Towards Efficient Explainability of Schedulability Properties in Real-Time Systems

Sanjoy Baruah
Washington University in Saint Louis

Pontus Ekberg
Uppsala University

ECRTS, Vienna 2023

The broader picture

The broader picture

System designer

Certification authority

The broader picture

System designer

Comes up with solution

Certification authority

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Comes up with solution

Sends *explanation* (or proof/certificate)

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Verifies solution

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Can spend lots of time/resources

Verification should be *efficient*

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Verifies solution

Can spend lots of time/resources

Verification should be *efficient*

Ideally, the explanation is *formally* verifiable

Foundational Response-Time Analysis as Explainable Evidence of Timeliness
by Maida, Bozhko and Brandenburg (ECRTS 2022)

CertiCAN certifying CAN analyses and their results
by Fradet, Guo and Quinton (Real-Time Systems 2023)

The broader picture

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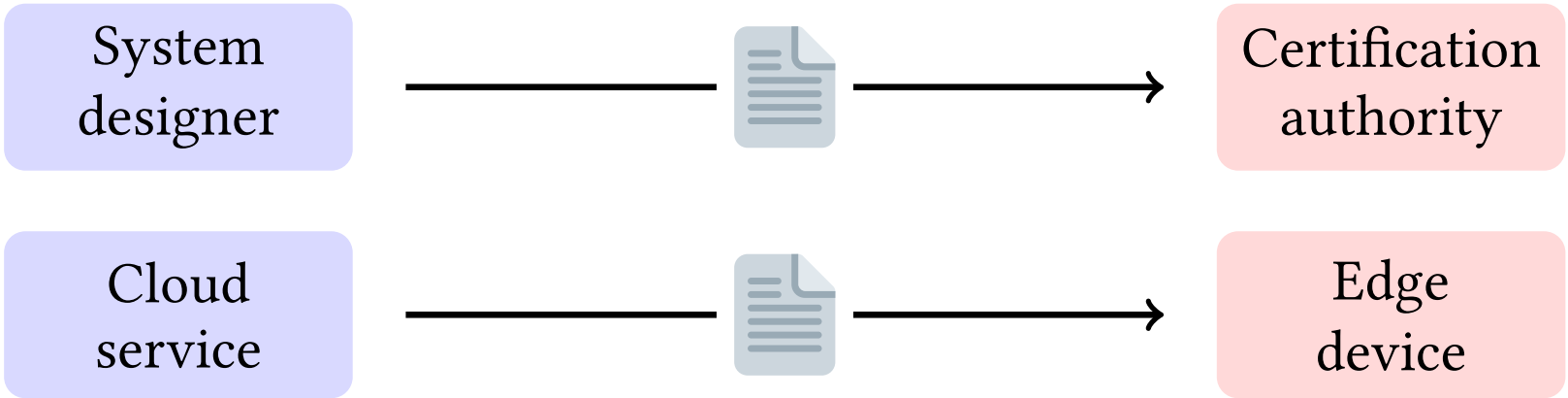

