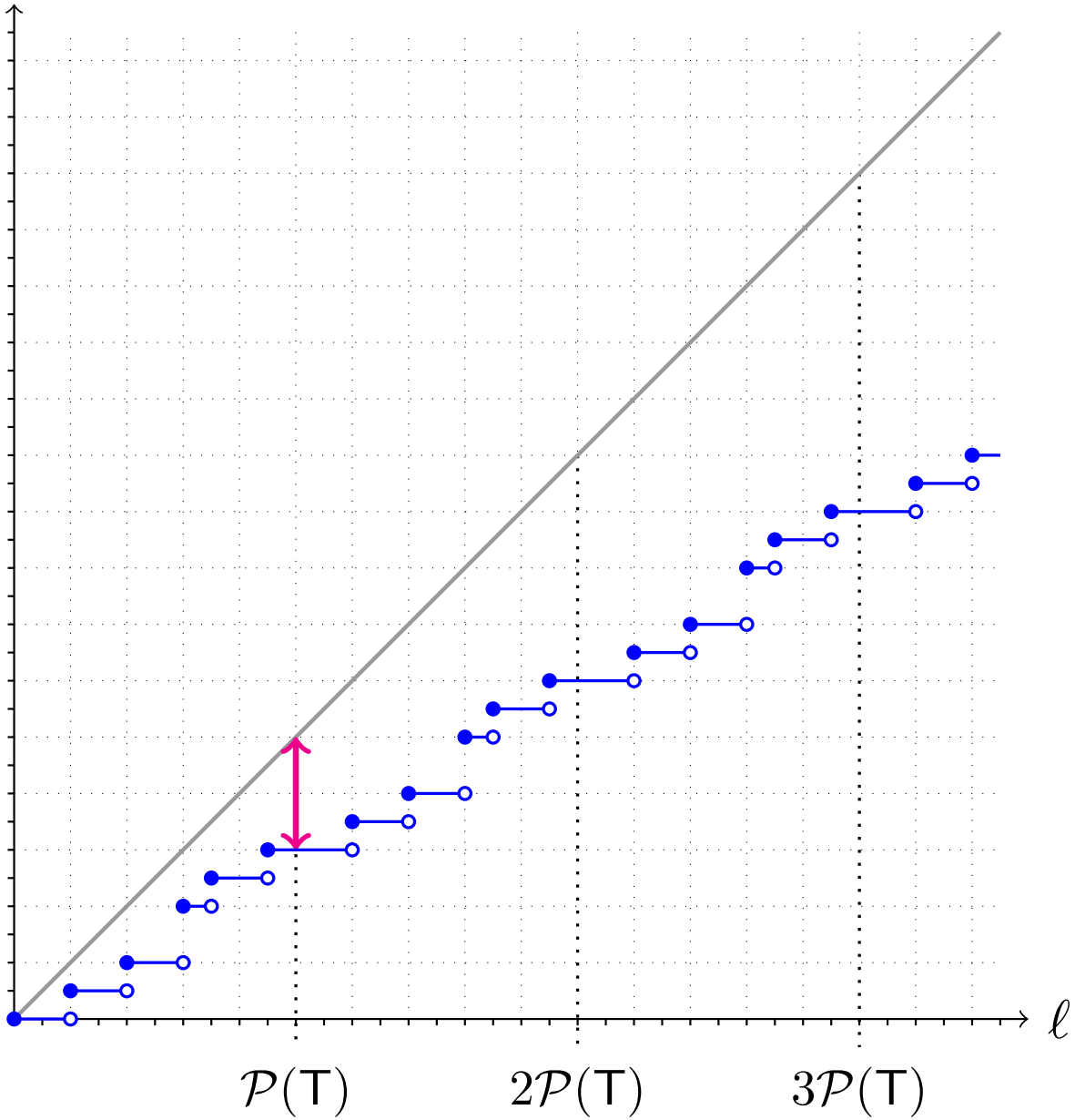

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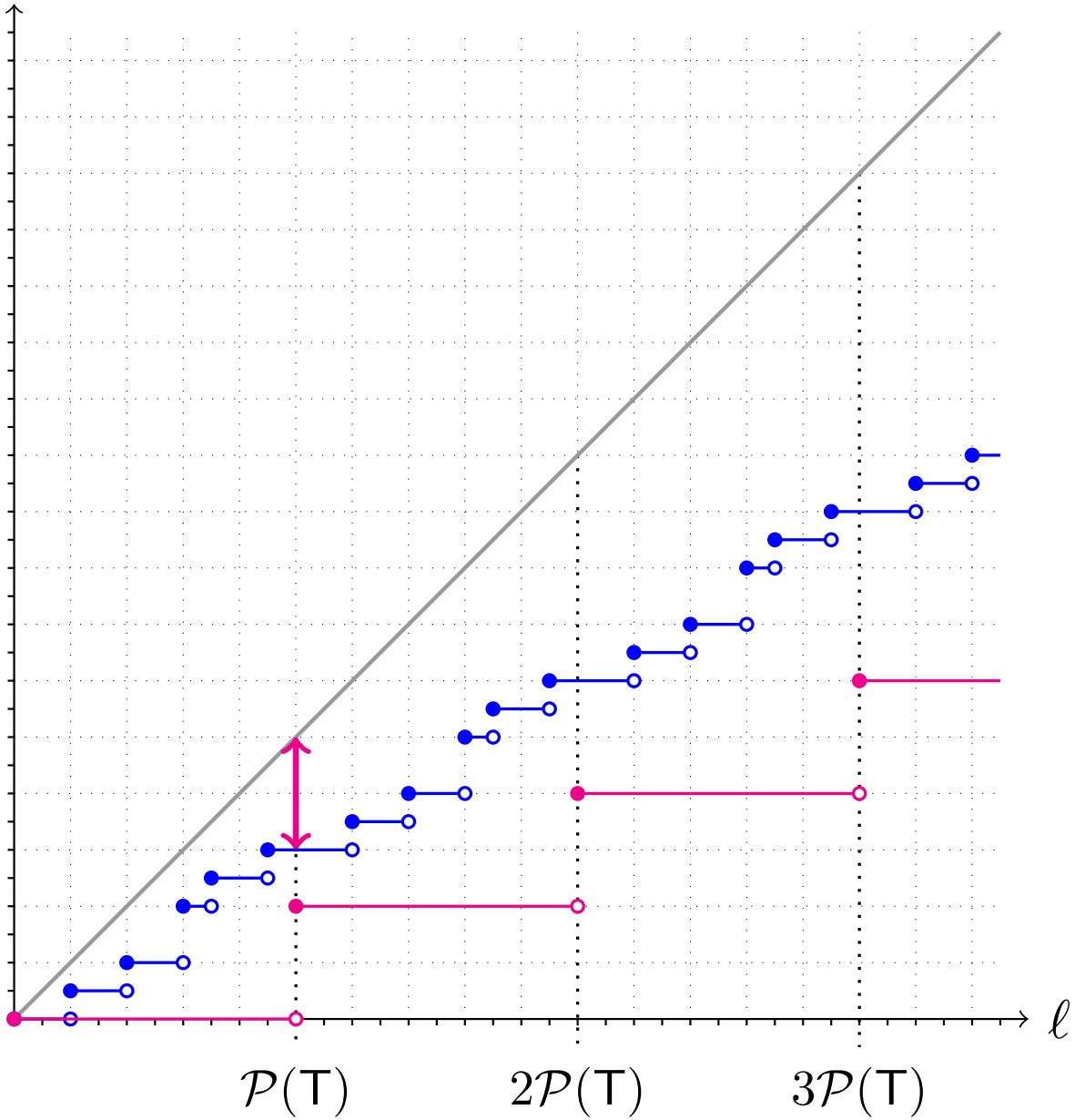

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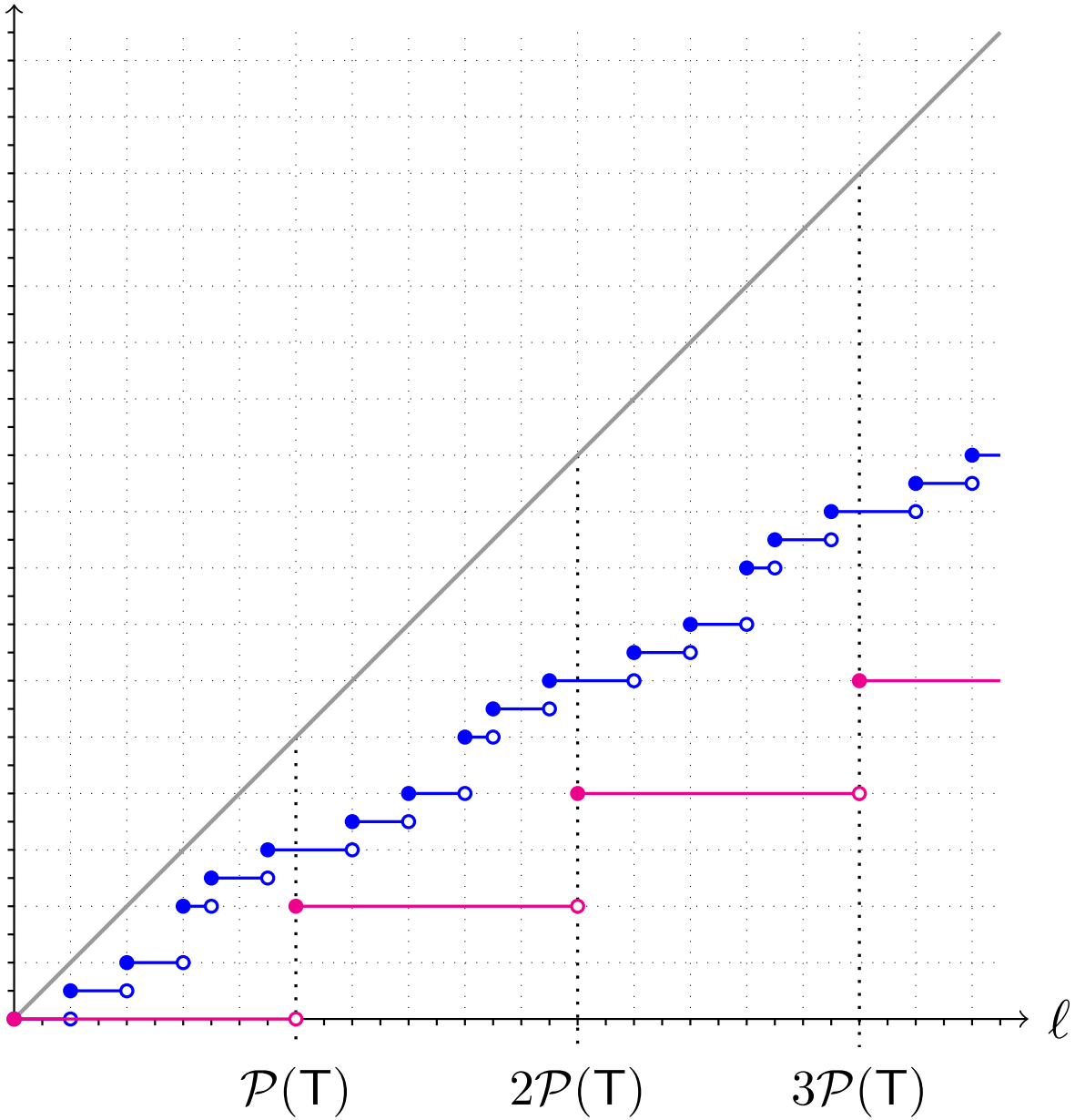

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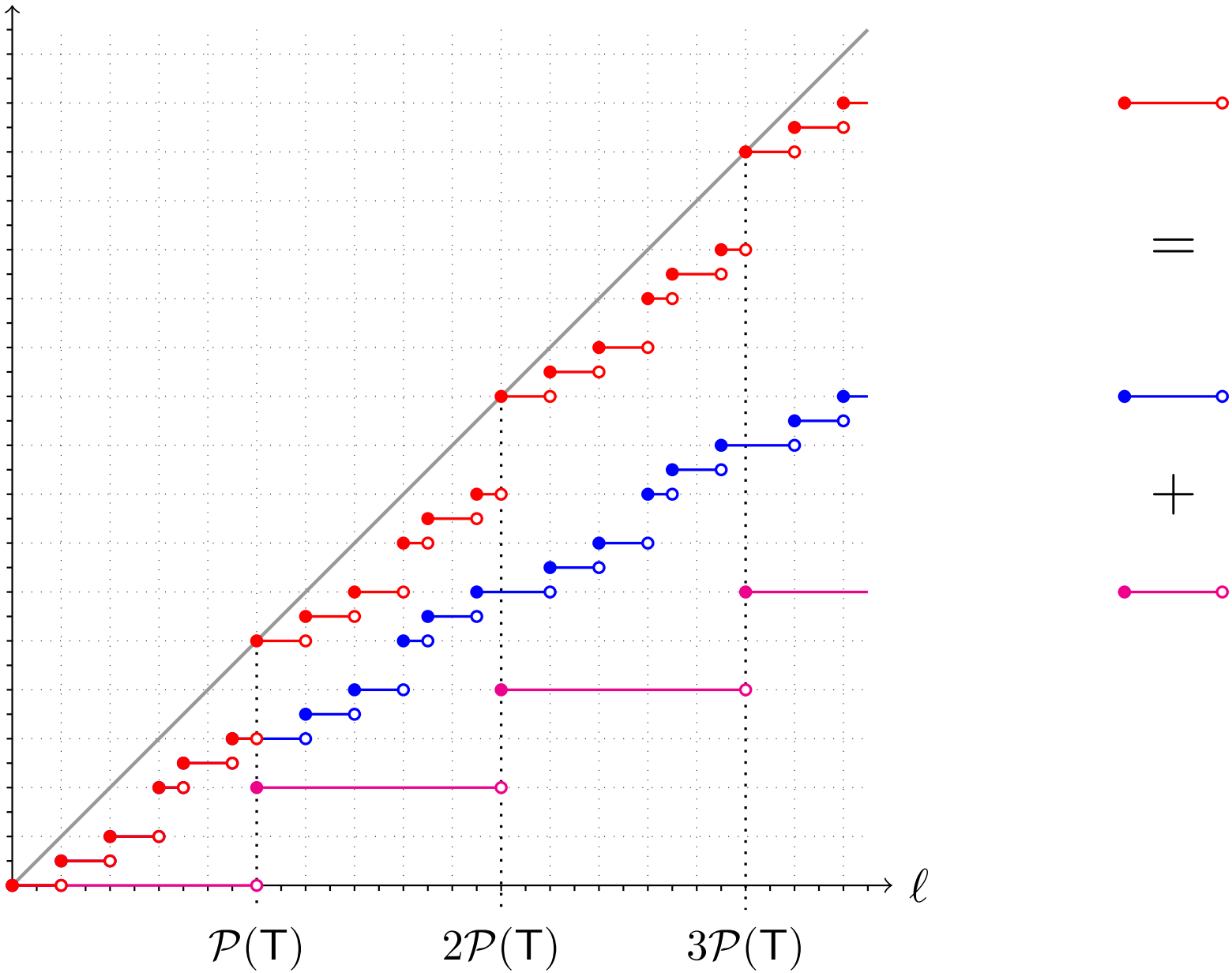

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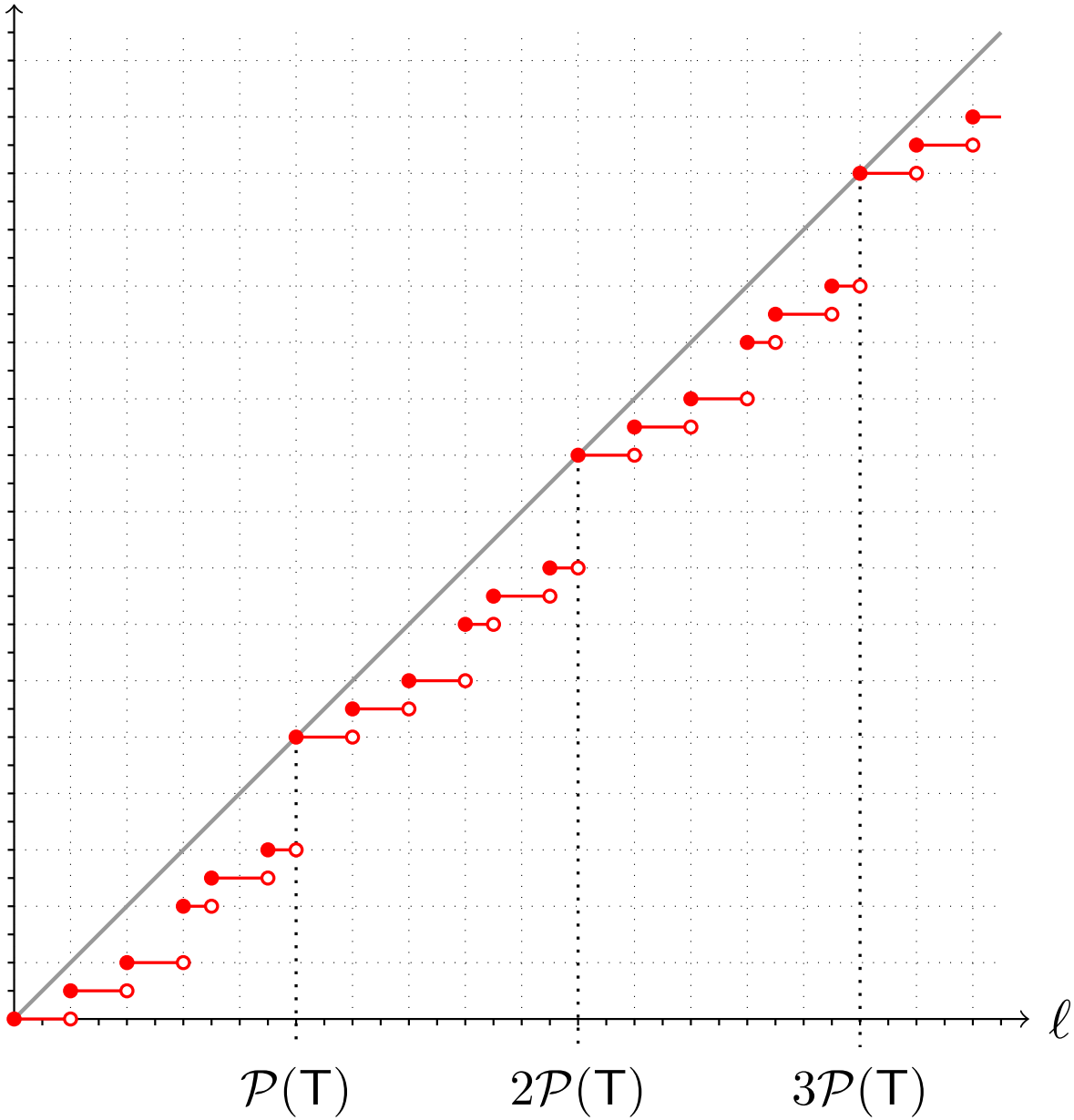

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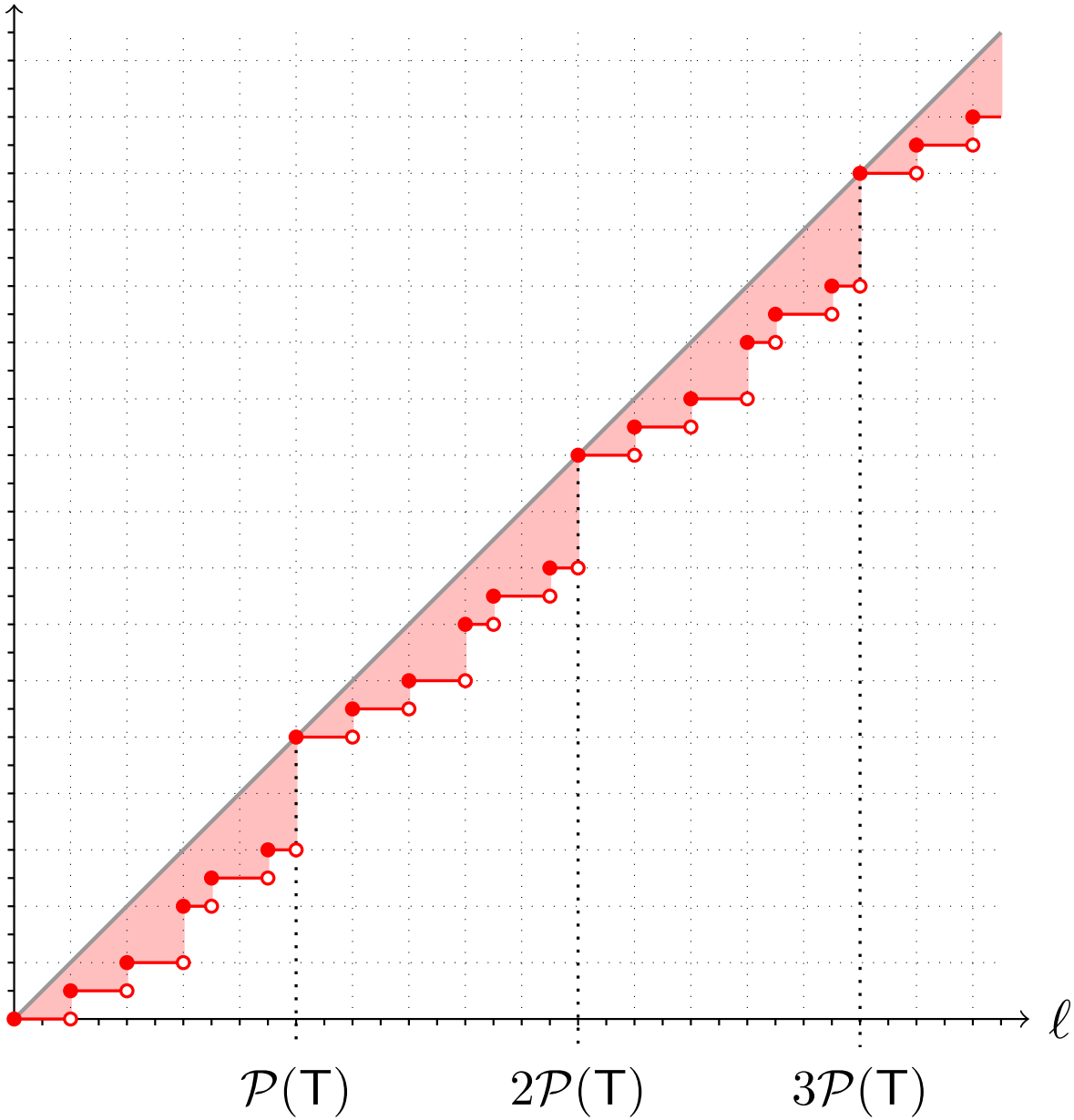

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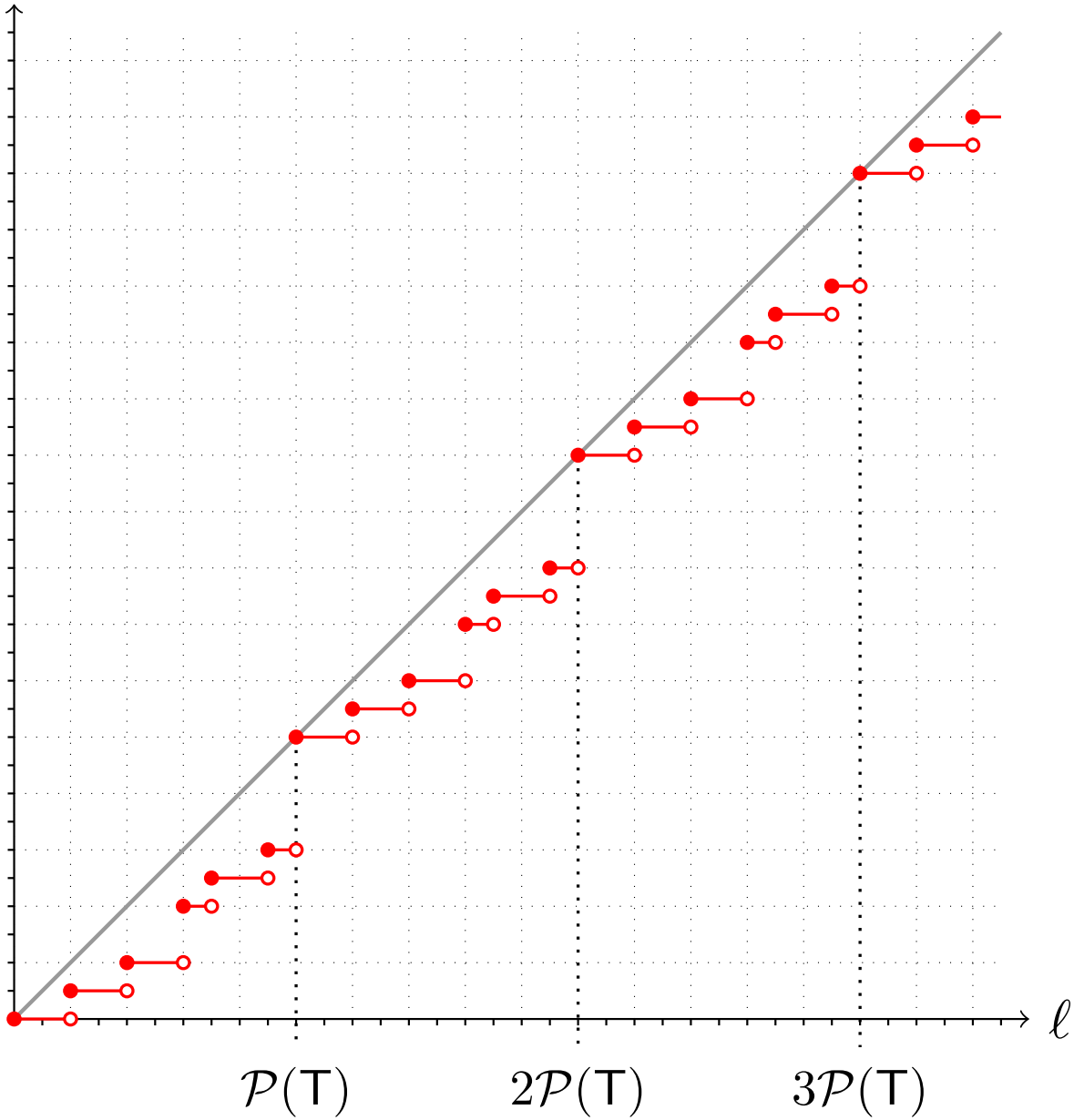