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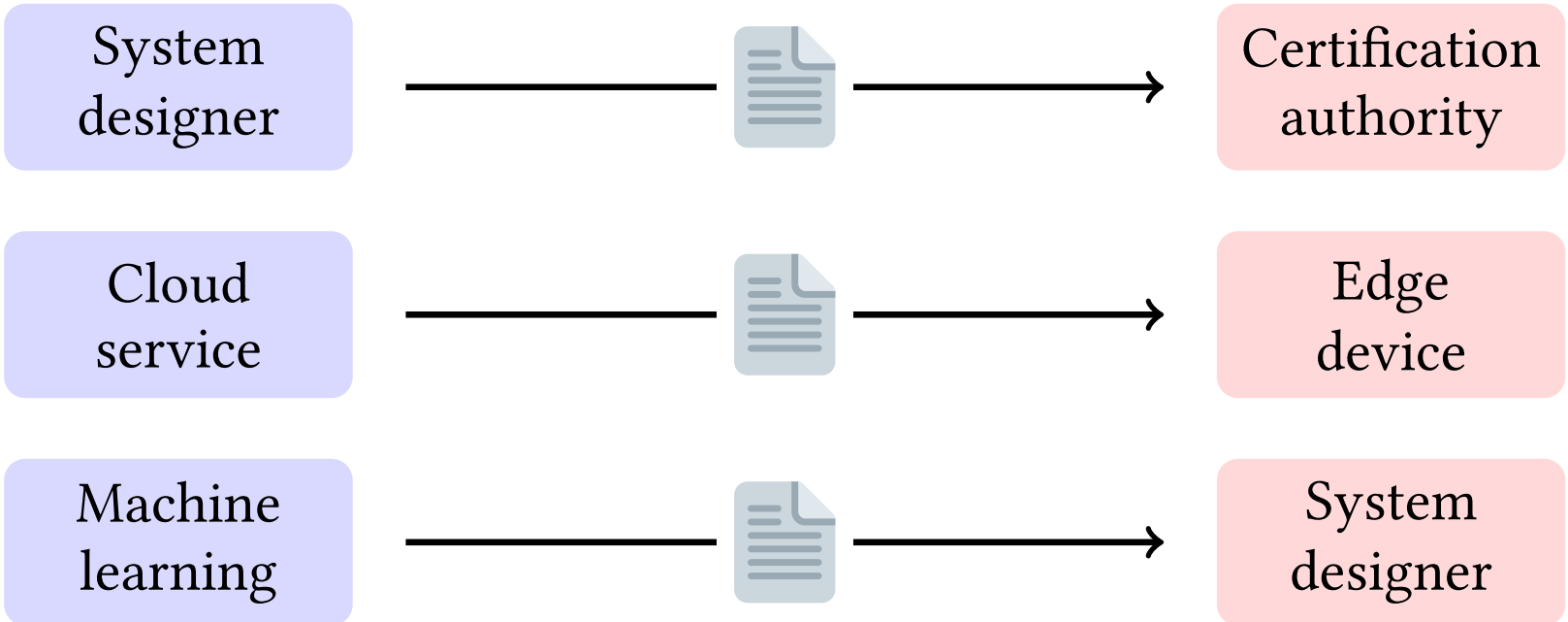

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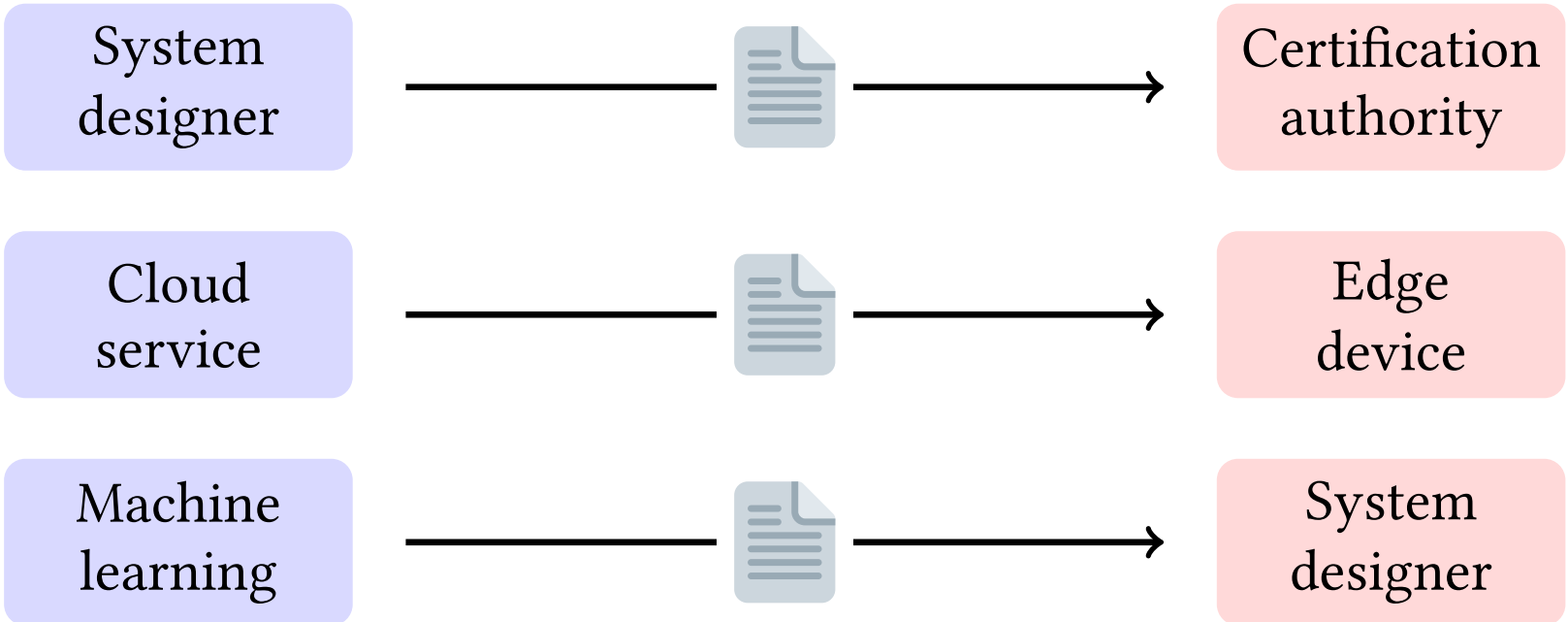