Feasibility preserved

Utilization $\leqslant c$

Computed in poly. time

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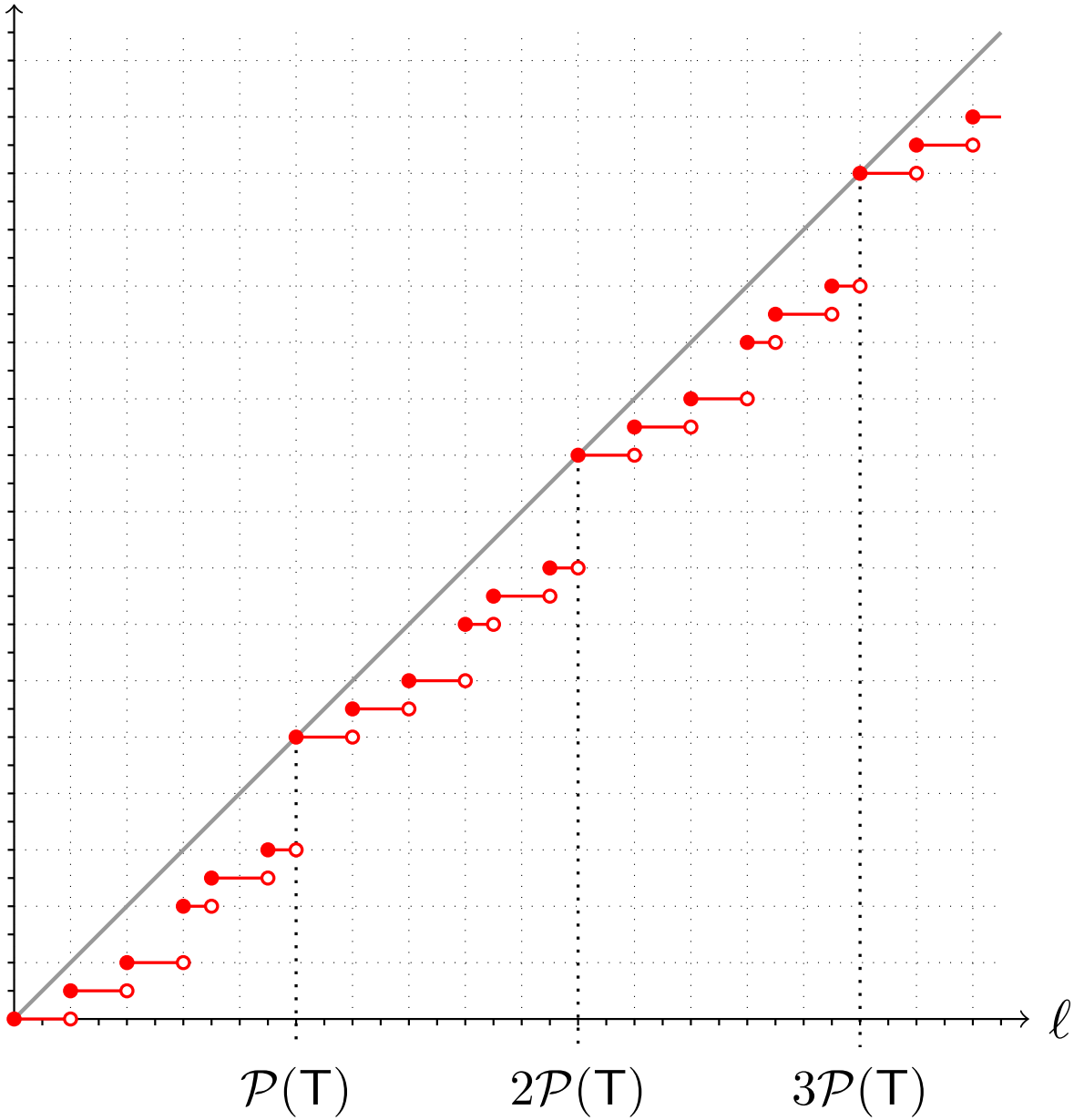

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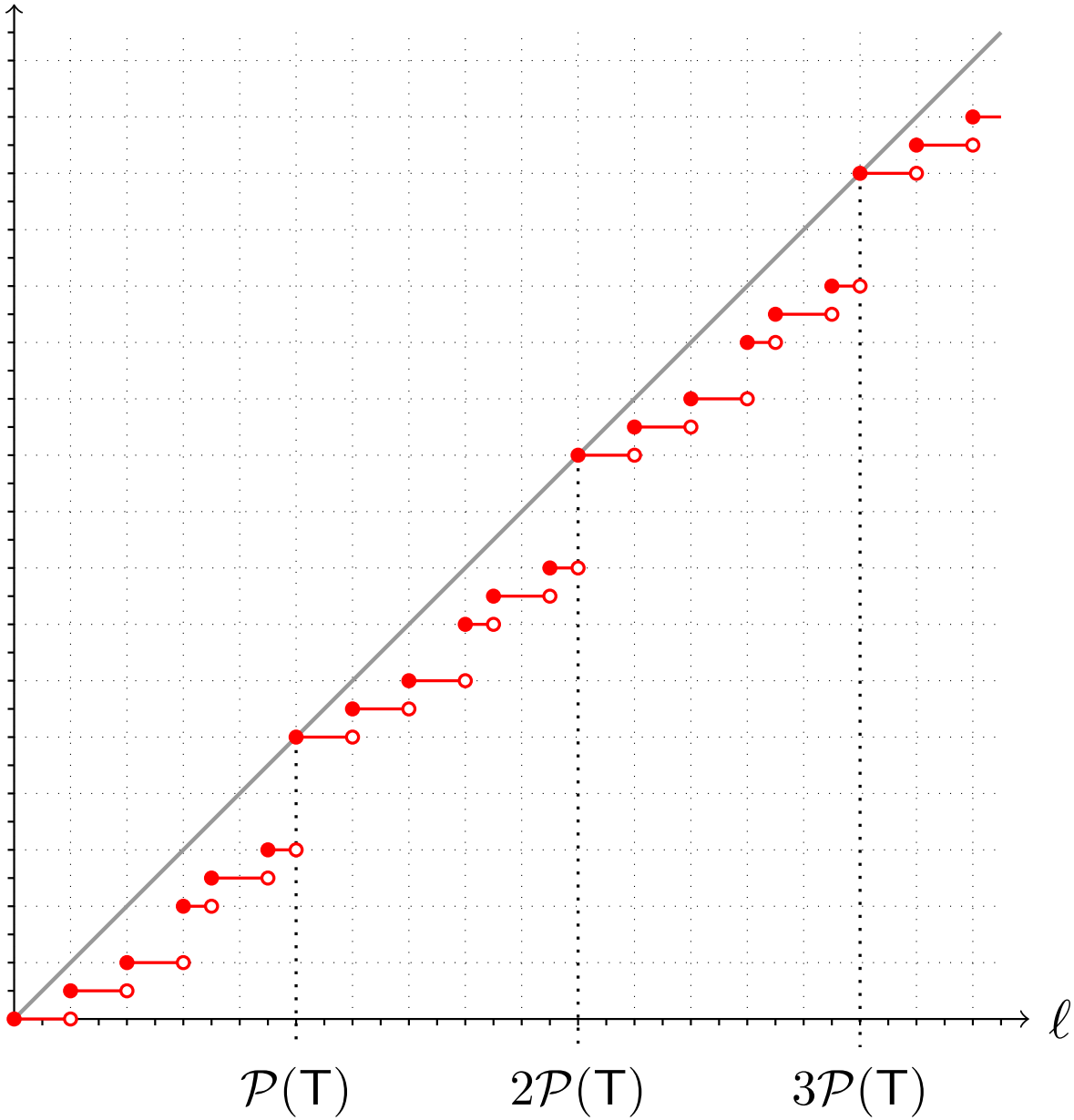

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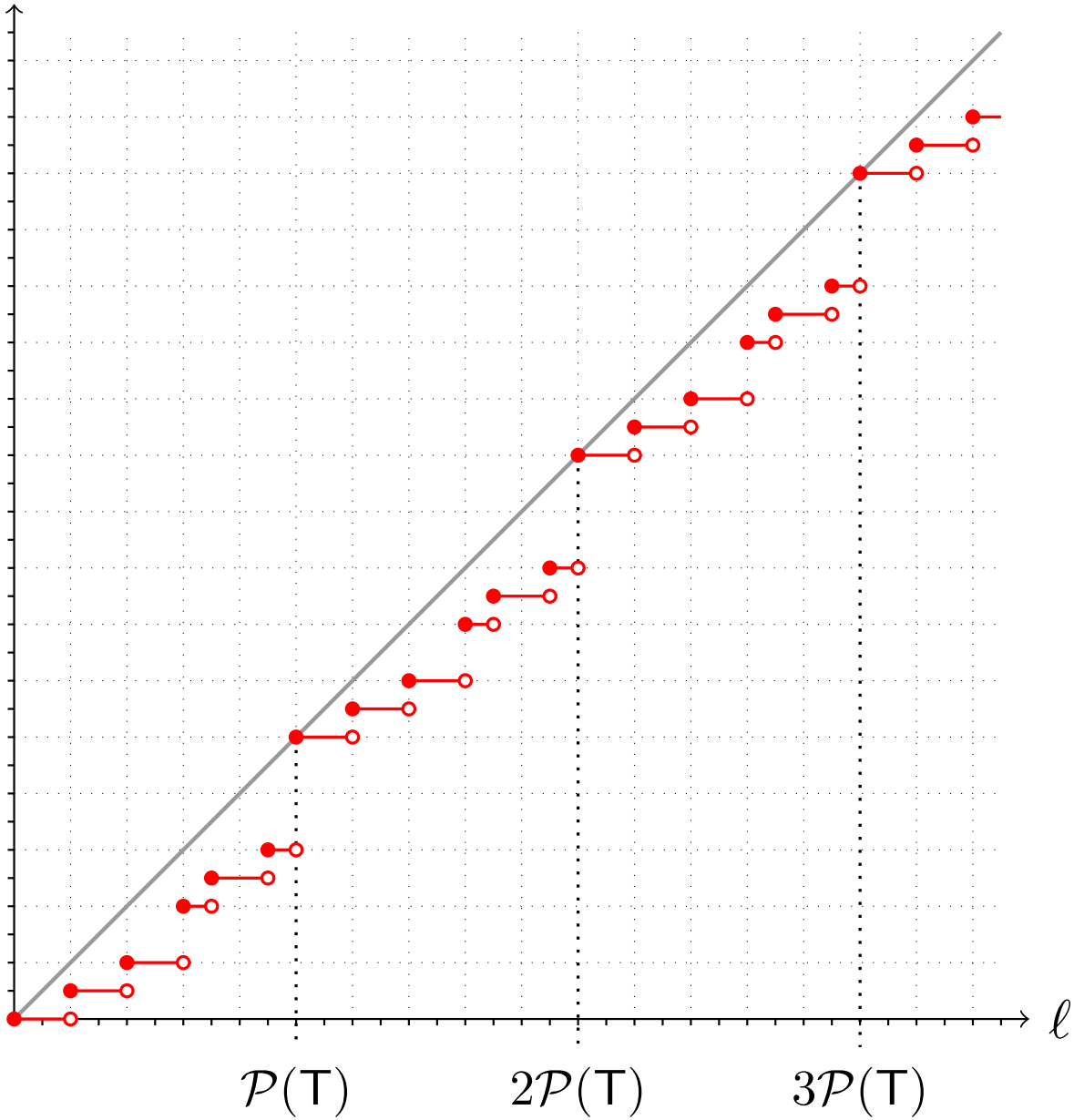

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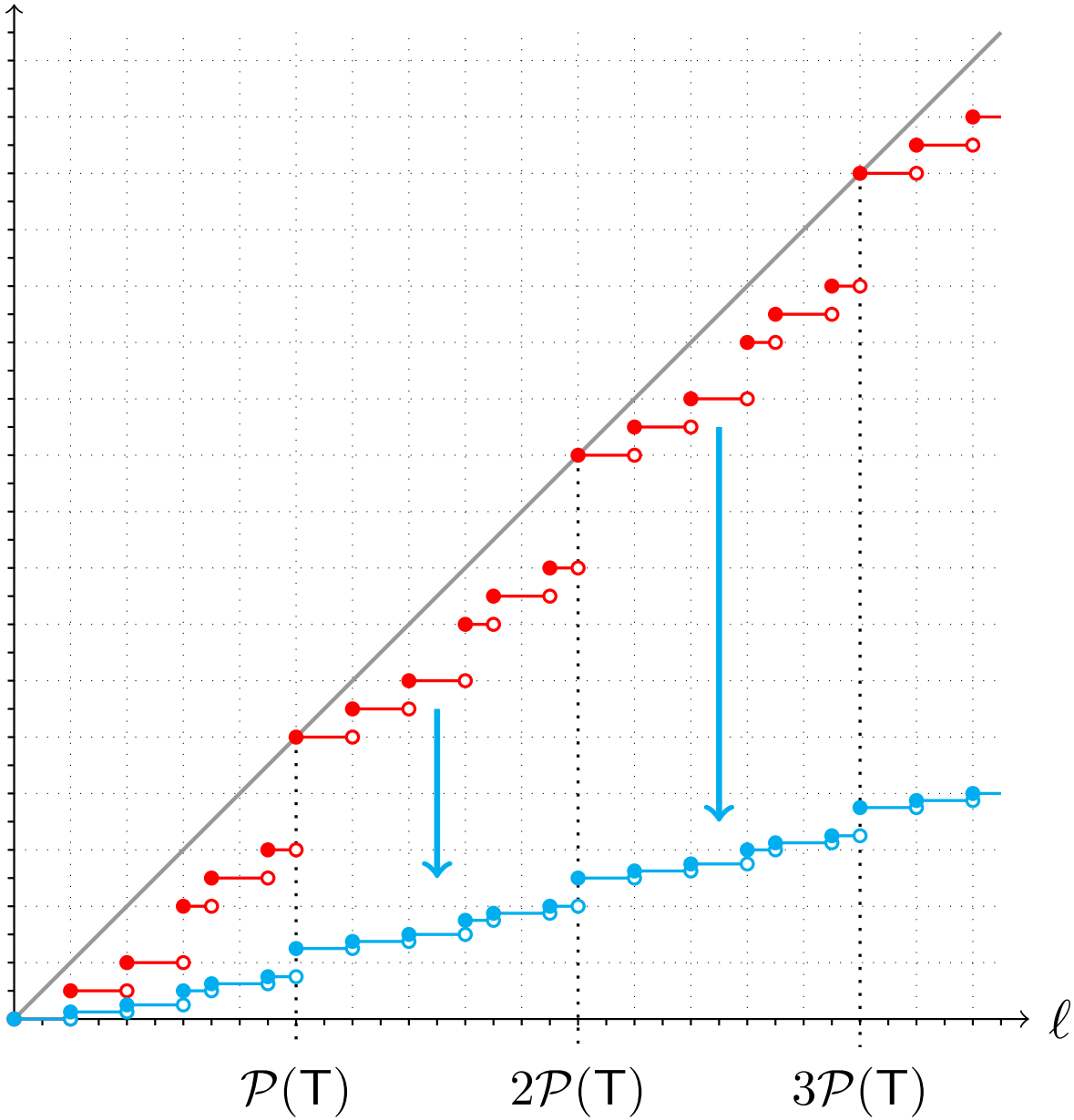
Feasibility $\propto$ c-Feasibility, Step 2



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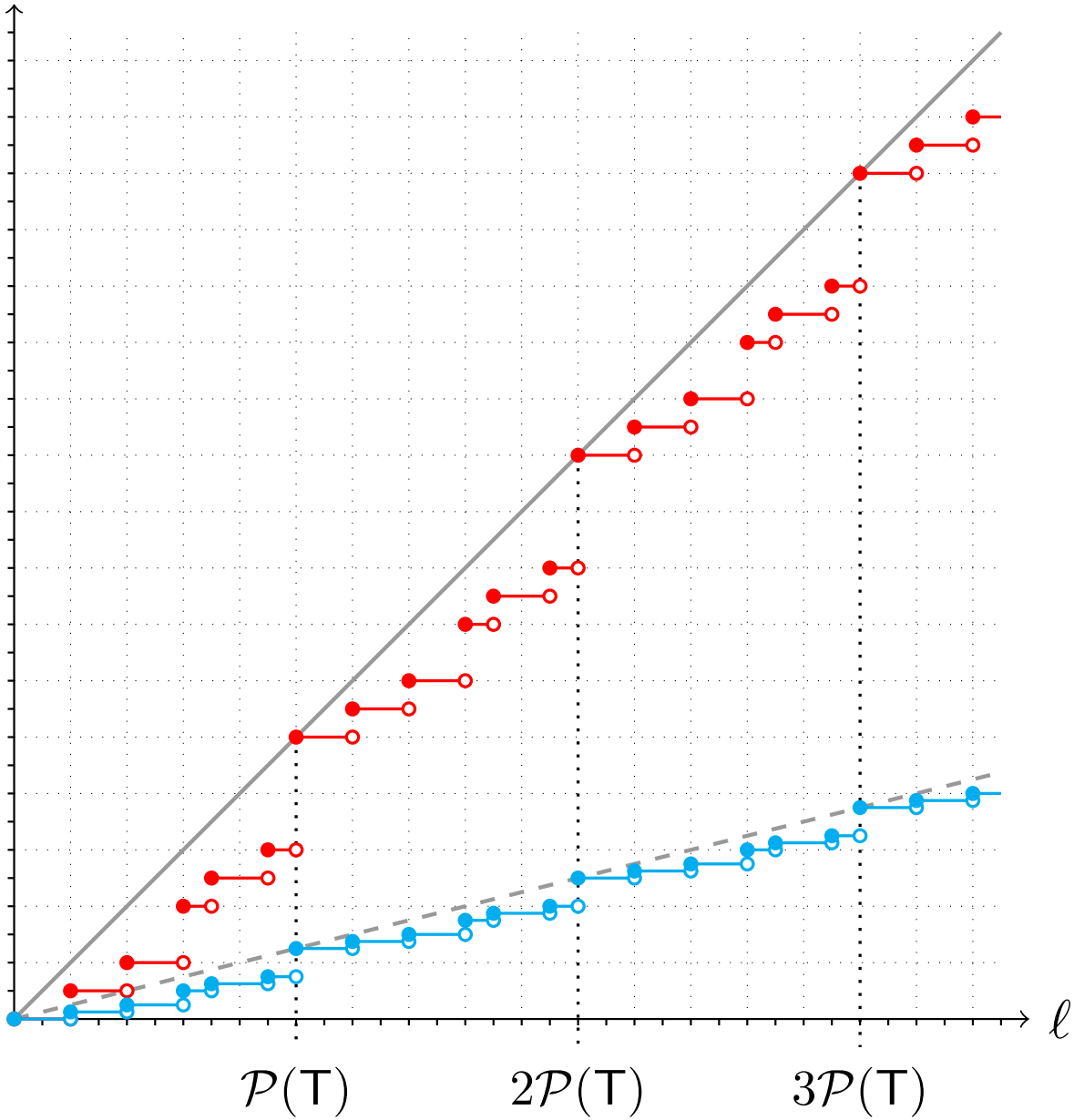

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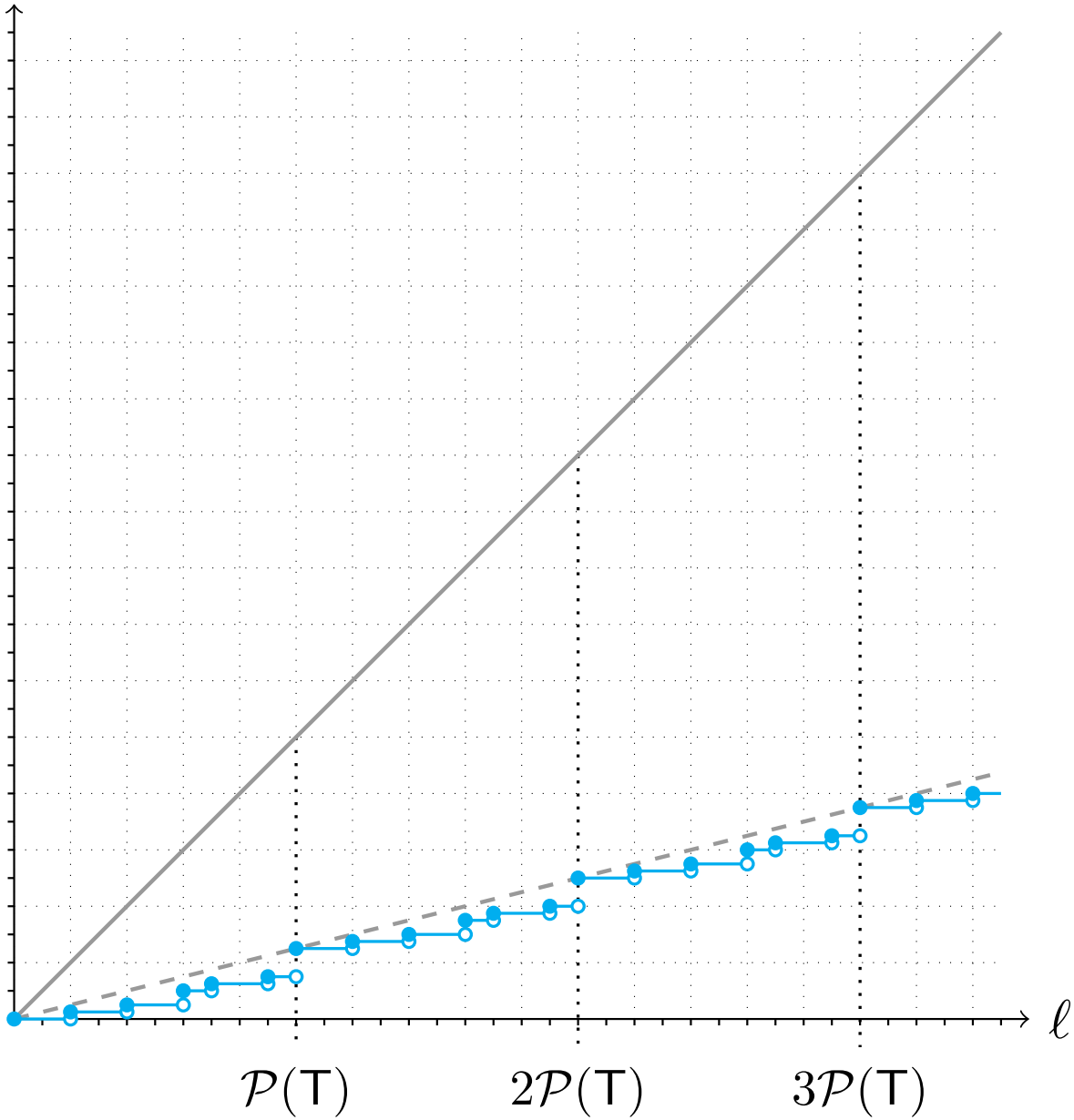

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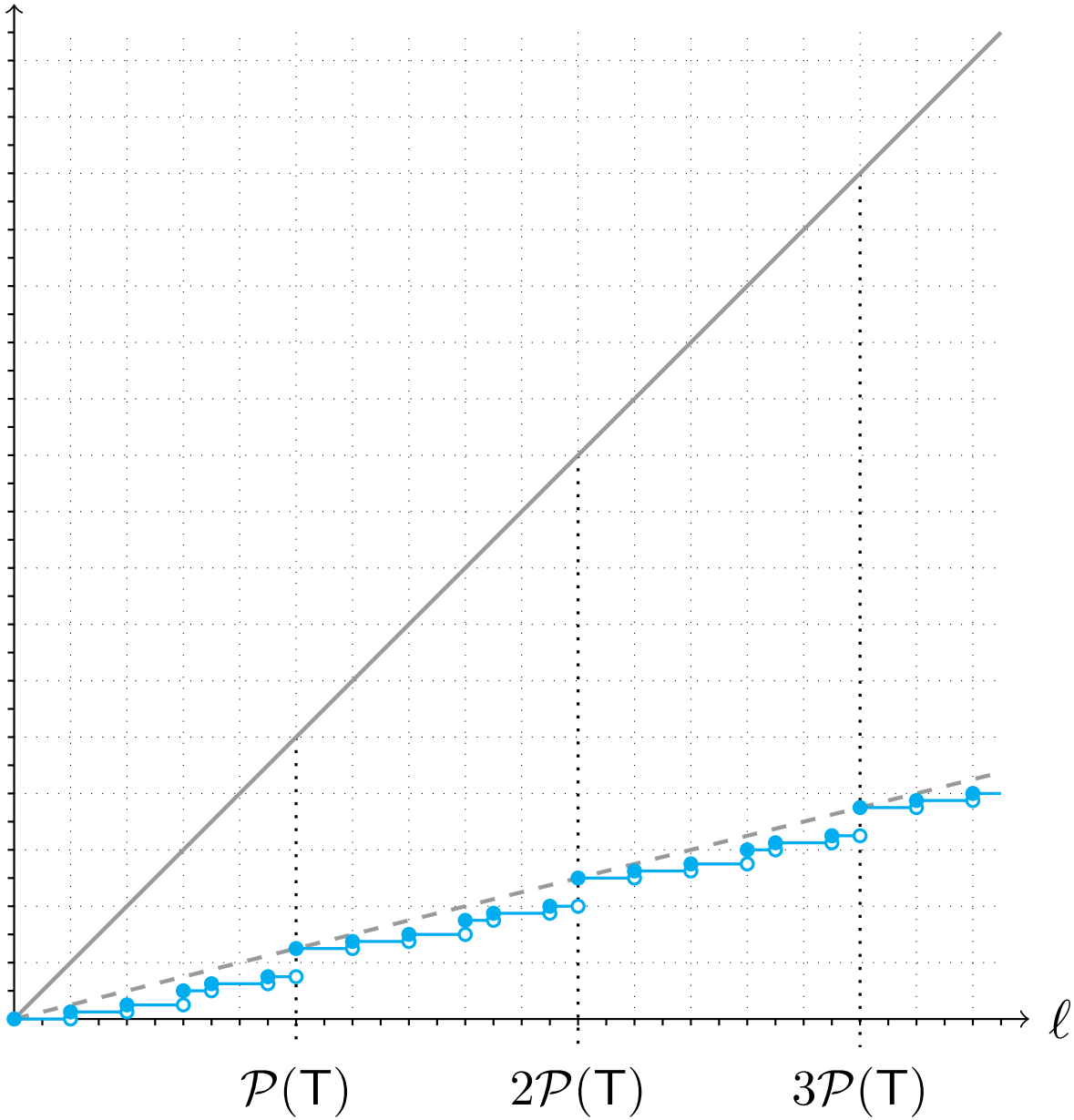

Feasibility preserved ✗

Utilization $\leqslant c$

Computed in poly. time

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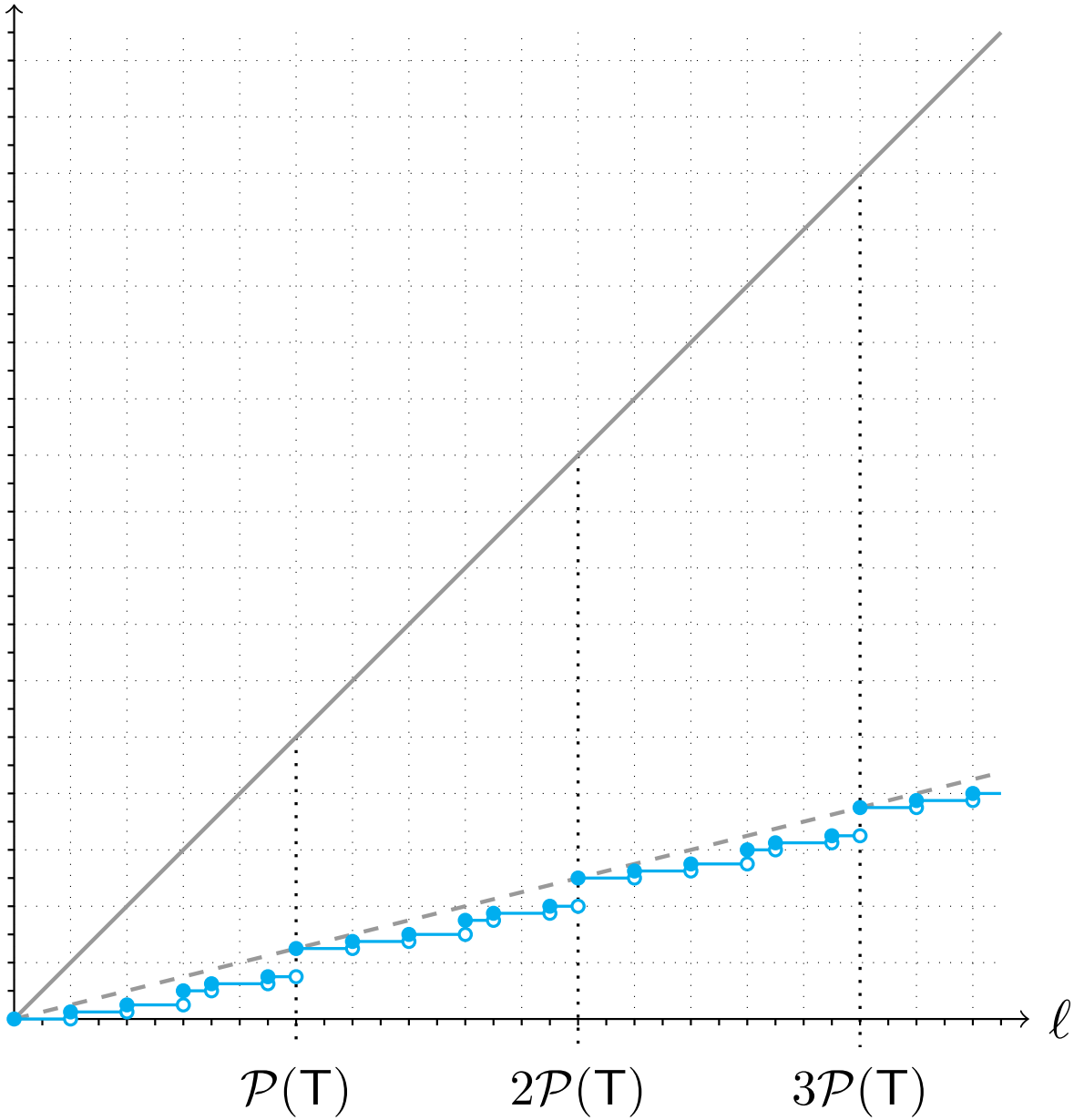

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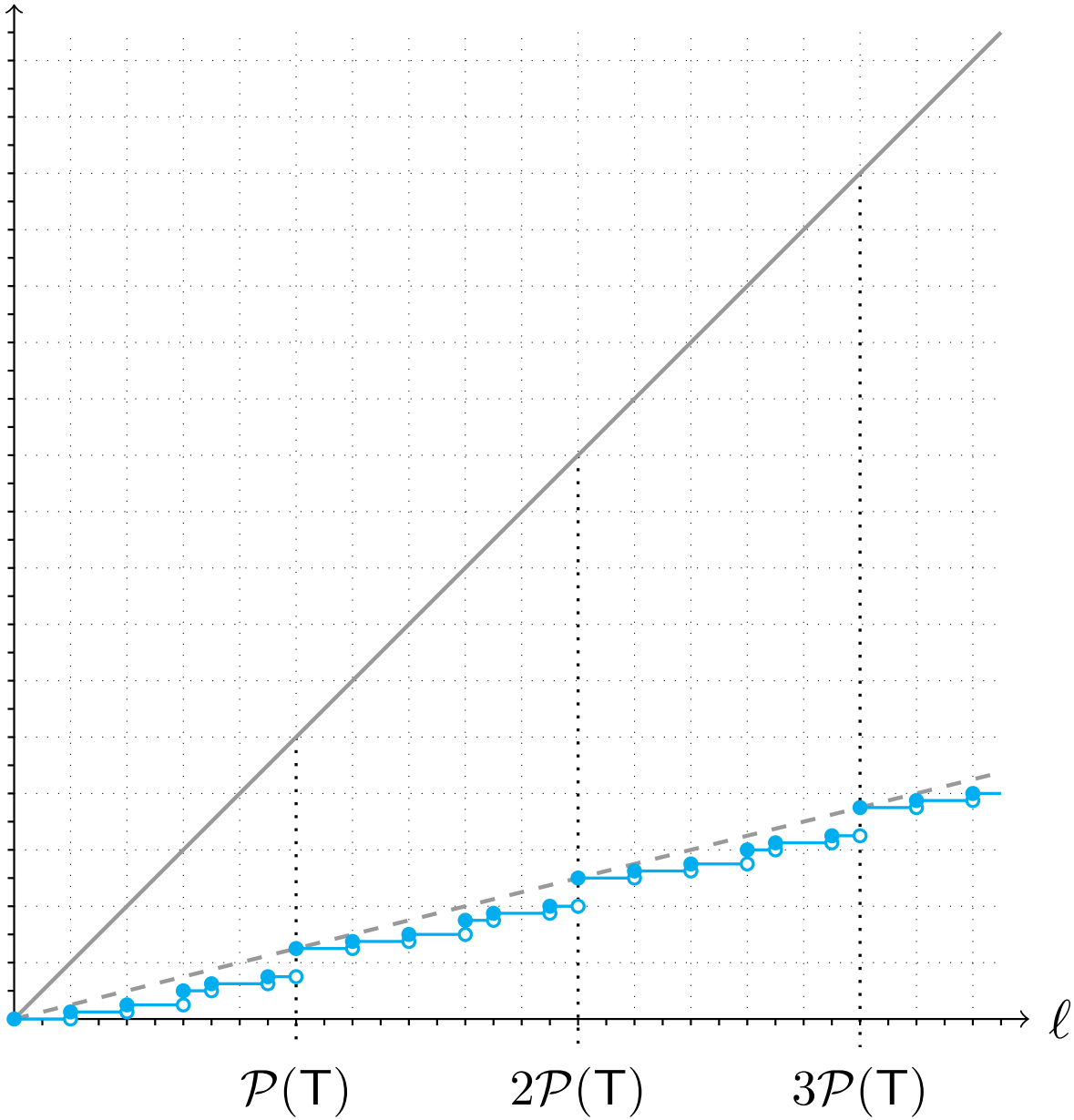

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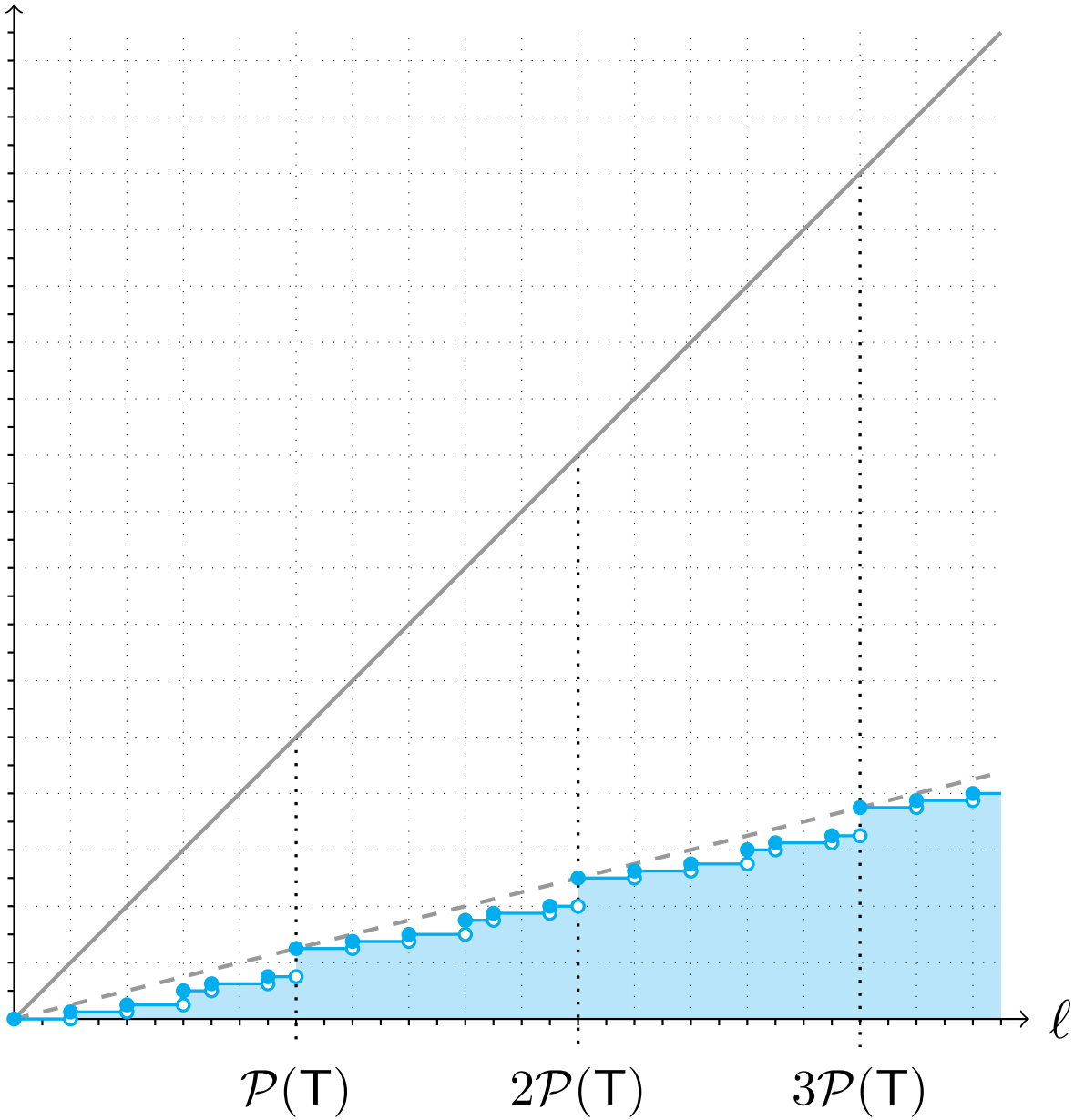

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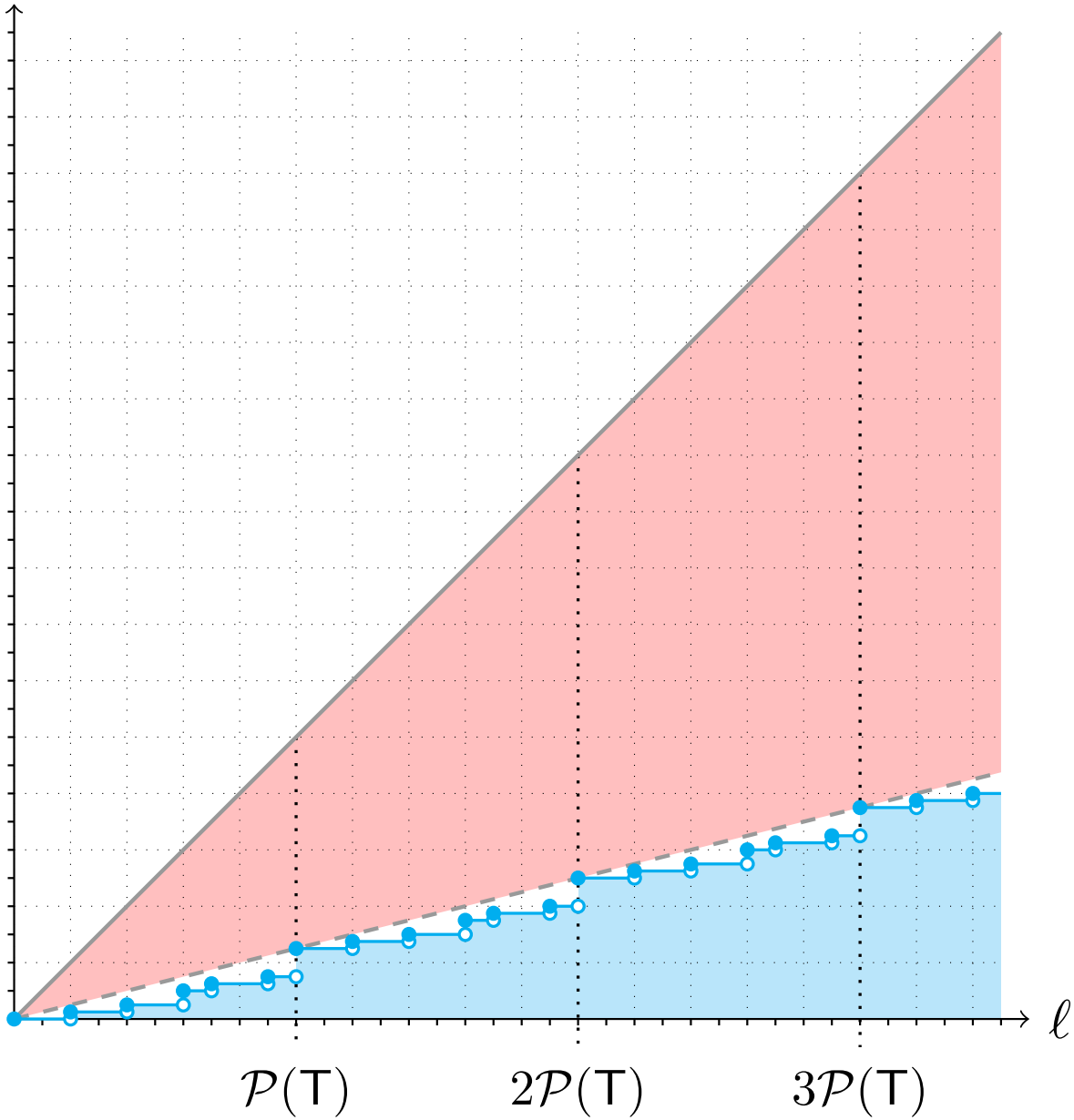

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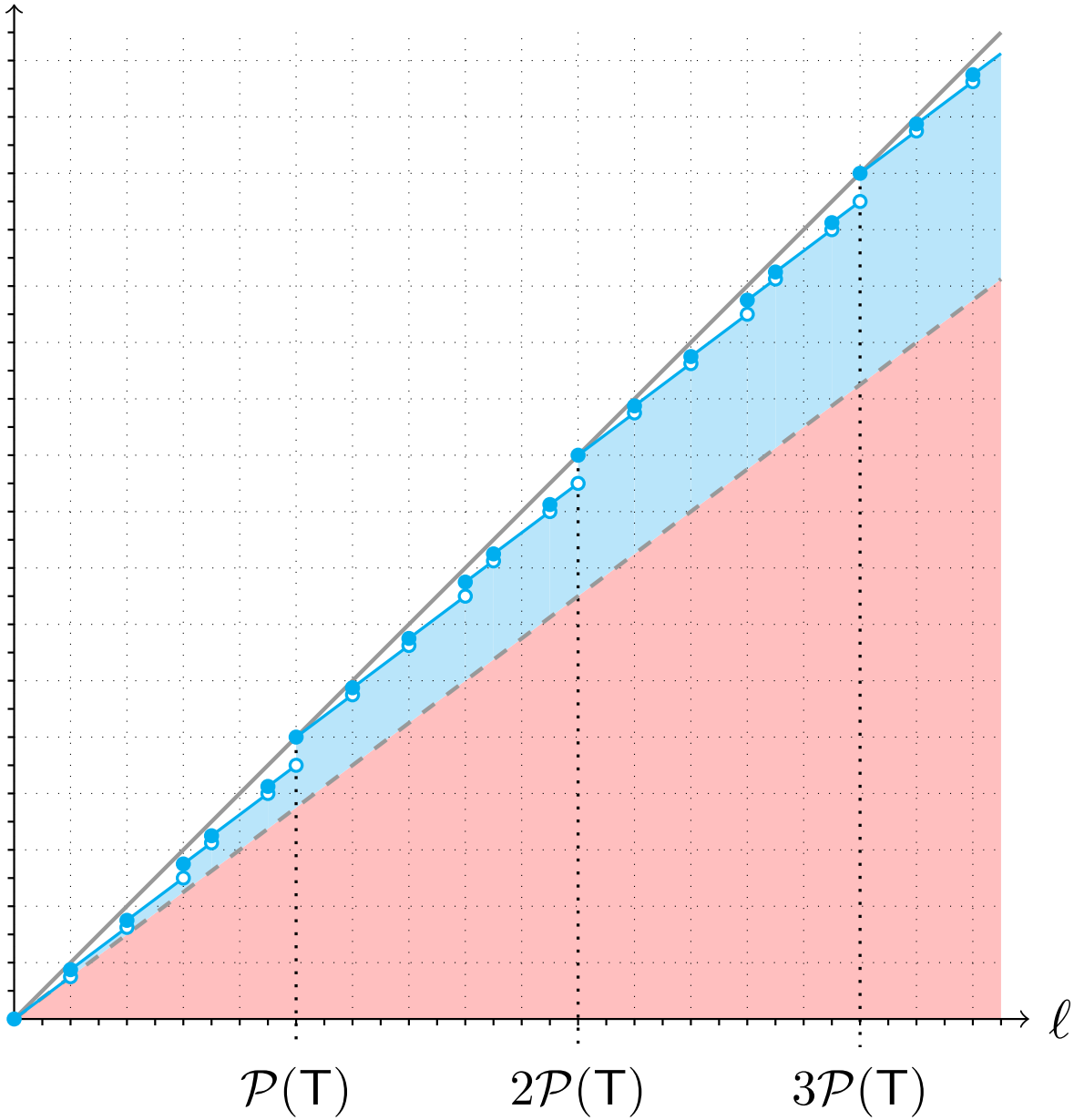

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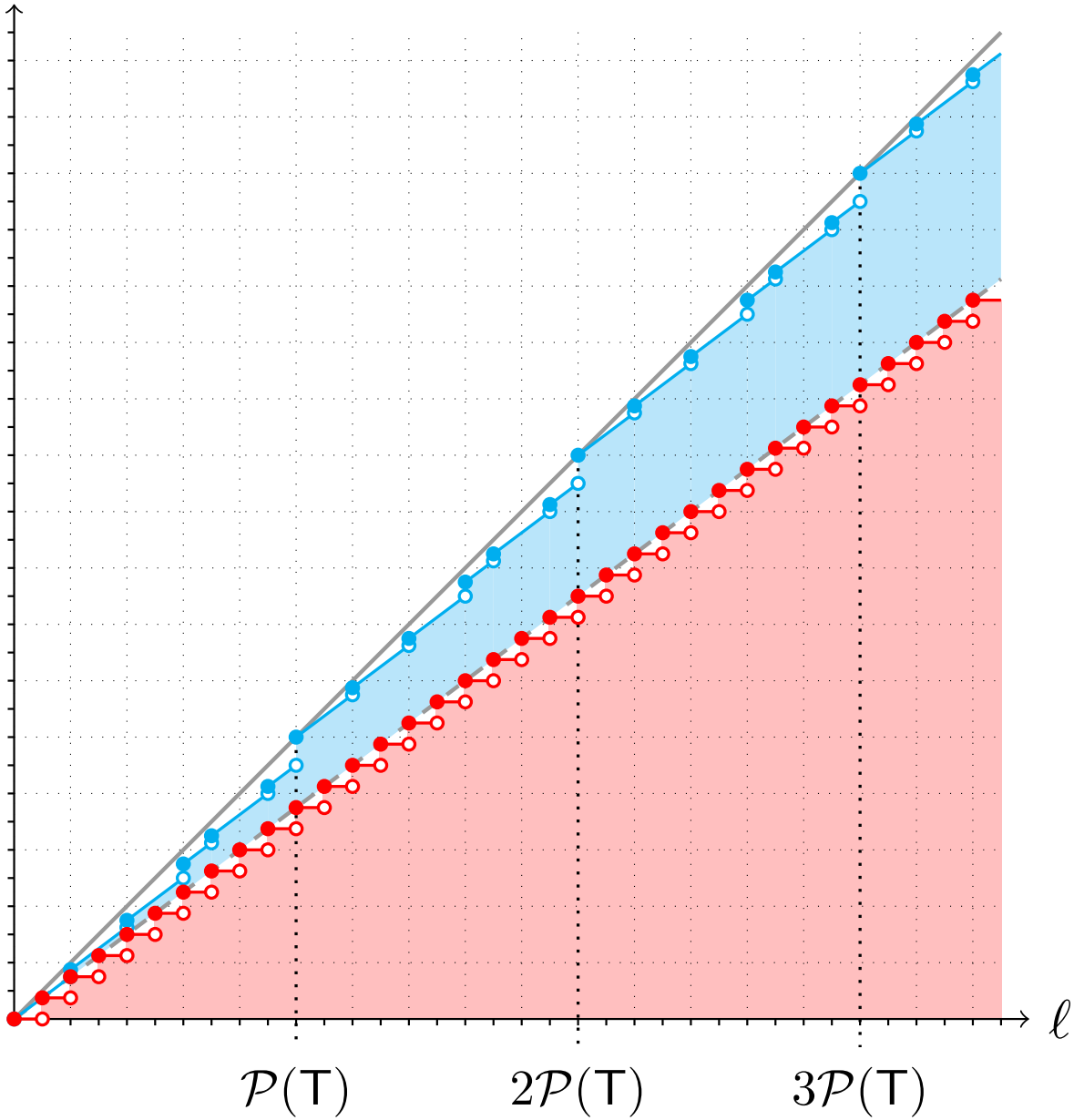