This work

- Classify schedulability problems as efficiently explainable or not

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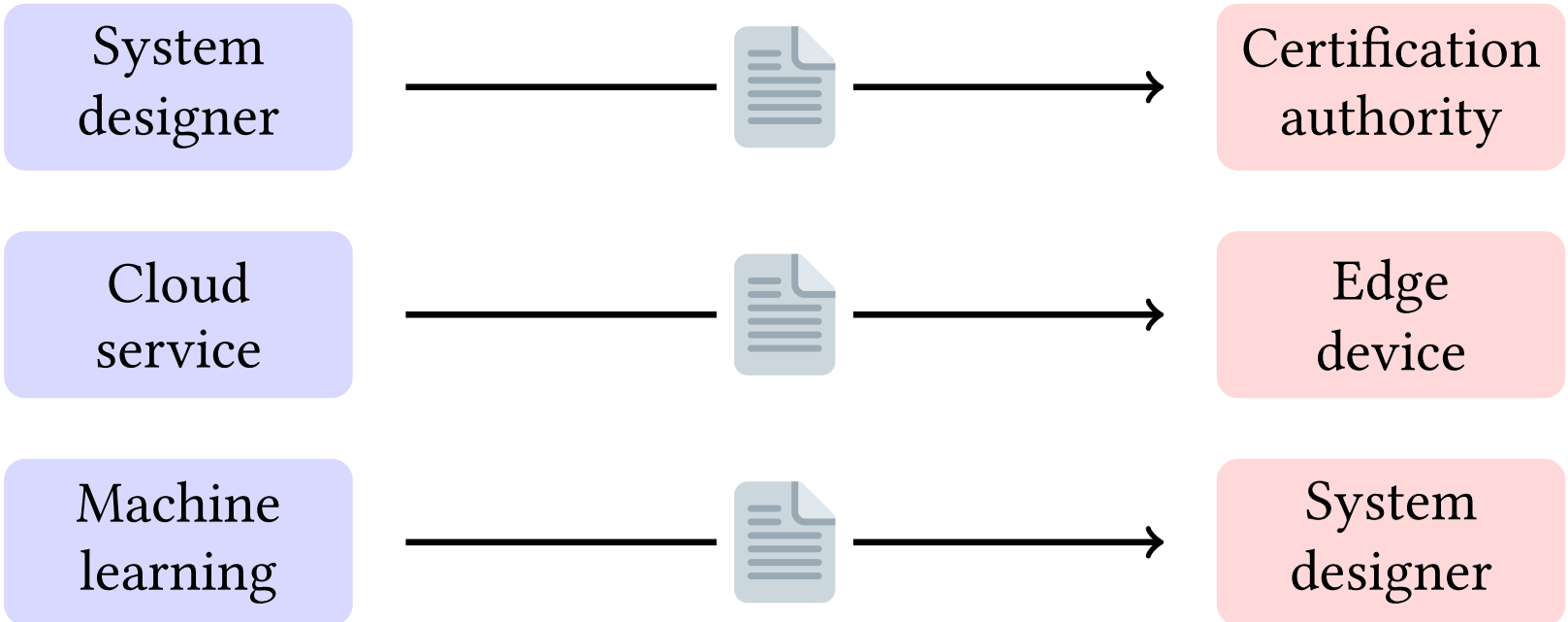

This work

- Classify schedulability problems as efficiently explainable or not
- Investigate techniques for dealing with the problems that are not

Example: Uniprocessor FP-schedulability

RTA (Joseph and Pandya, 1986, and others)

Constrained-deadline task system $\Gamma = \{\tau_1, \ldots, \tau_n\}$ is FP-schedulable iff the smallest positive fixed-point R_i,

$$R_i = C_i + \sum_{j \in \mathrm{hp}(i)} \left\lceil \frac{R_i}{T_j} \right\rceil \times C_j,$$

is $\leqslant D_i$ for each $\tau_i \in \Gamma$.