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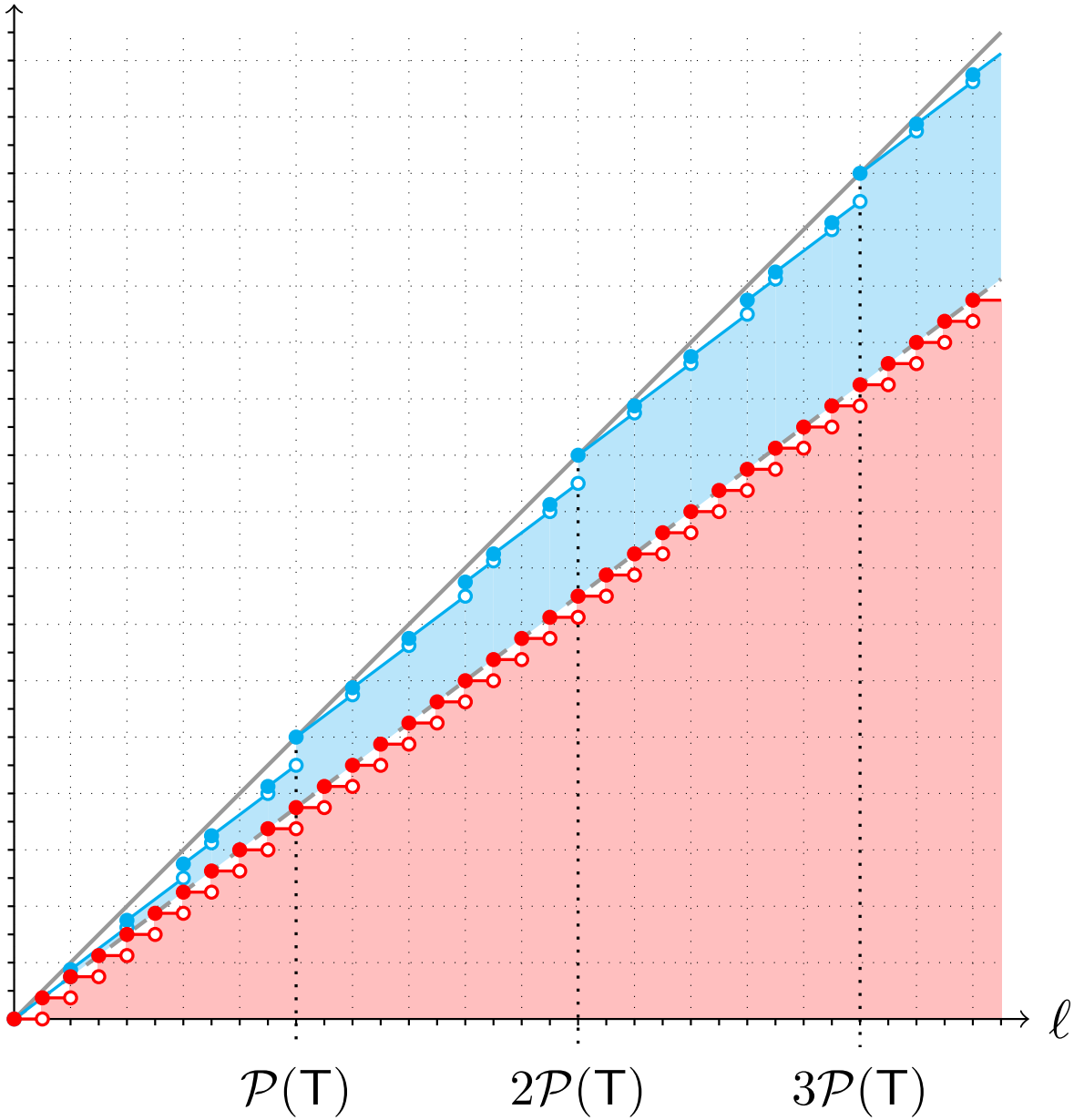

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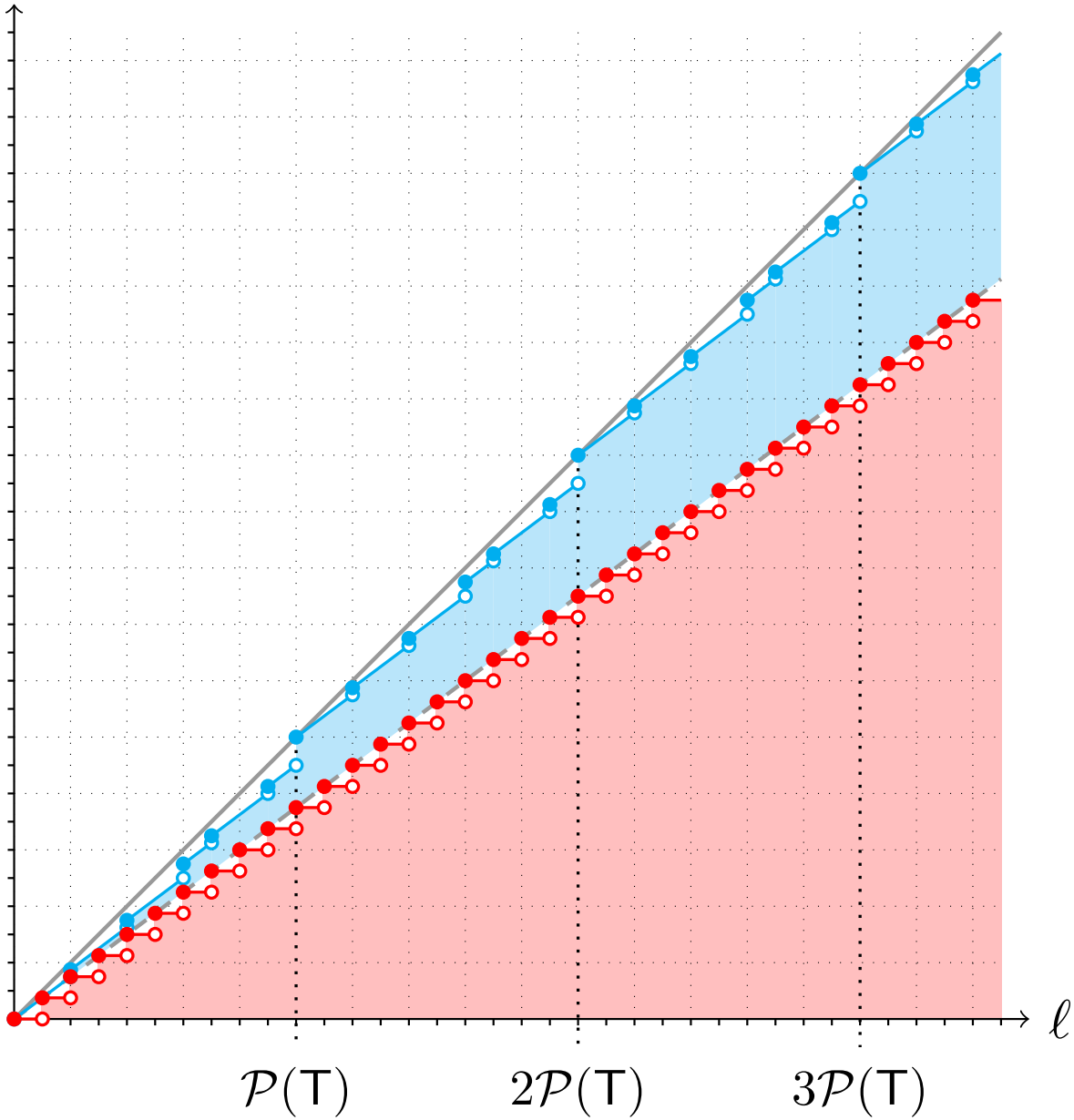

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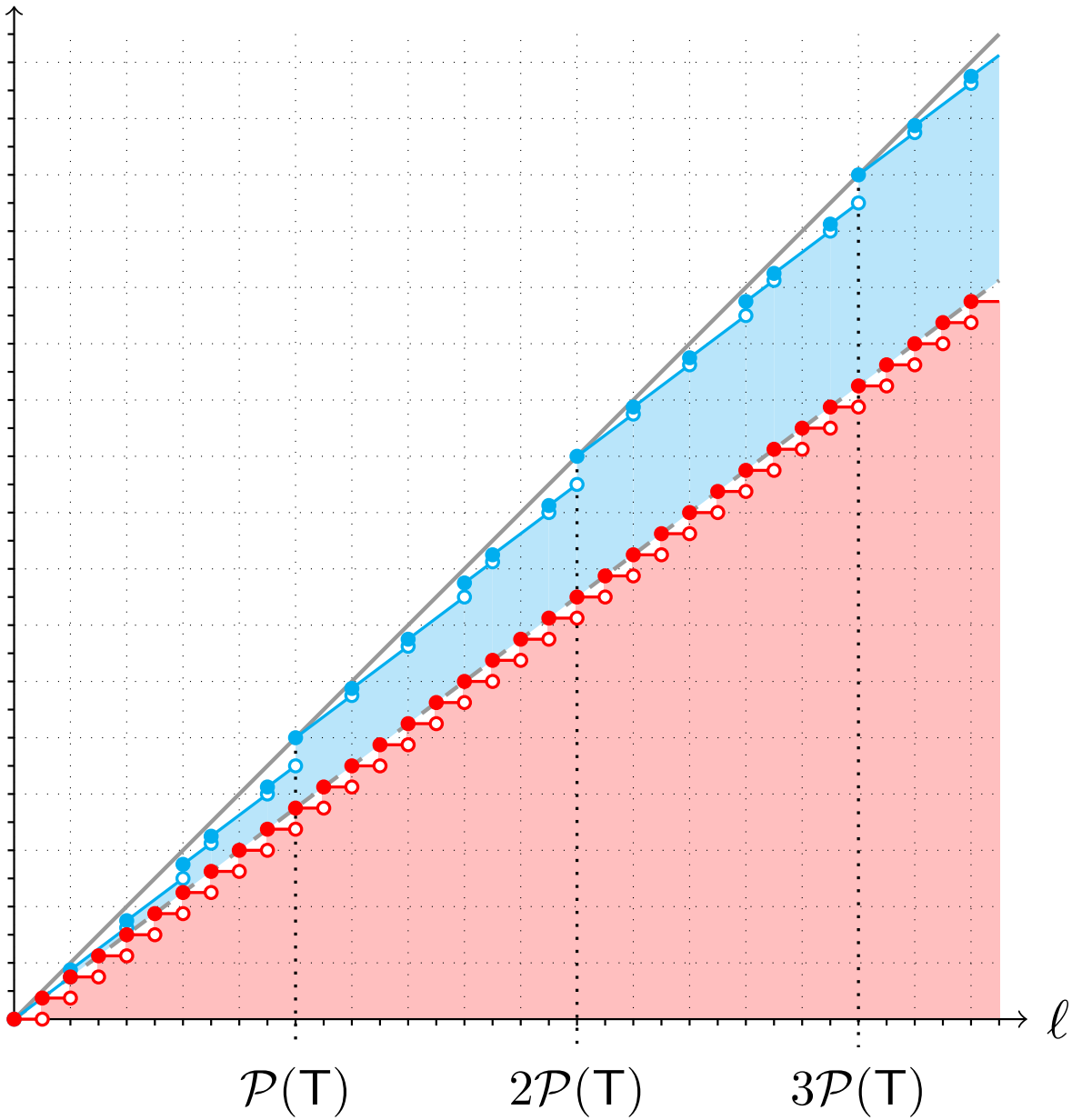

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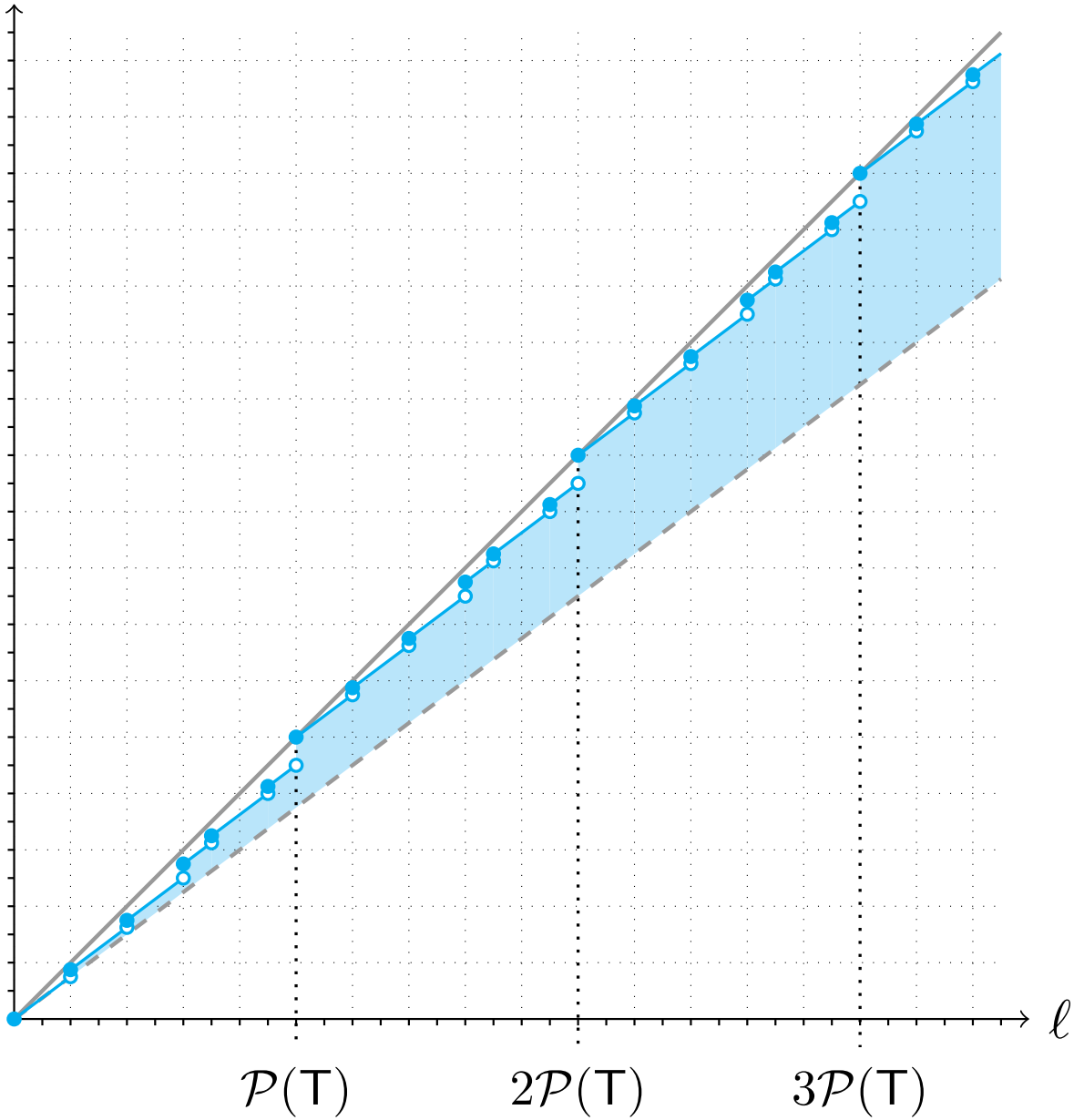

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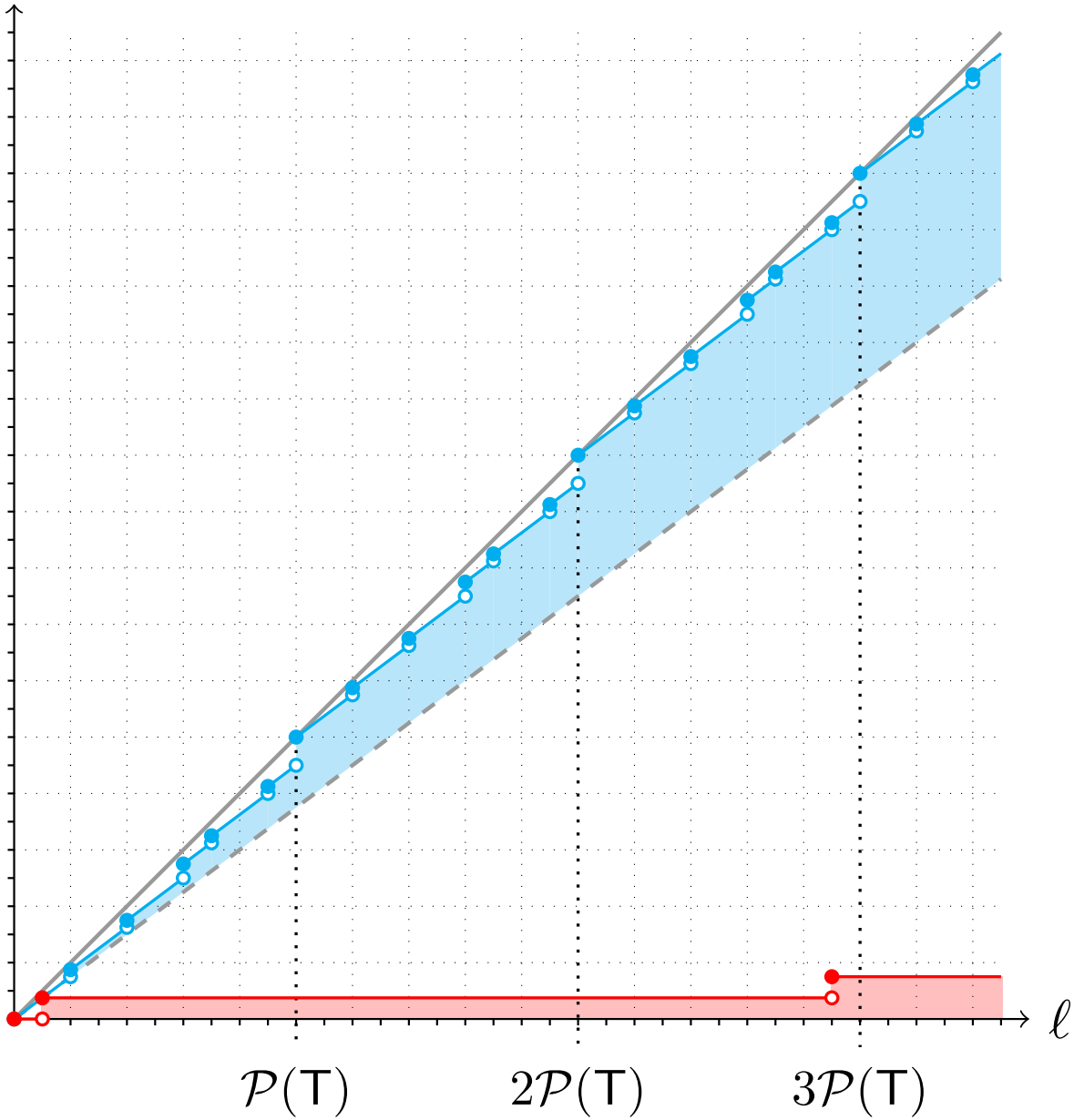

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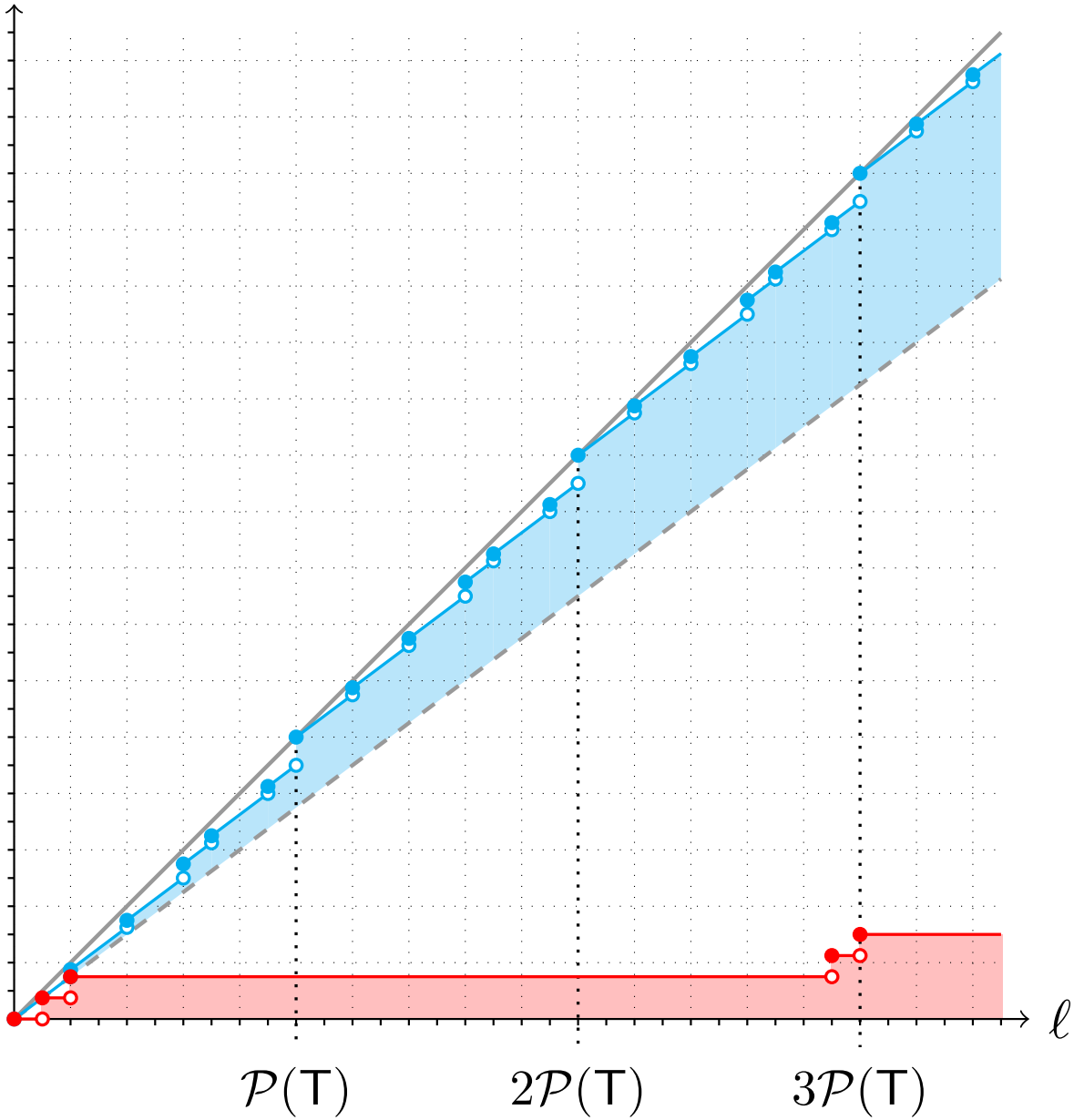

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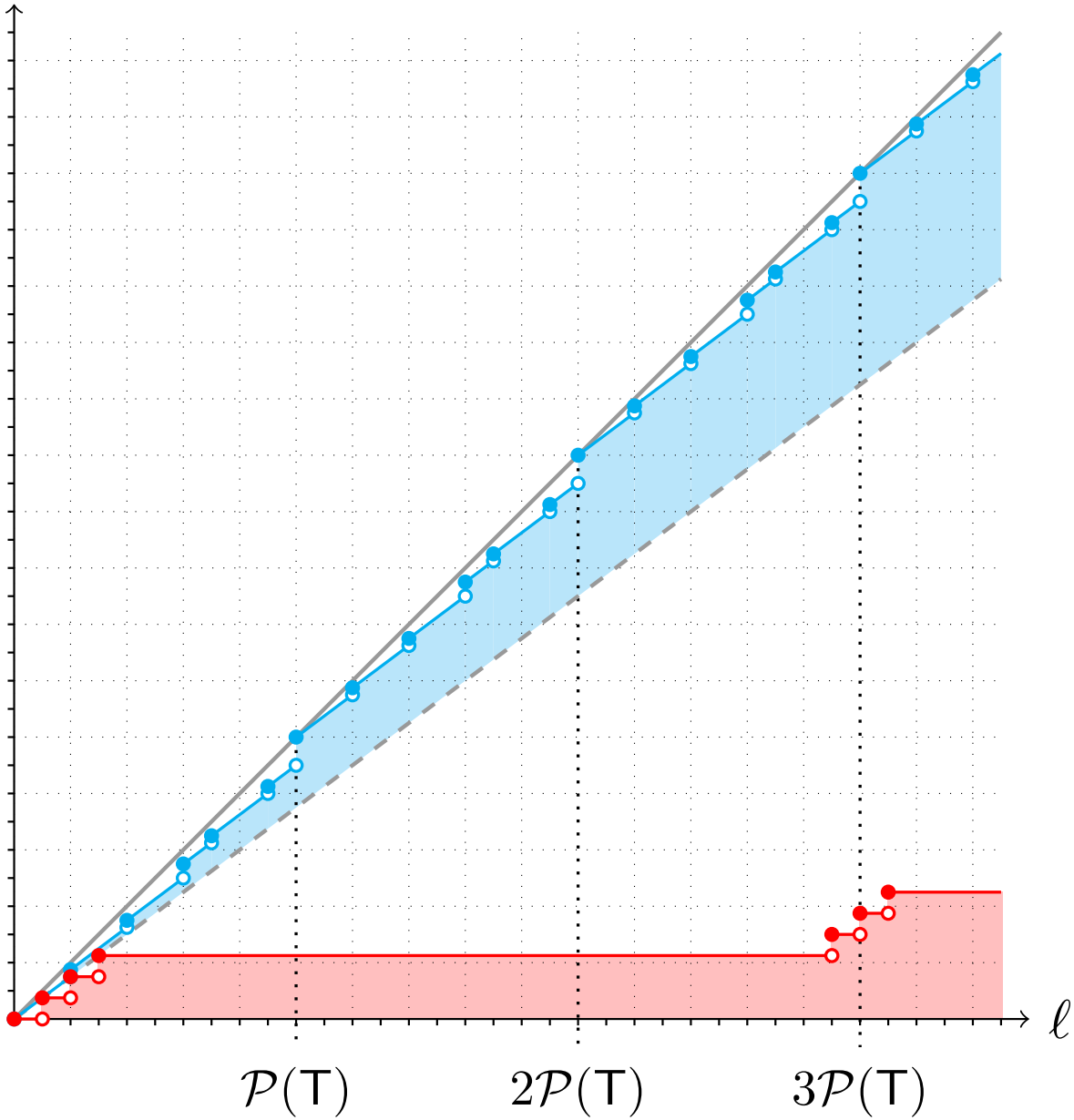

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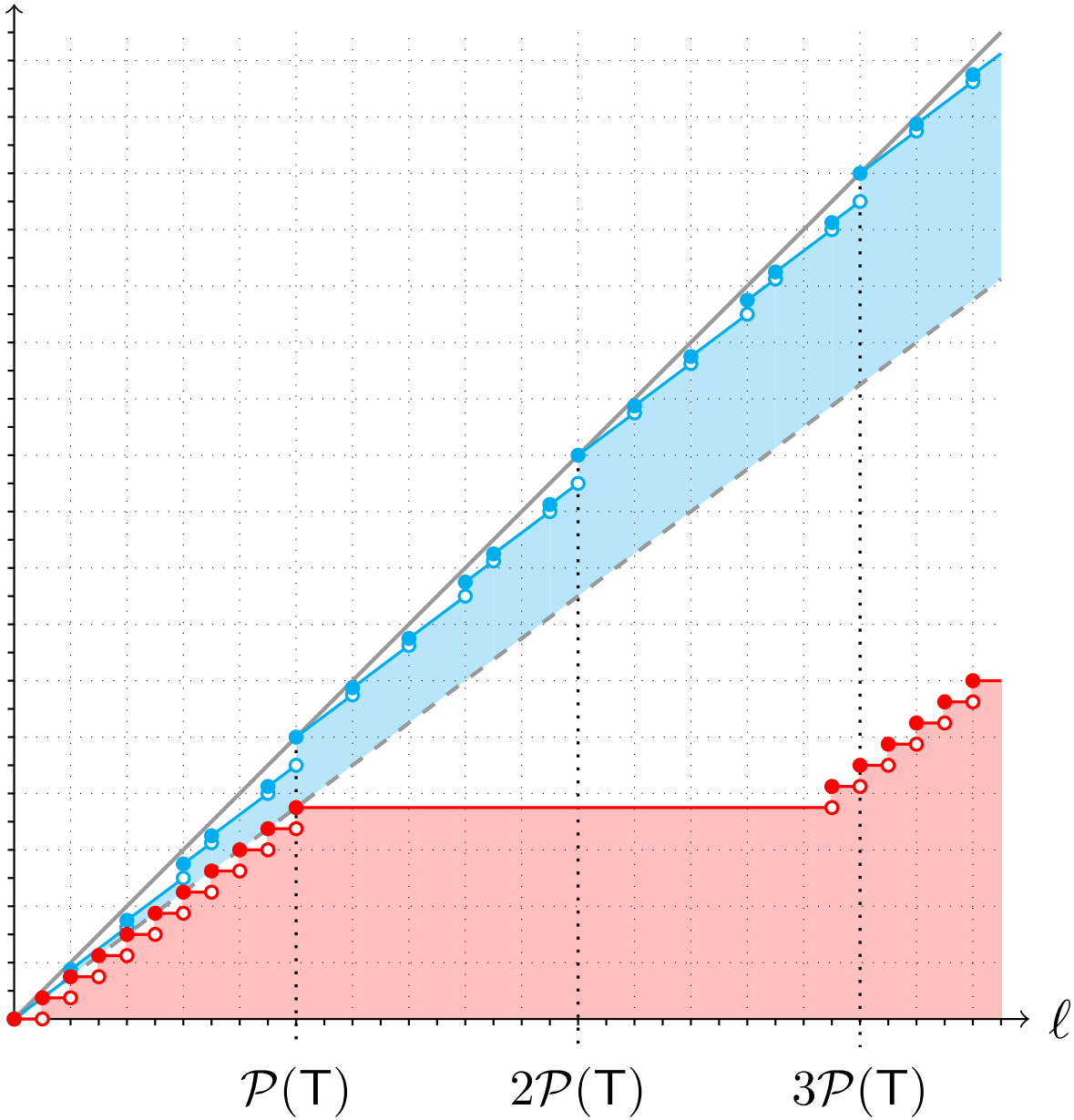

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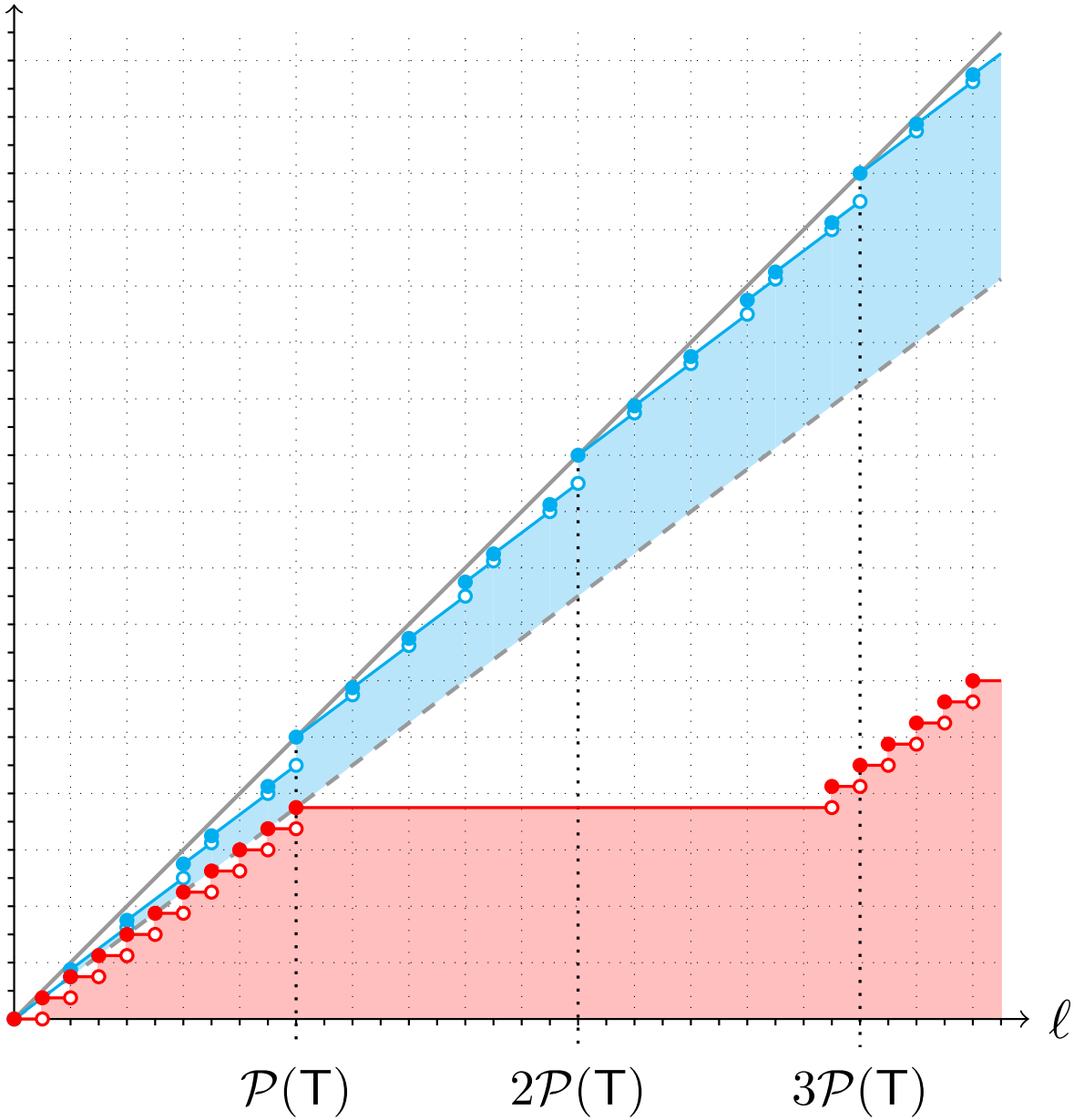

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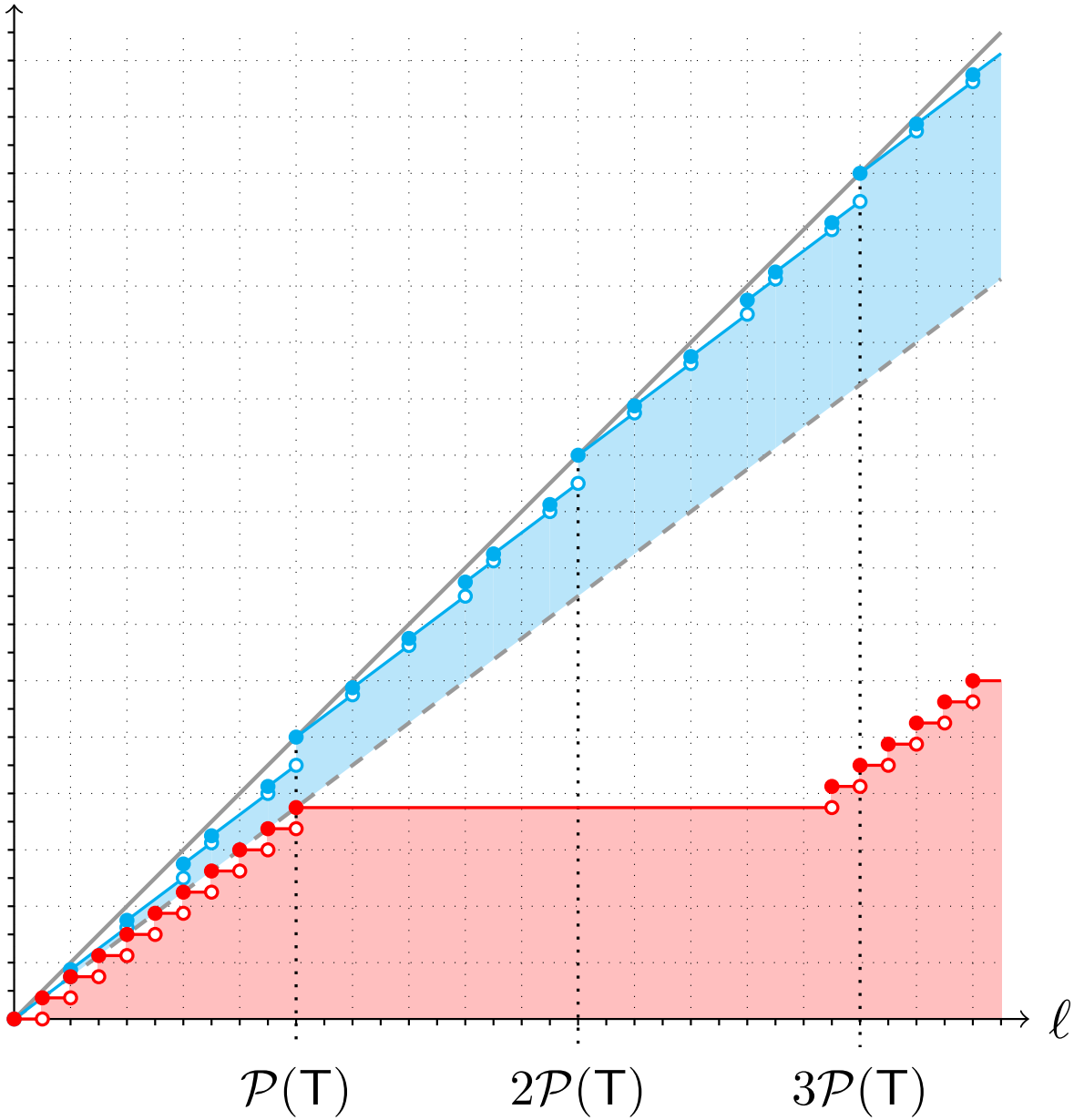

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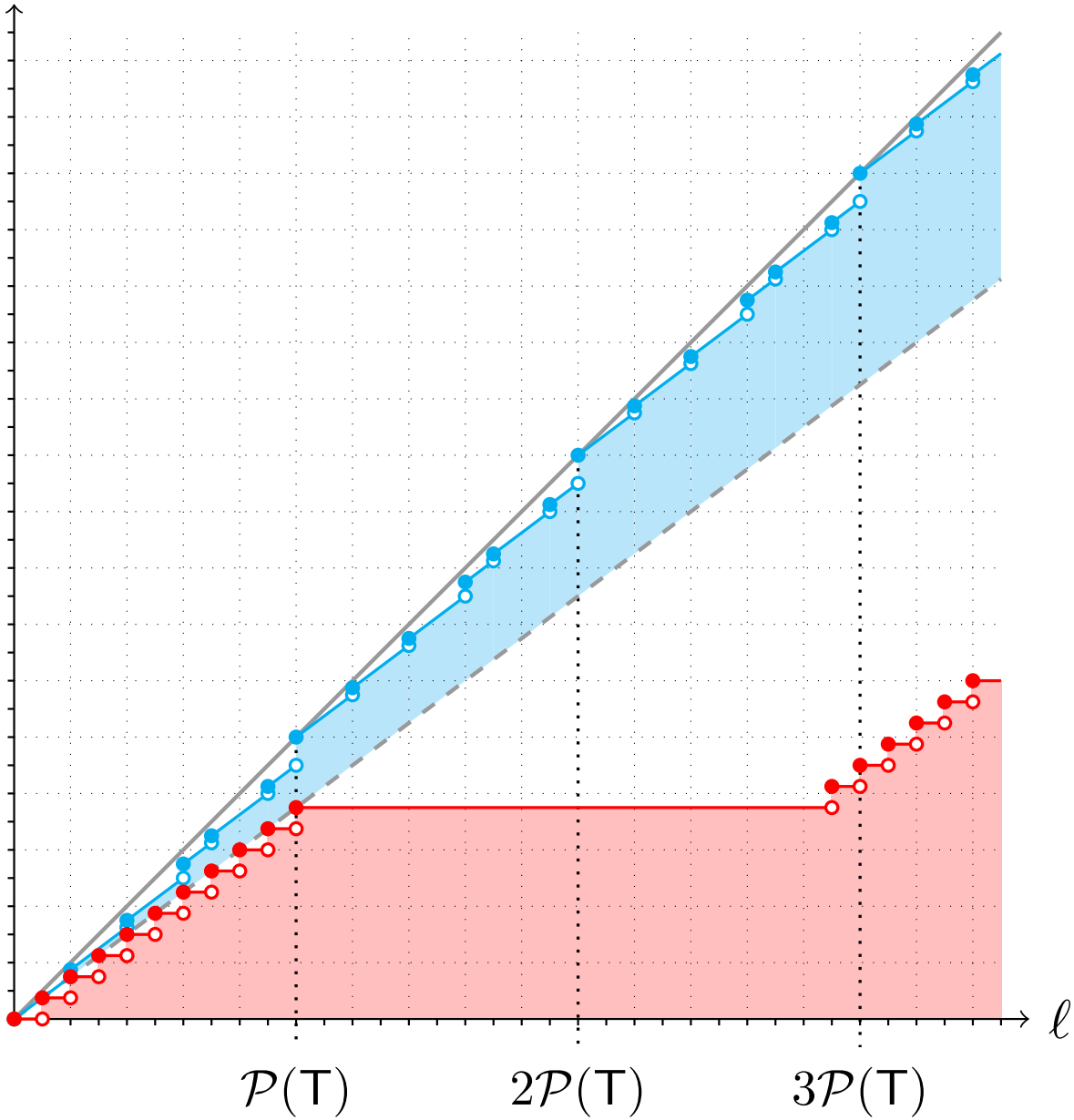

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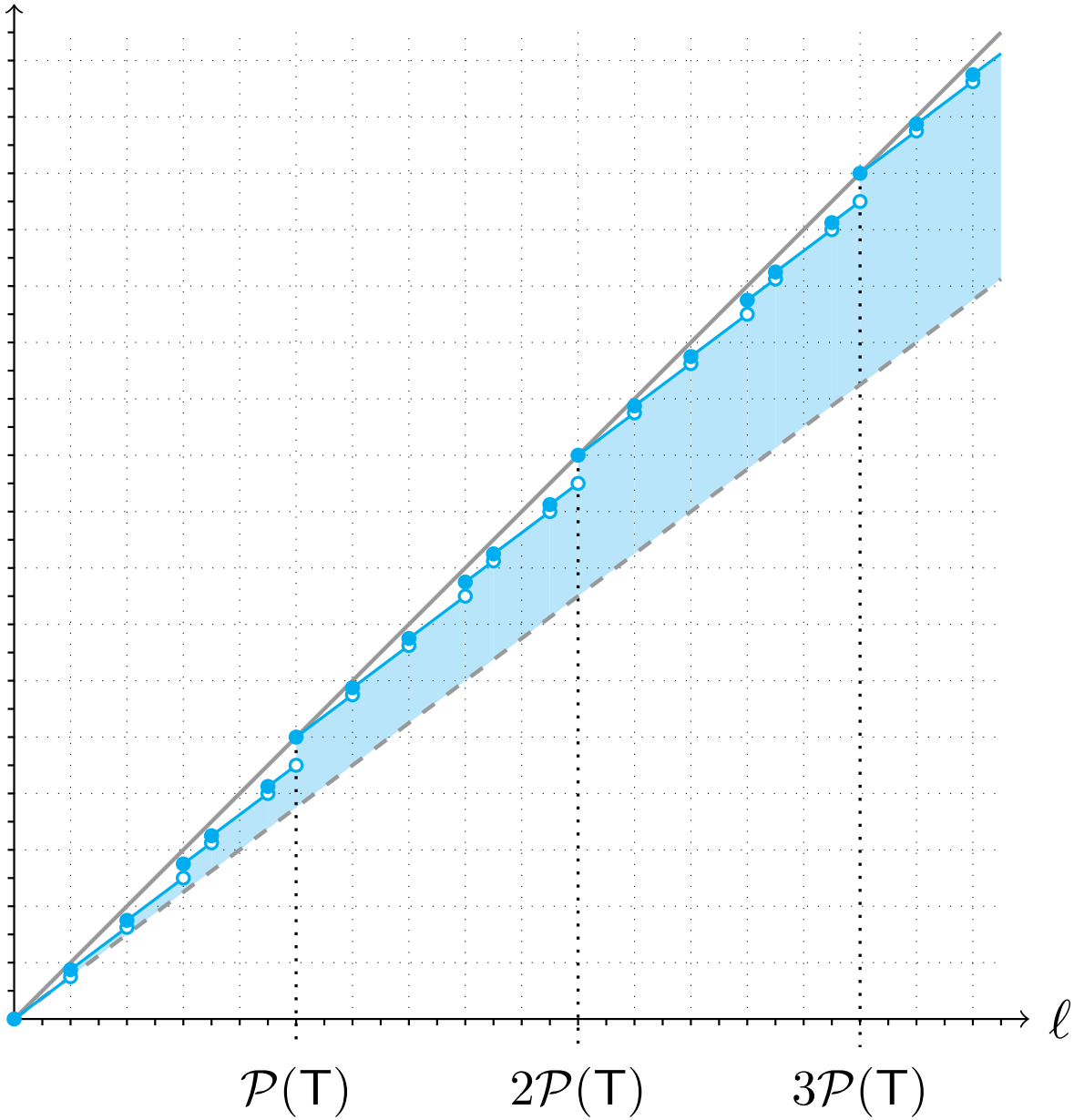

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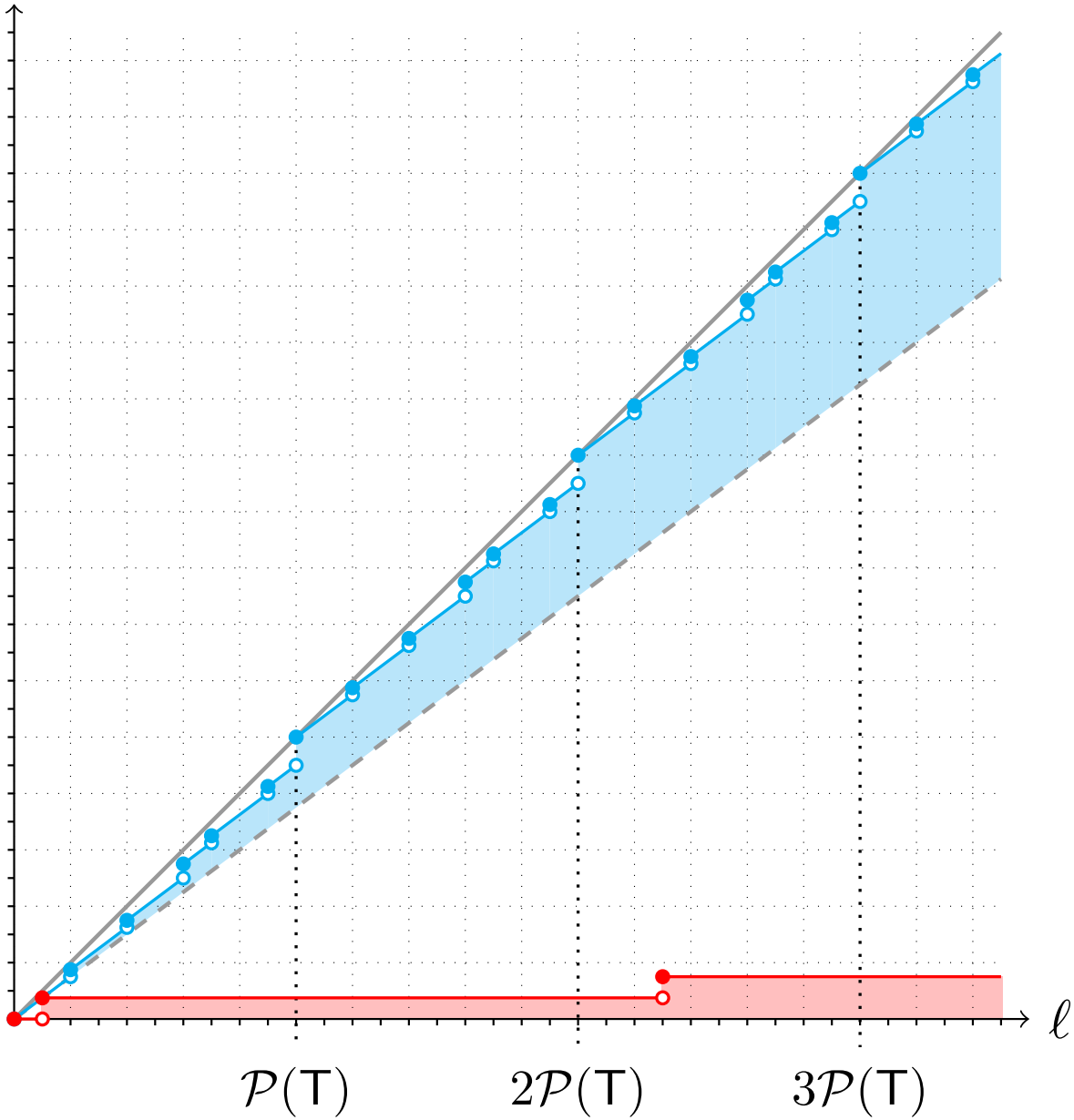

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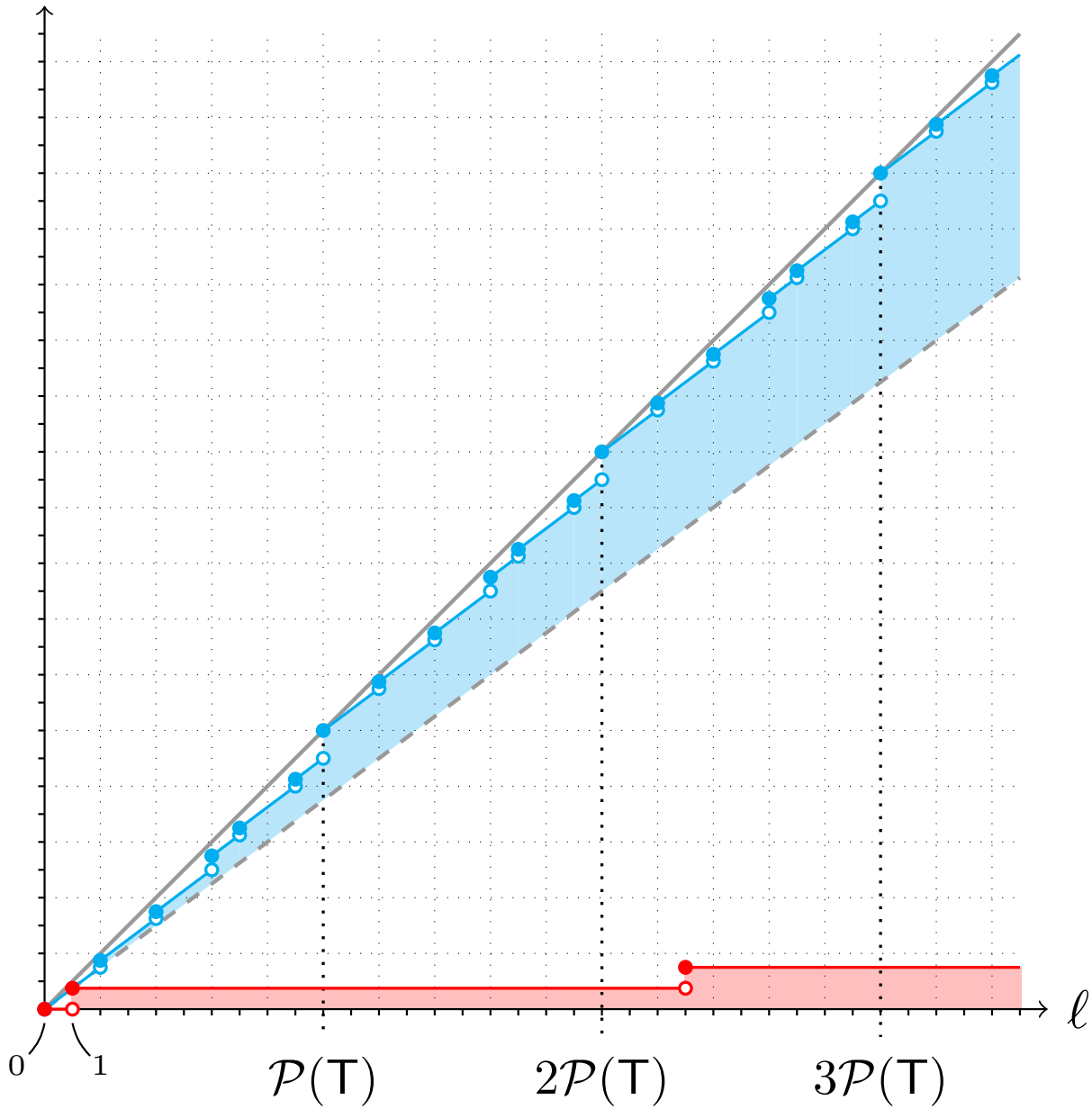