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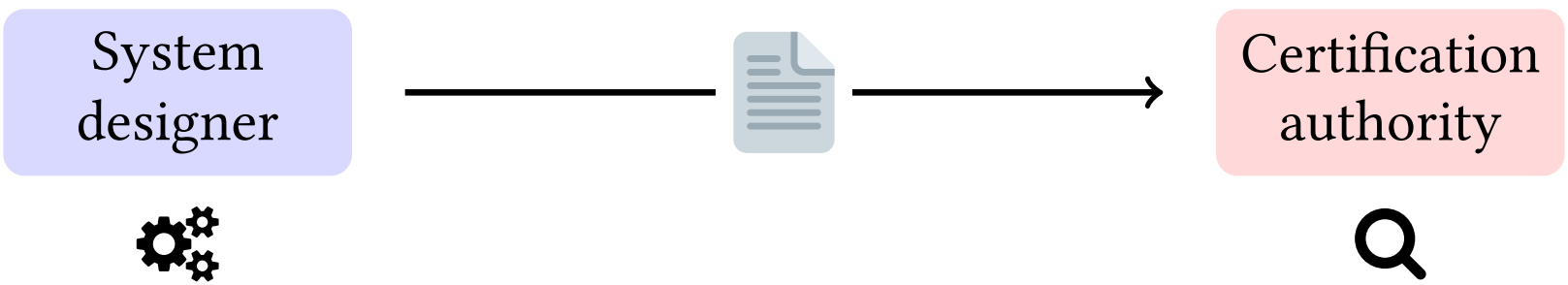

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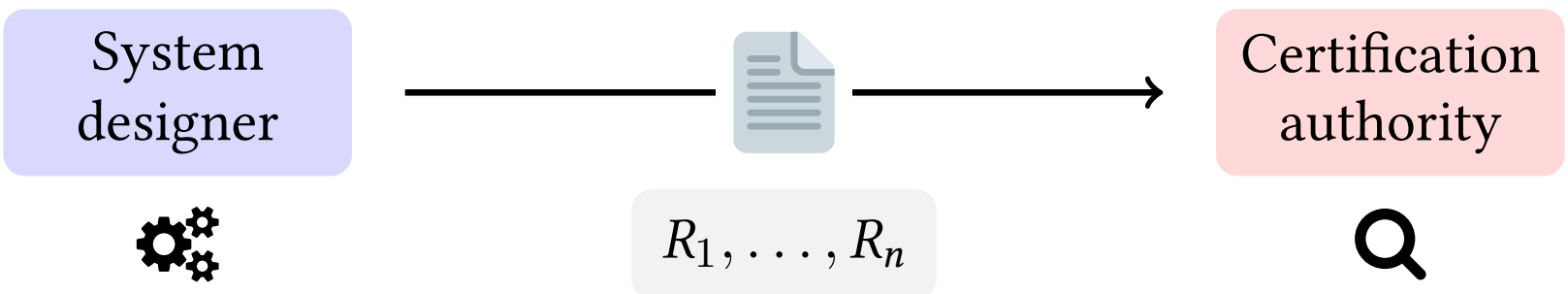

Example 2: Uniprocessor EDF-schedulability

PDA (Baruah et al., 1990)

Task system $\Gamma = \{\tau_1, \ldots, \tau_n\}$ is EDF-schedulable iff

$$\sum_{\tau_i \in \Gamma} \mathrm{dbf}_i(t) \leqslant t,$$

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$$\mathrm{dbf}_i(t) \stackrel{\text{def}}{=} \max\left(\left\lfloor \frac{t - D_i}{T_i} \right\rfloor + 1, 0\right) \times C_i$$