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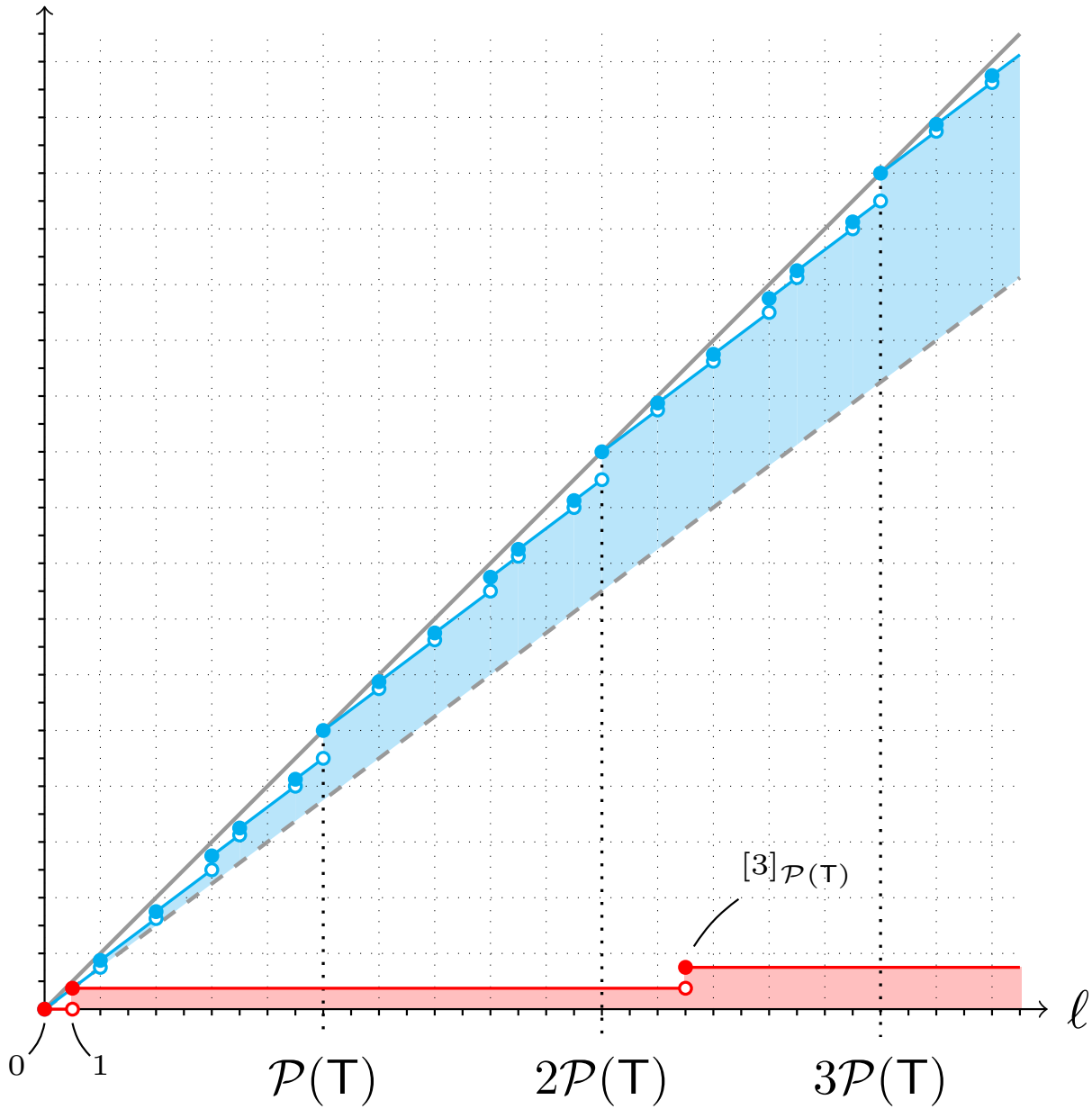

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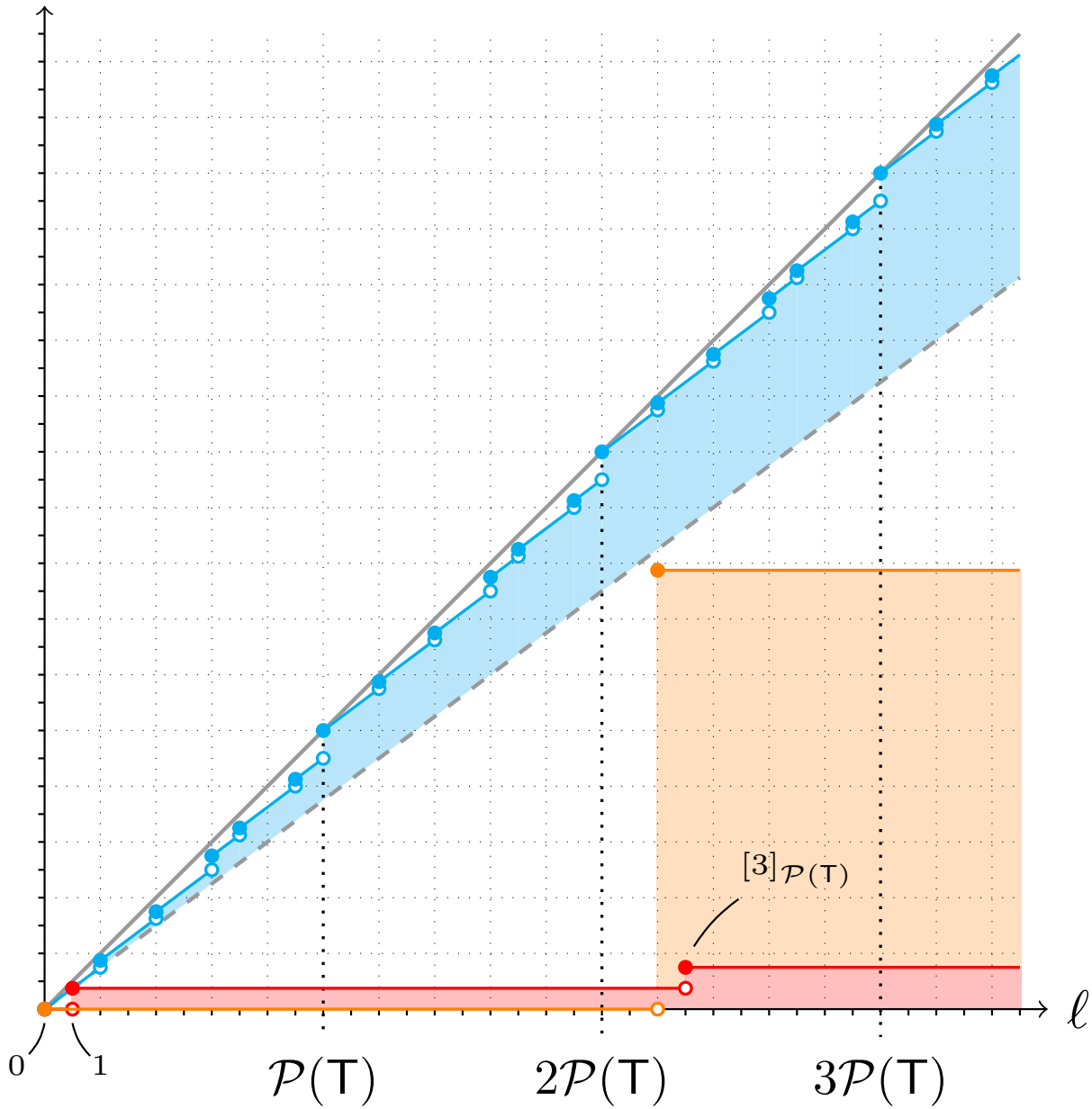

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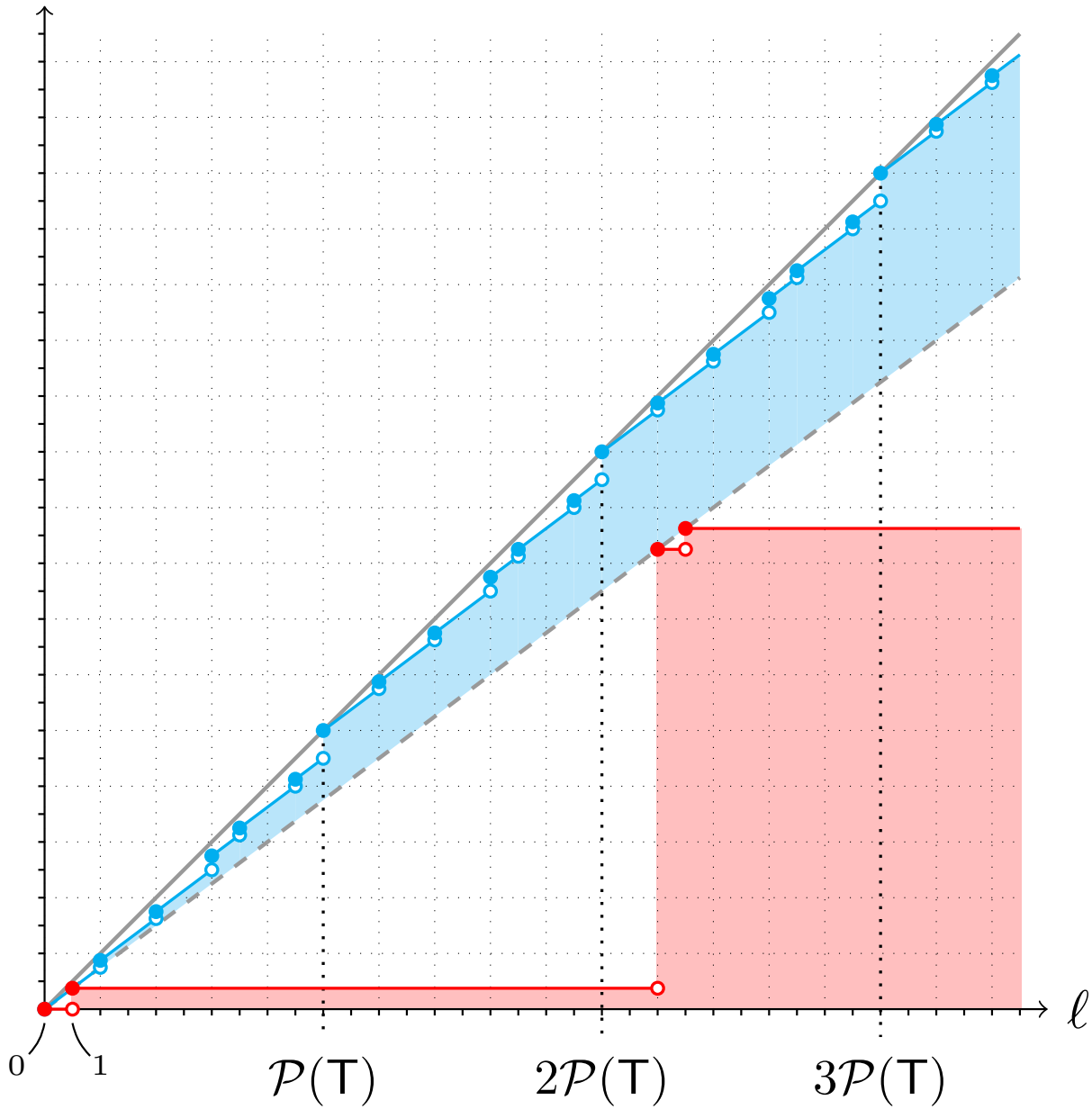

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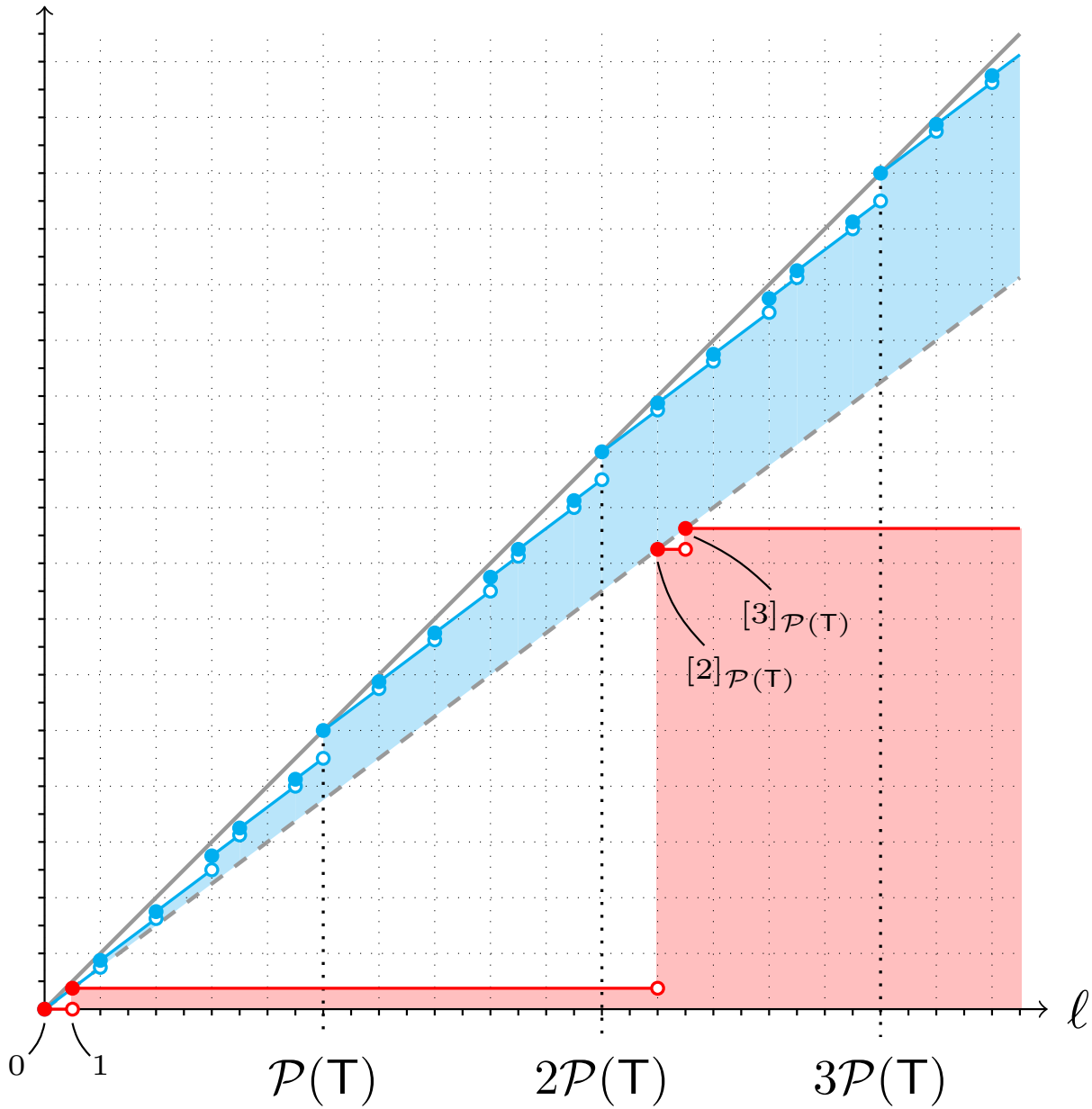

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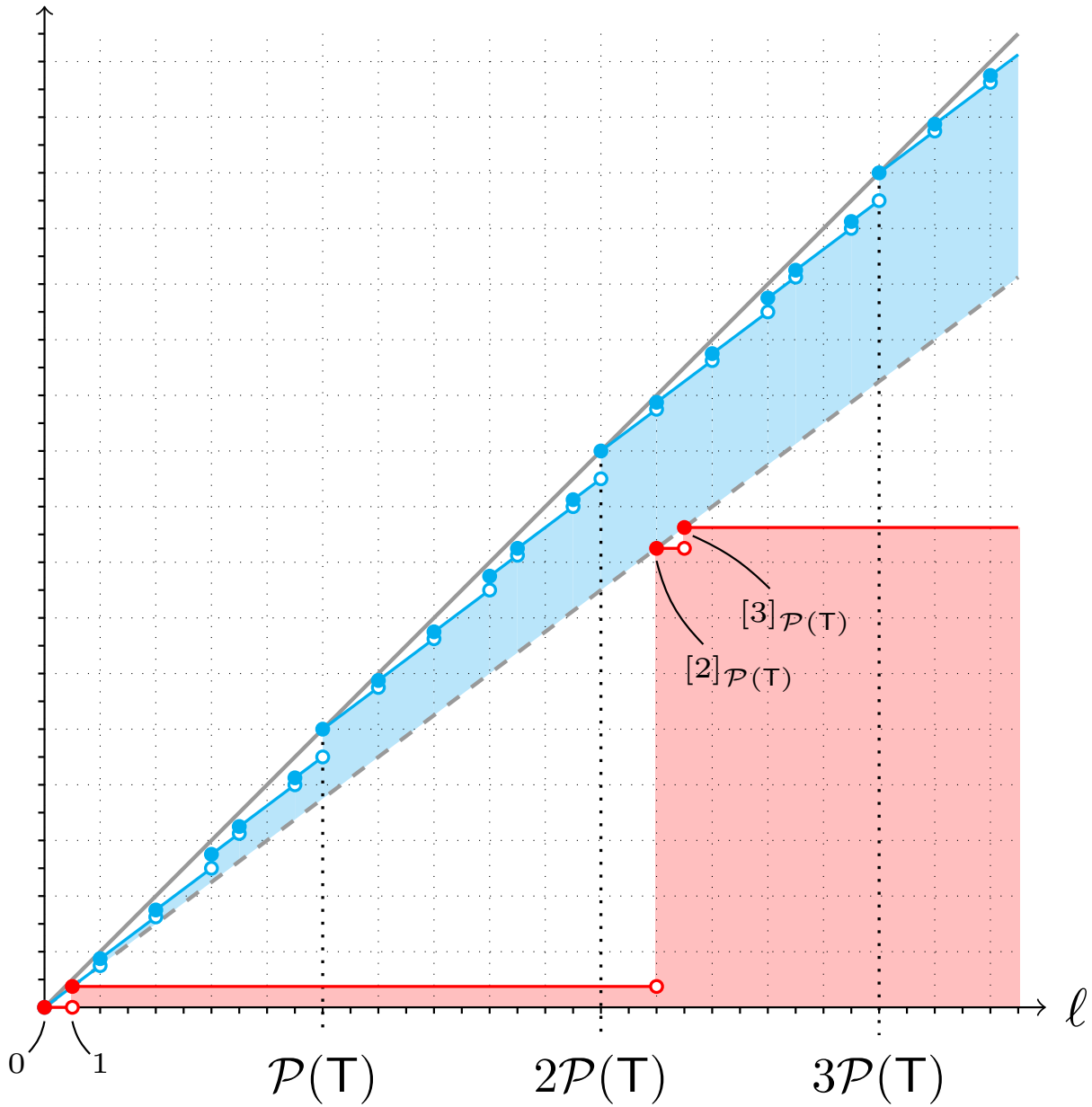

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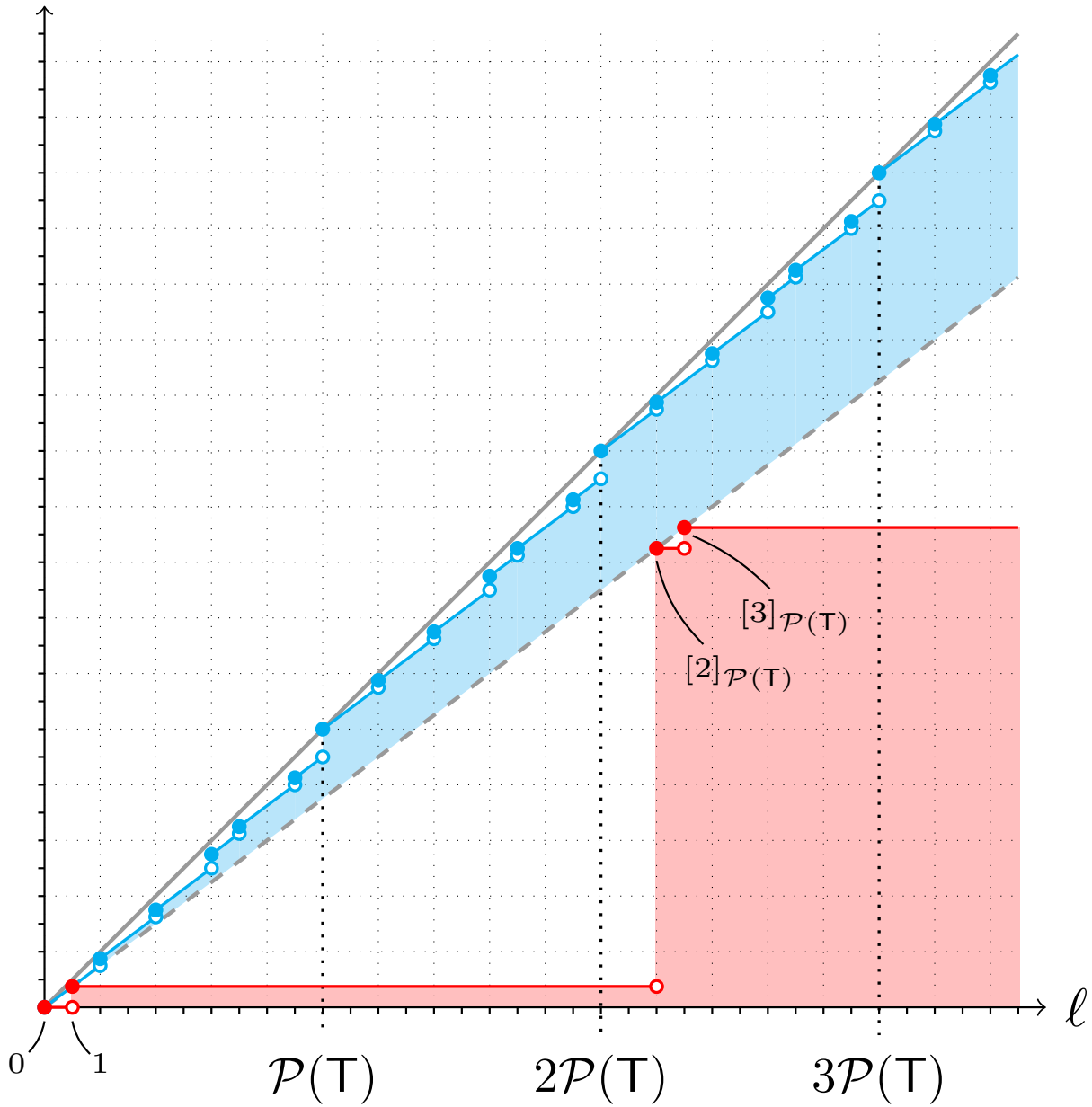

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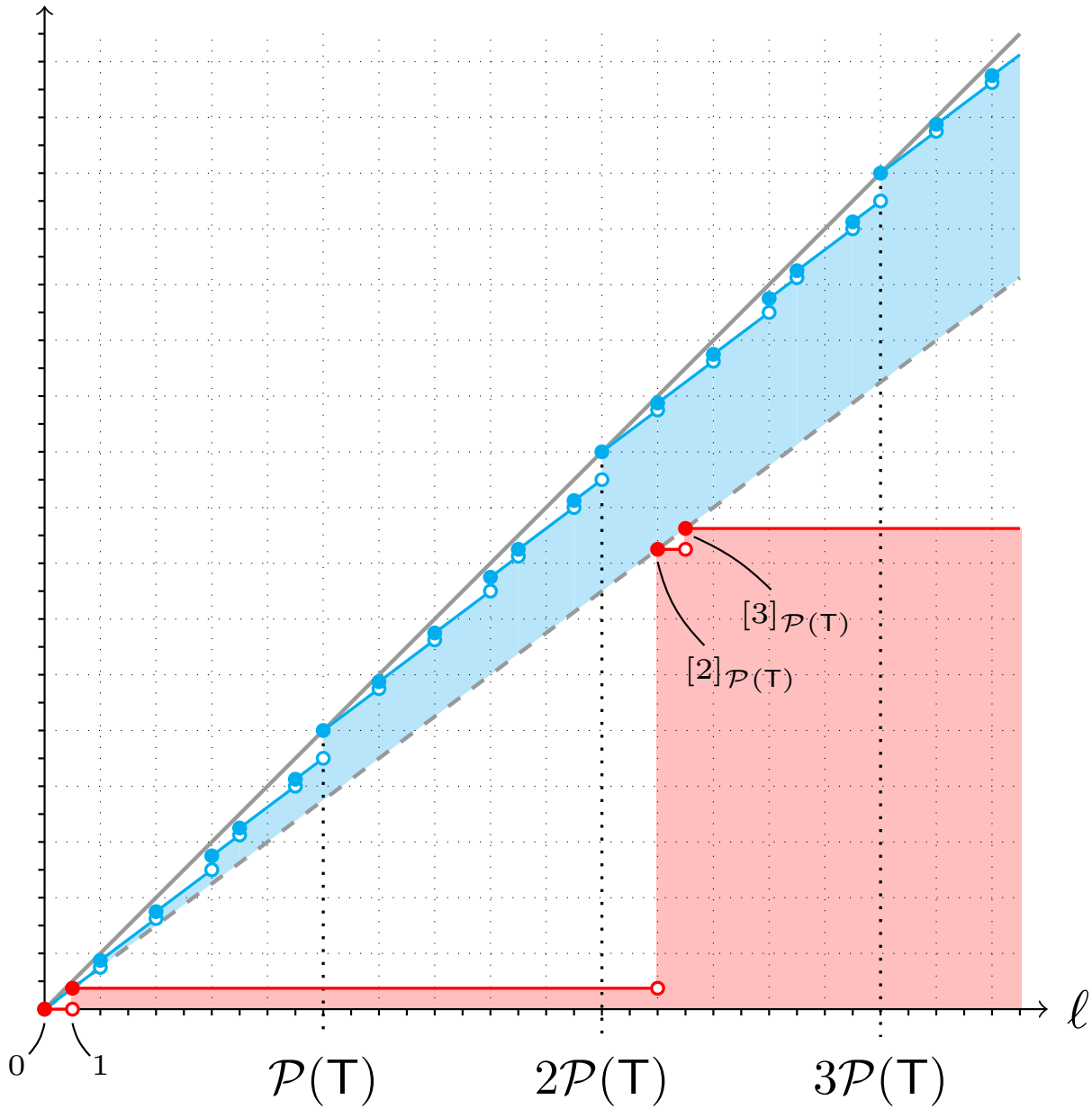

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Feasibility $\propto$ c-Feasibility, With log-scale Glasses

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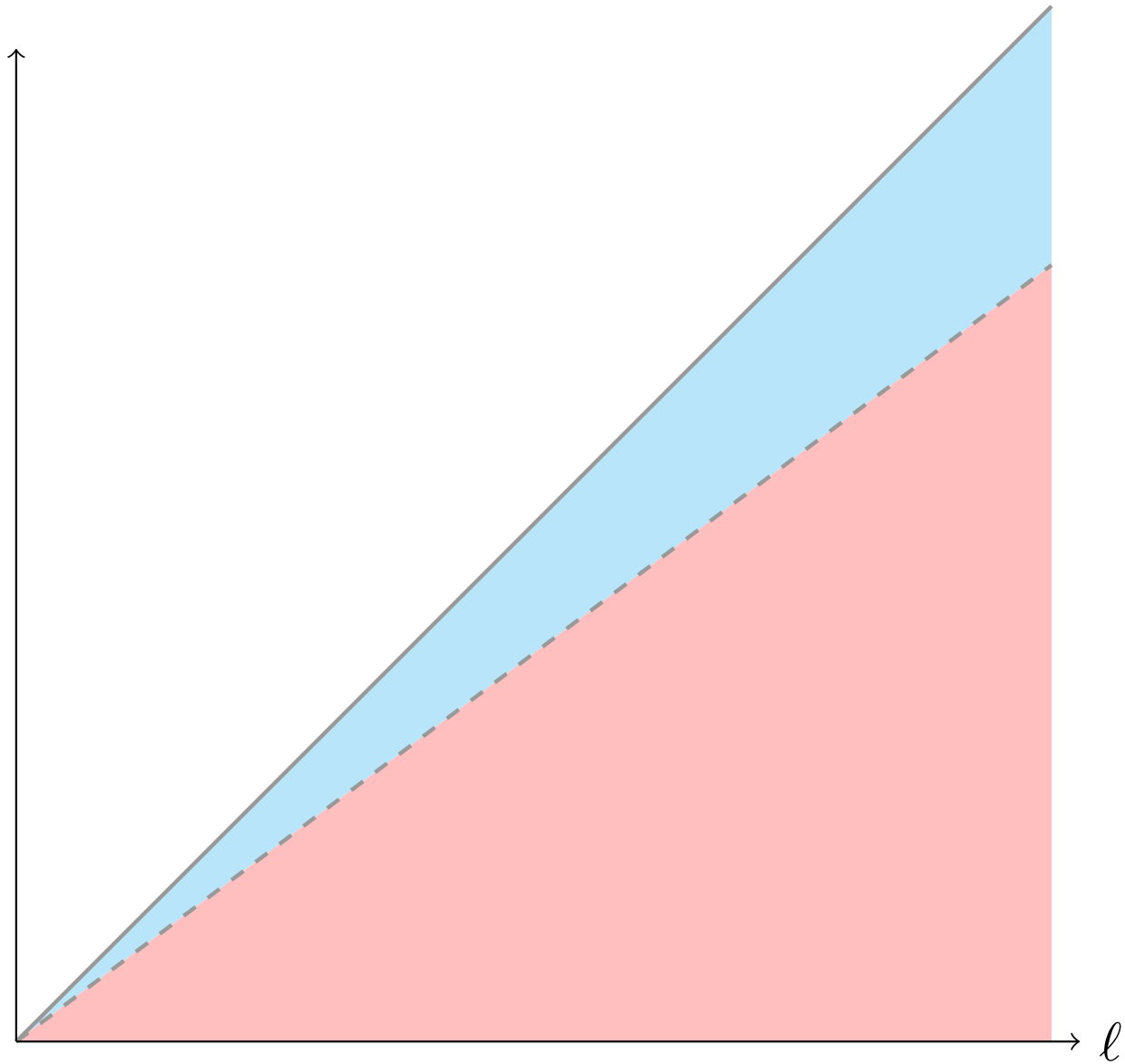

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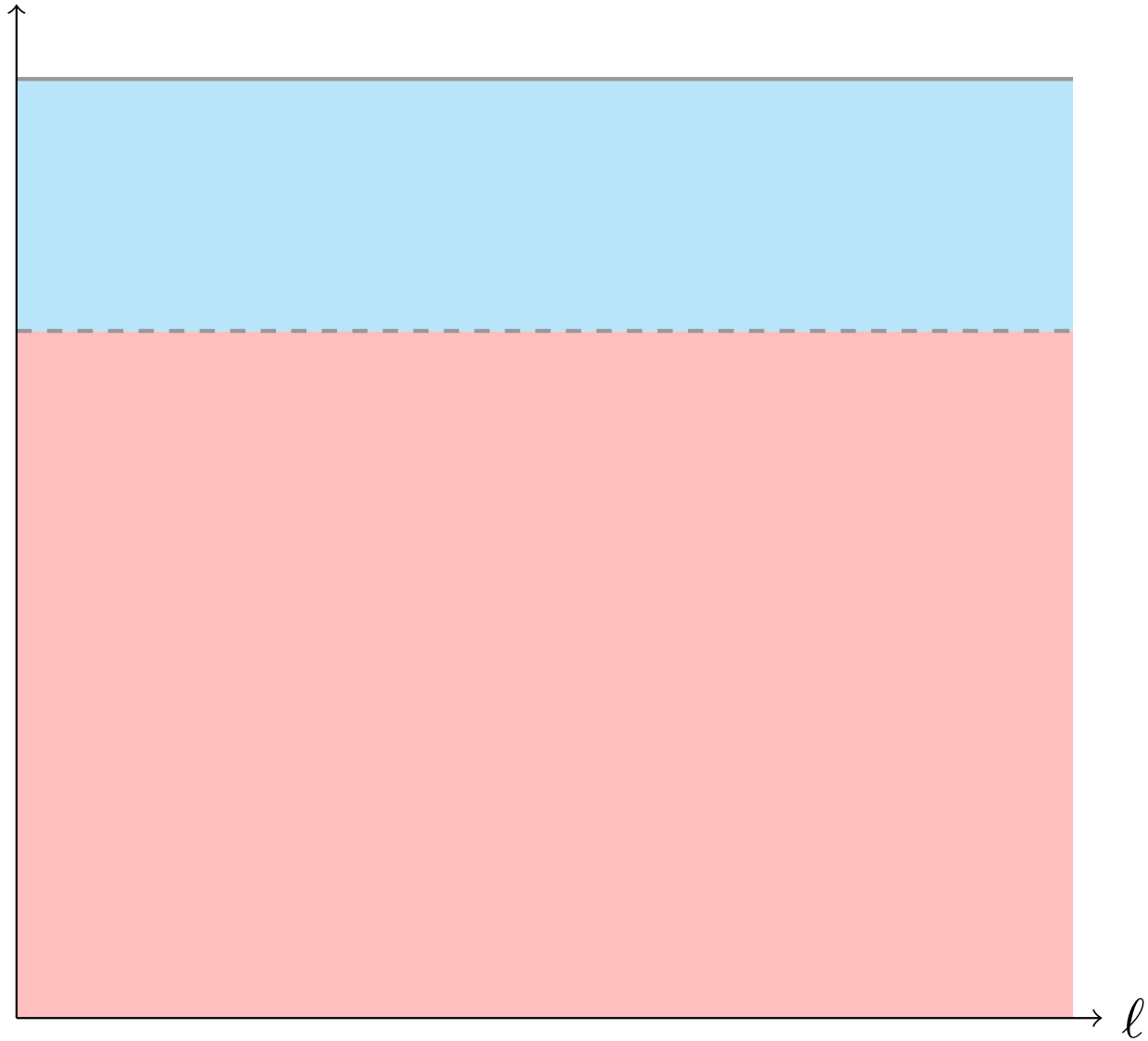

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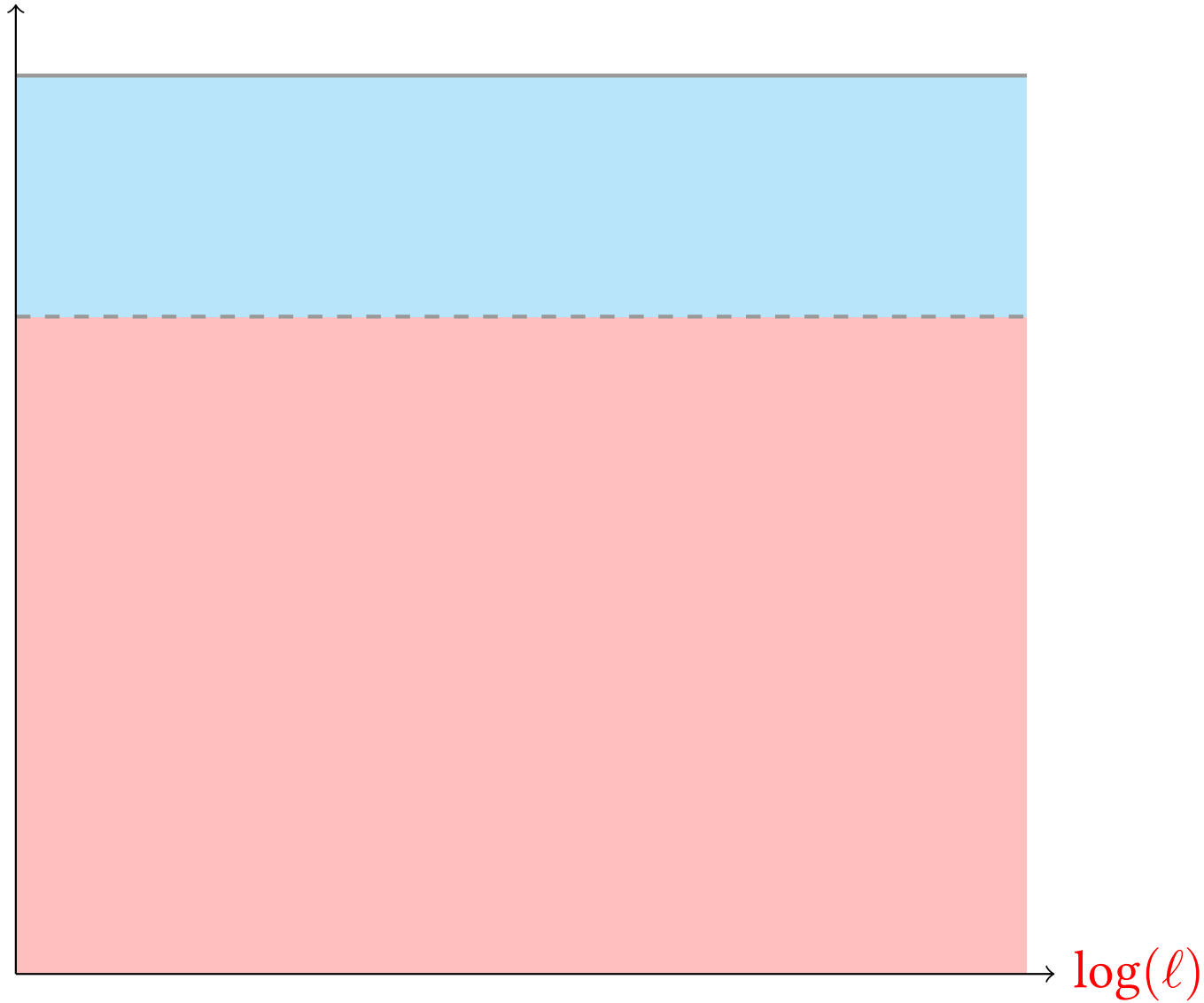

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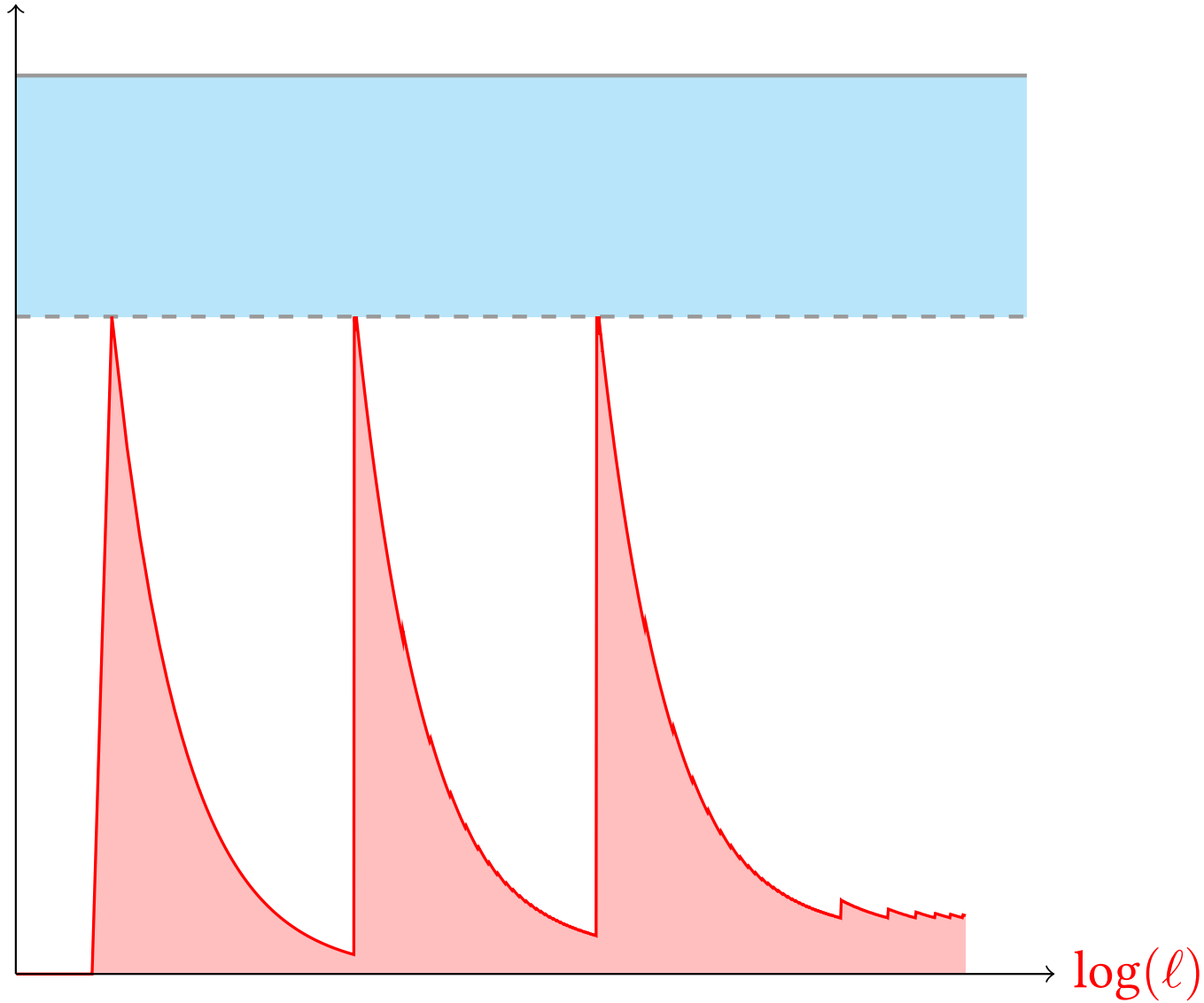

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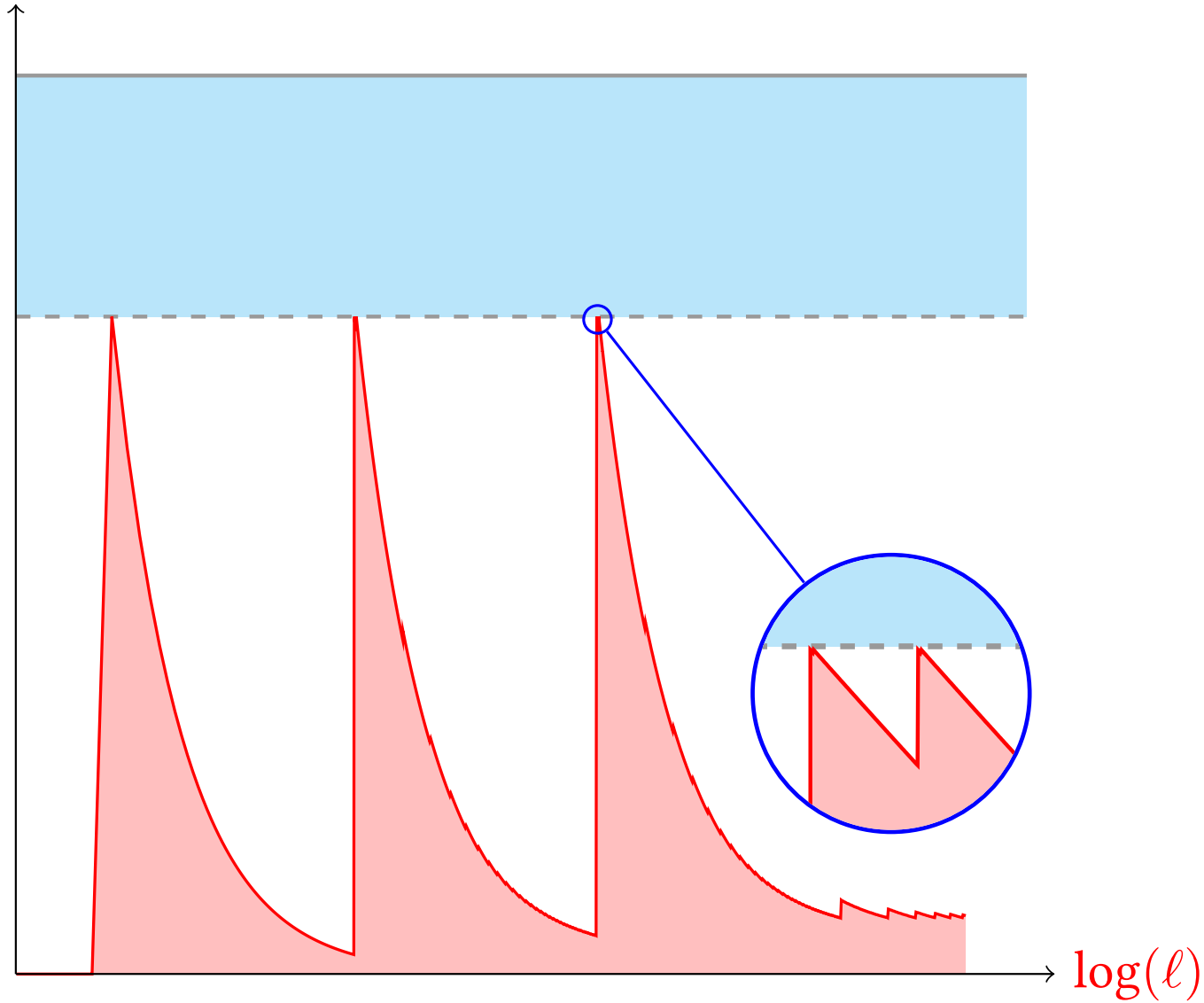

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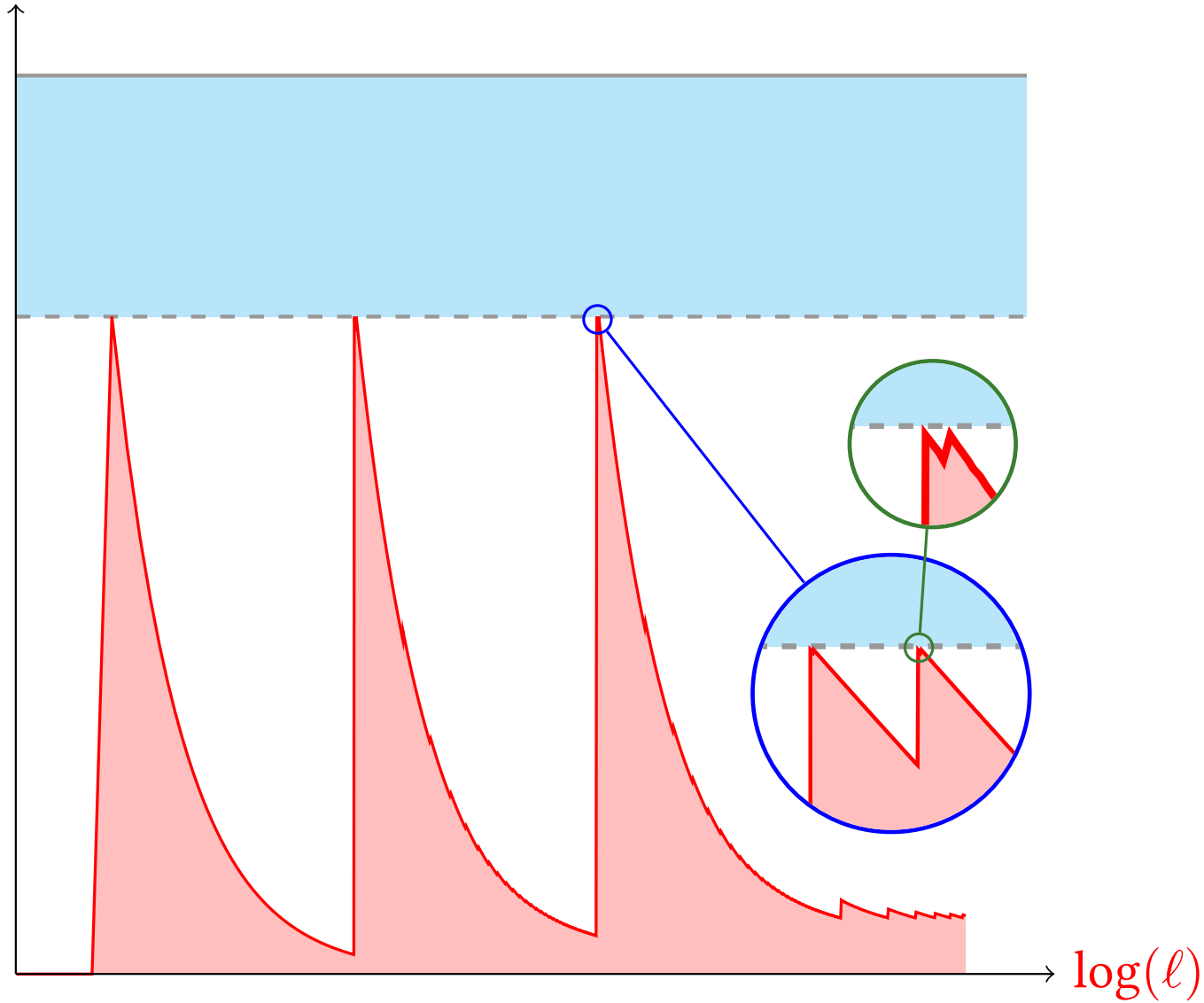