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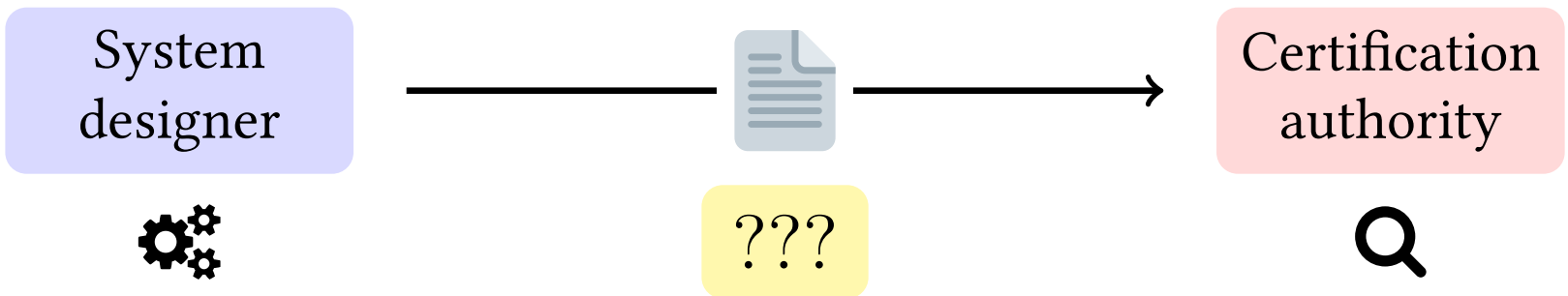
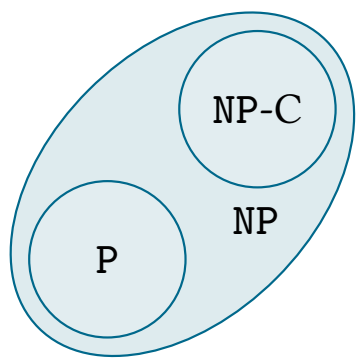


Does this relate to complexity?



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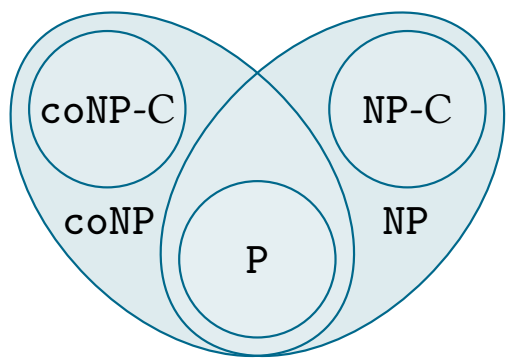

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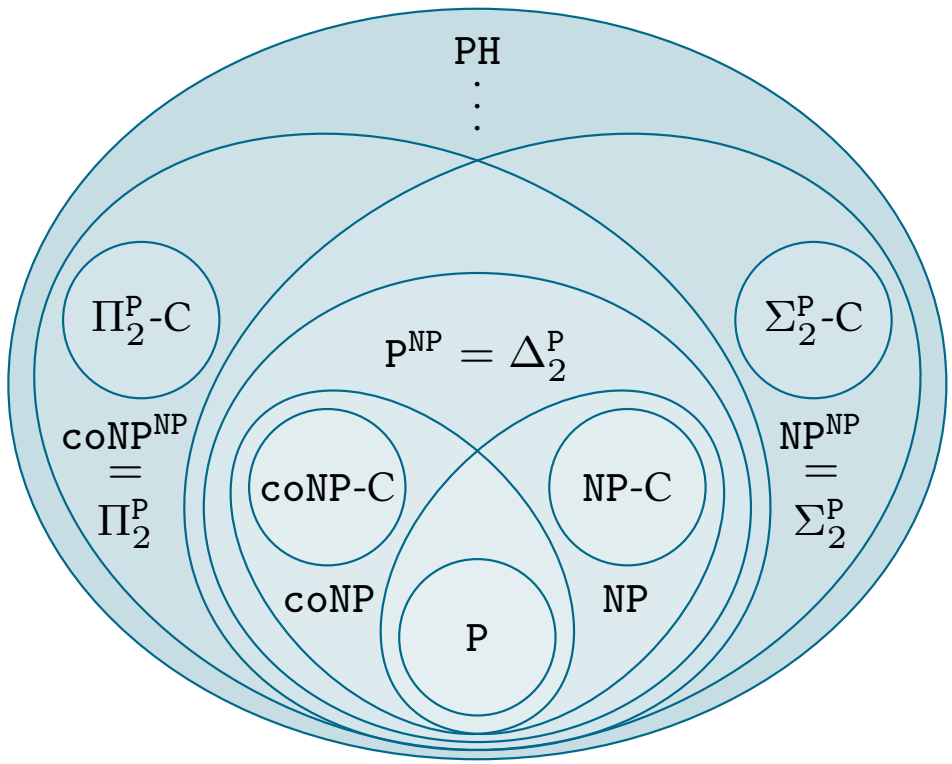