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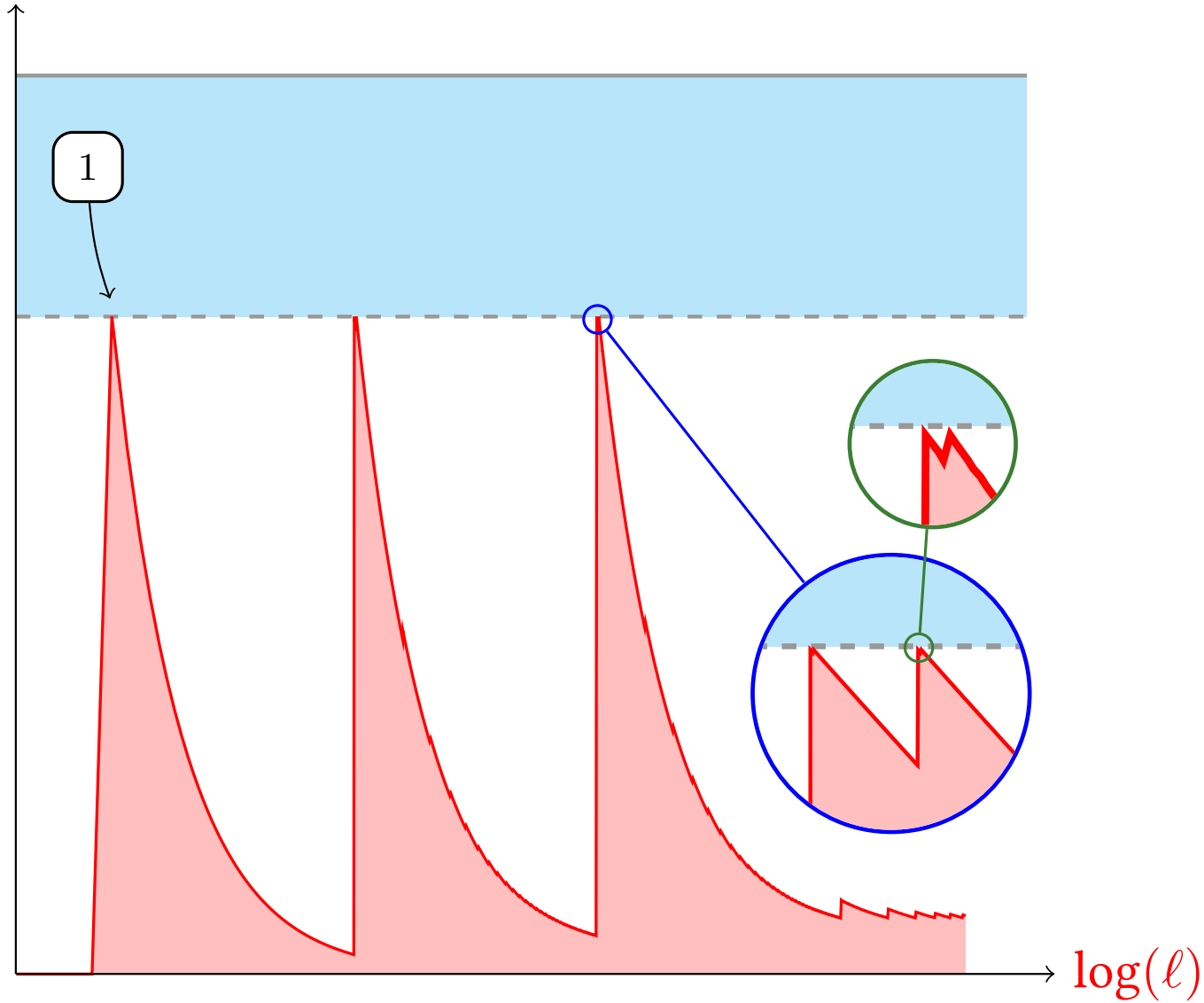

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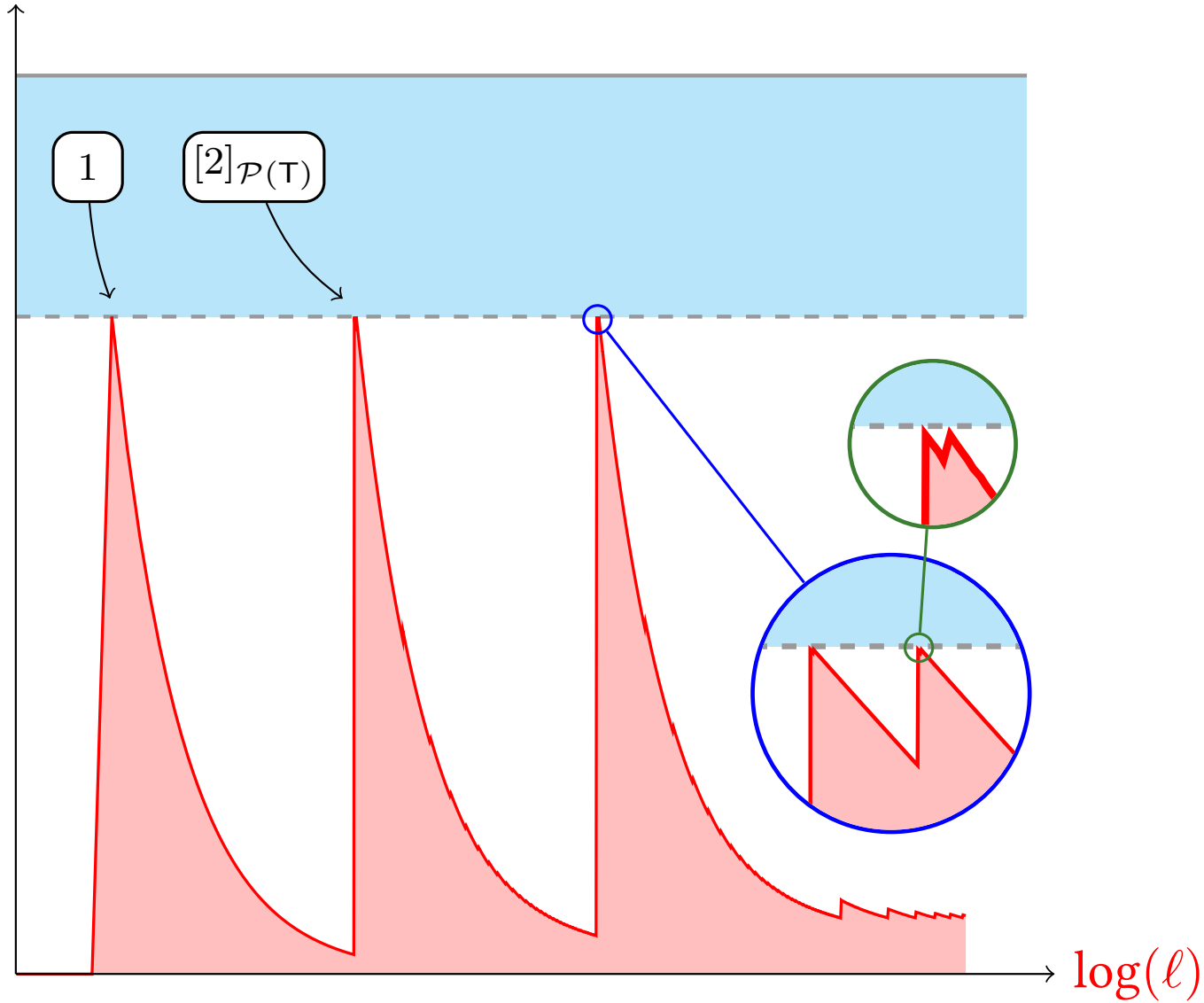

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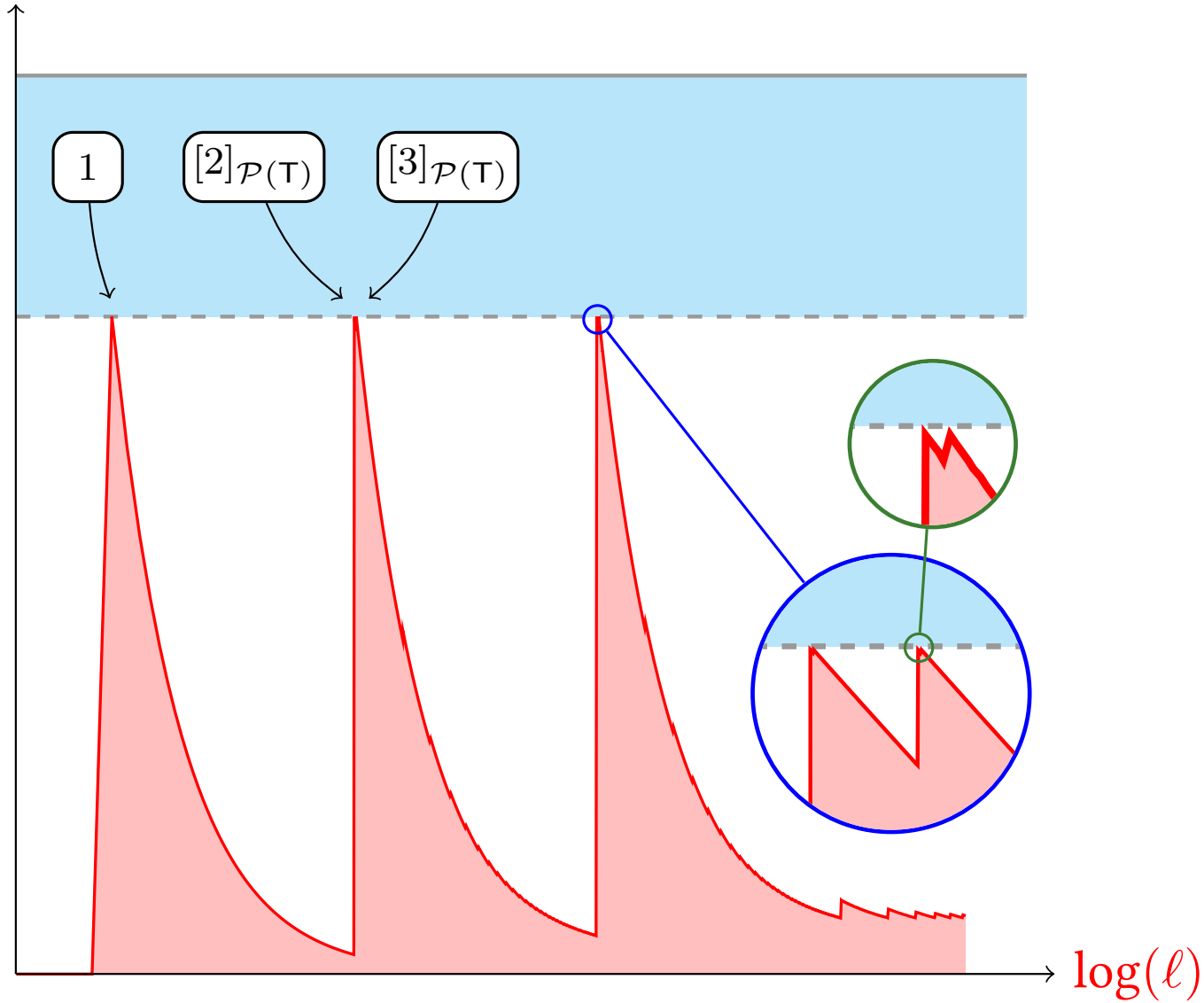

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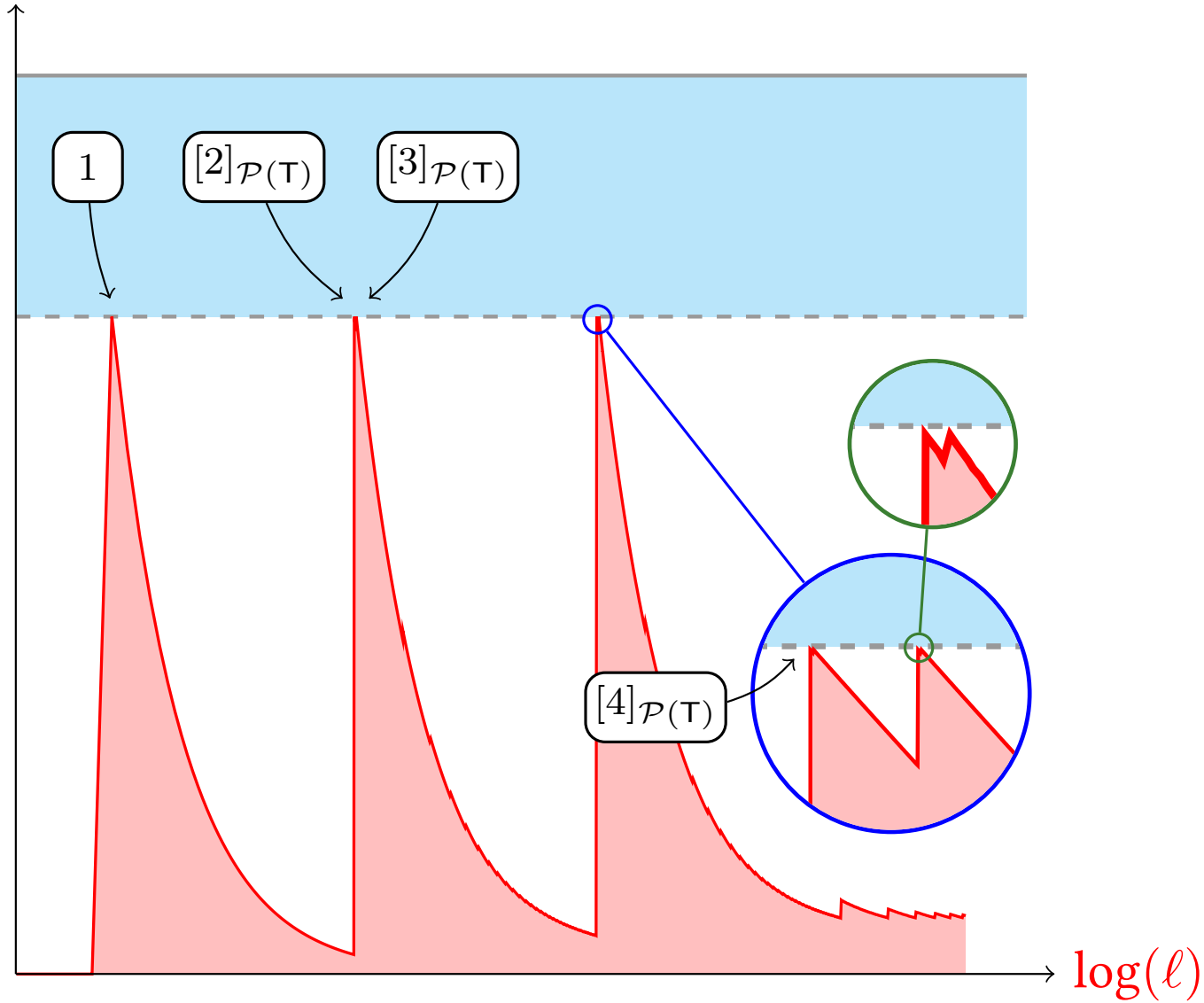

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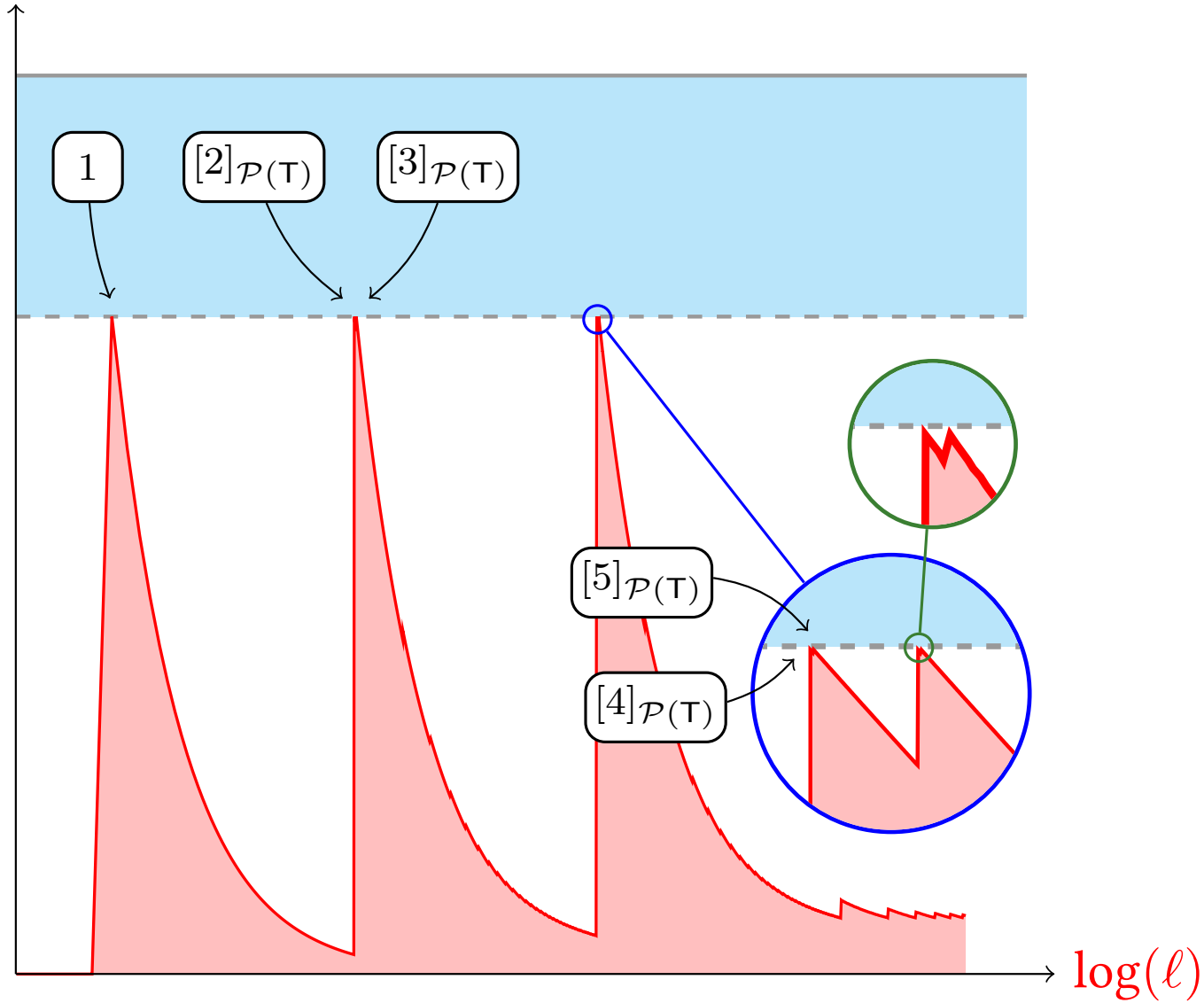

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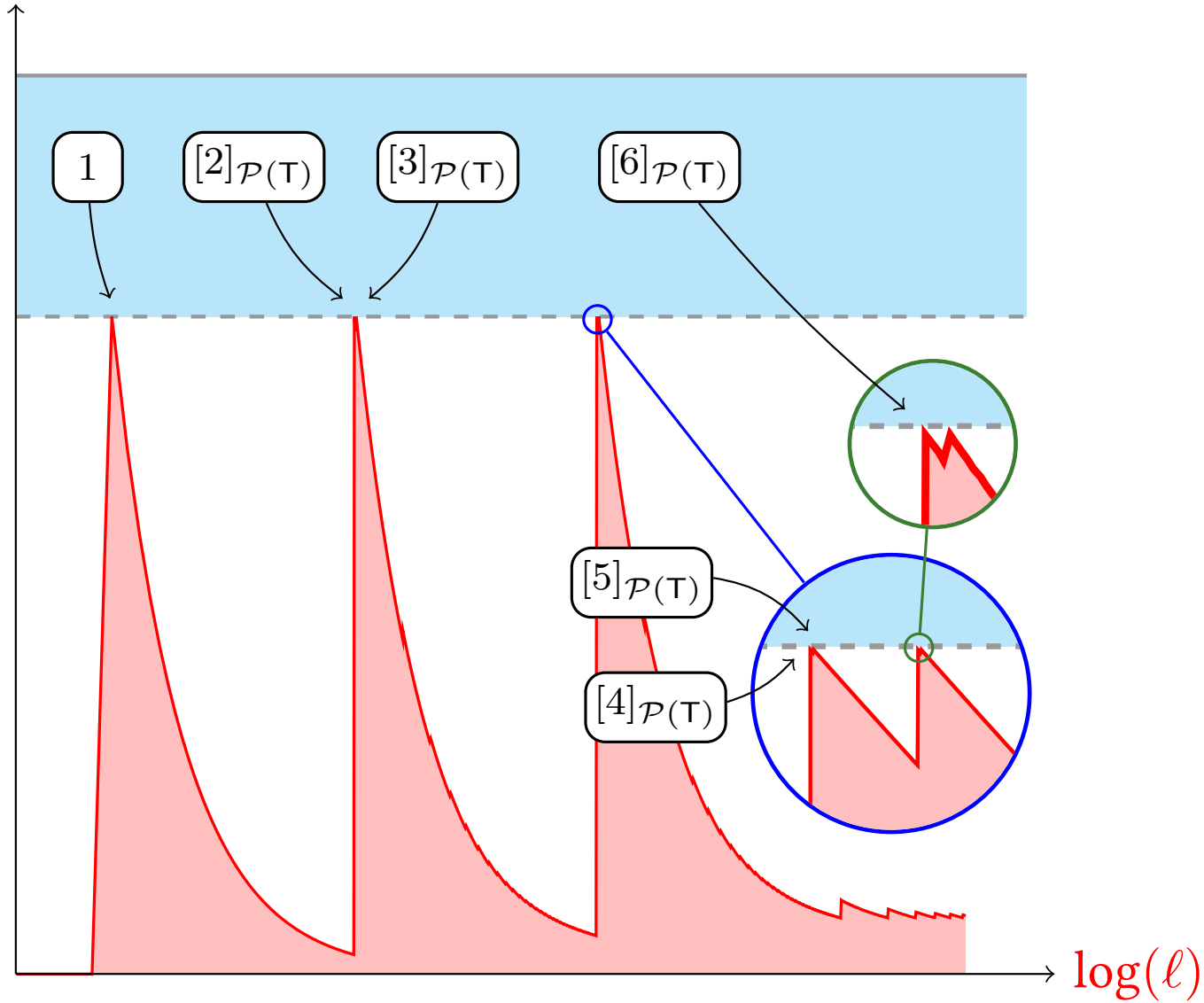

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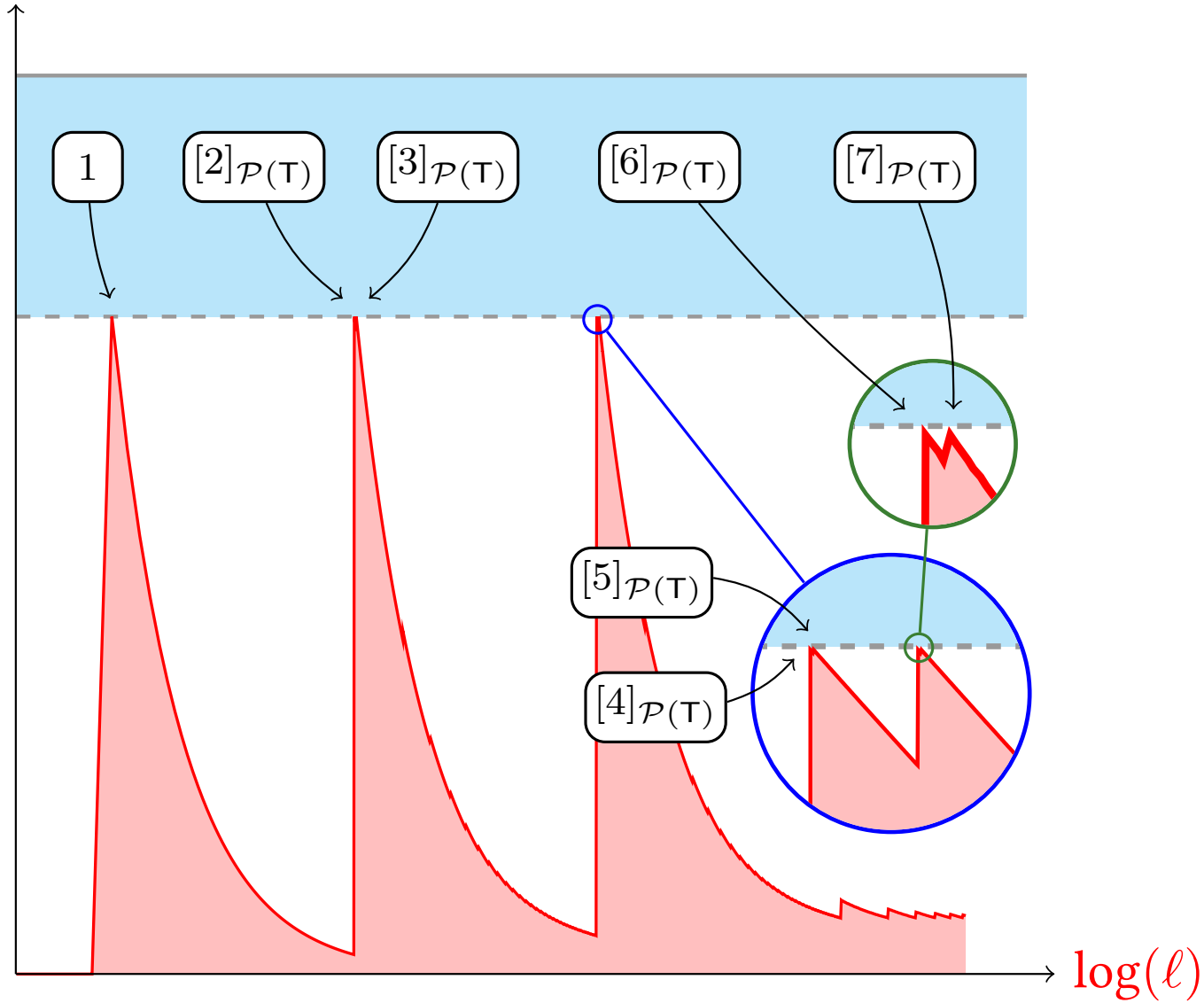

When is Feasibility Decidable in Poly. Time?

1. If deadlines are implicit. [Liu & Layland, 1973]
2. If deadlines are constrained and periods are harmonic. [Bonifaci et al., 2013]
3. If $\mathrm{U}(\Gamma) \leqslant c < 1$ and $\dfrac{\text{max period}}{\text{min period}} \leqslant q(n)$.

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Conclusion

	General case	Utilization bounded by a constant c, $0 < c < 1$
Asynchronous periodic	Strongly coNP-complete	Strongly coNP-complete
Synchronous periodic (or sporadic)	Strongly coNP-complete	**Weakly coNP-complete**

$\forall$Thank you!

$\exists$Questions?