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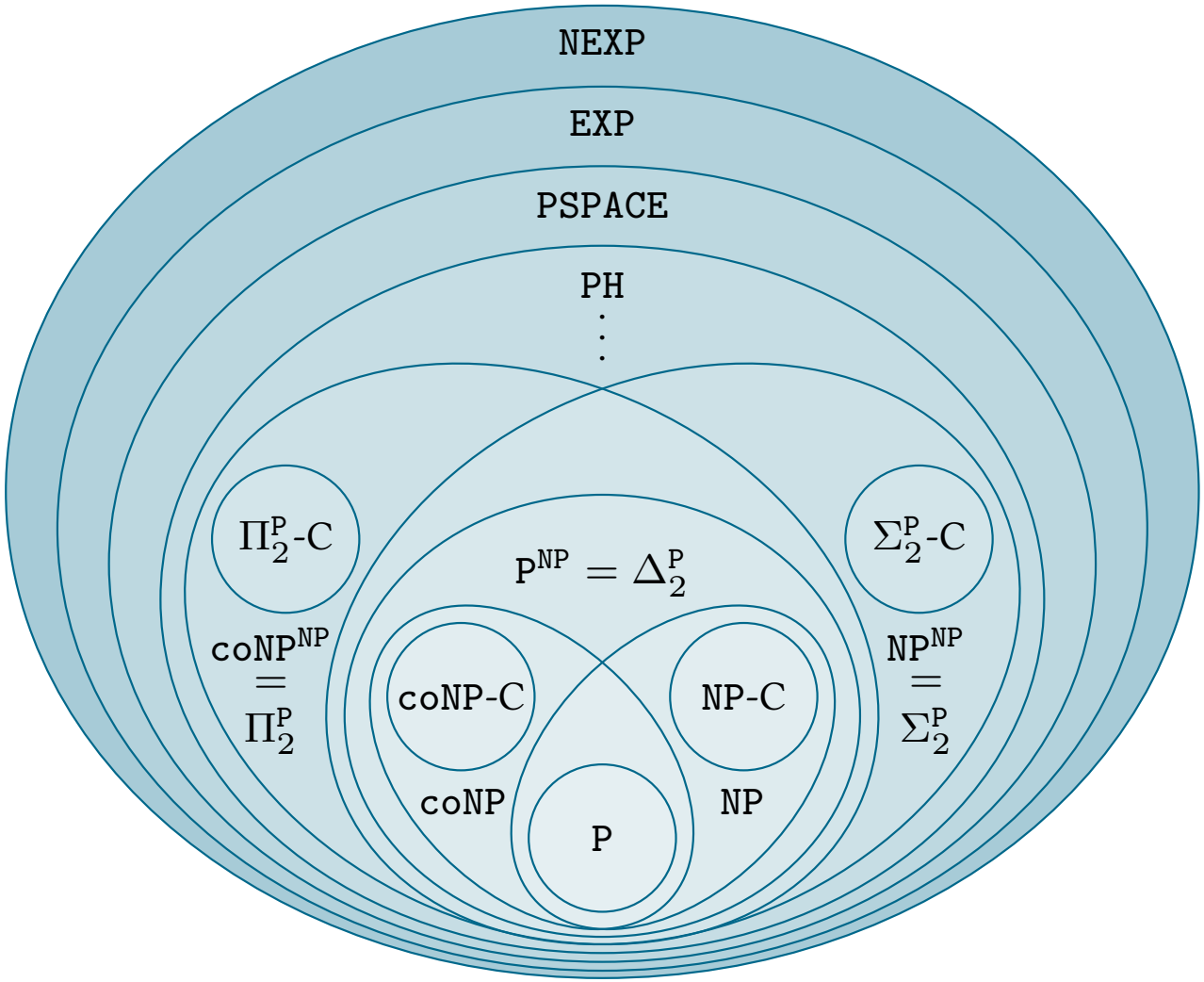

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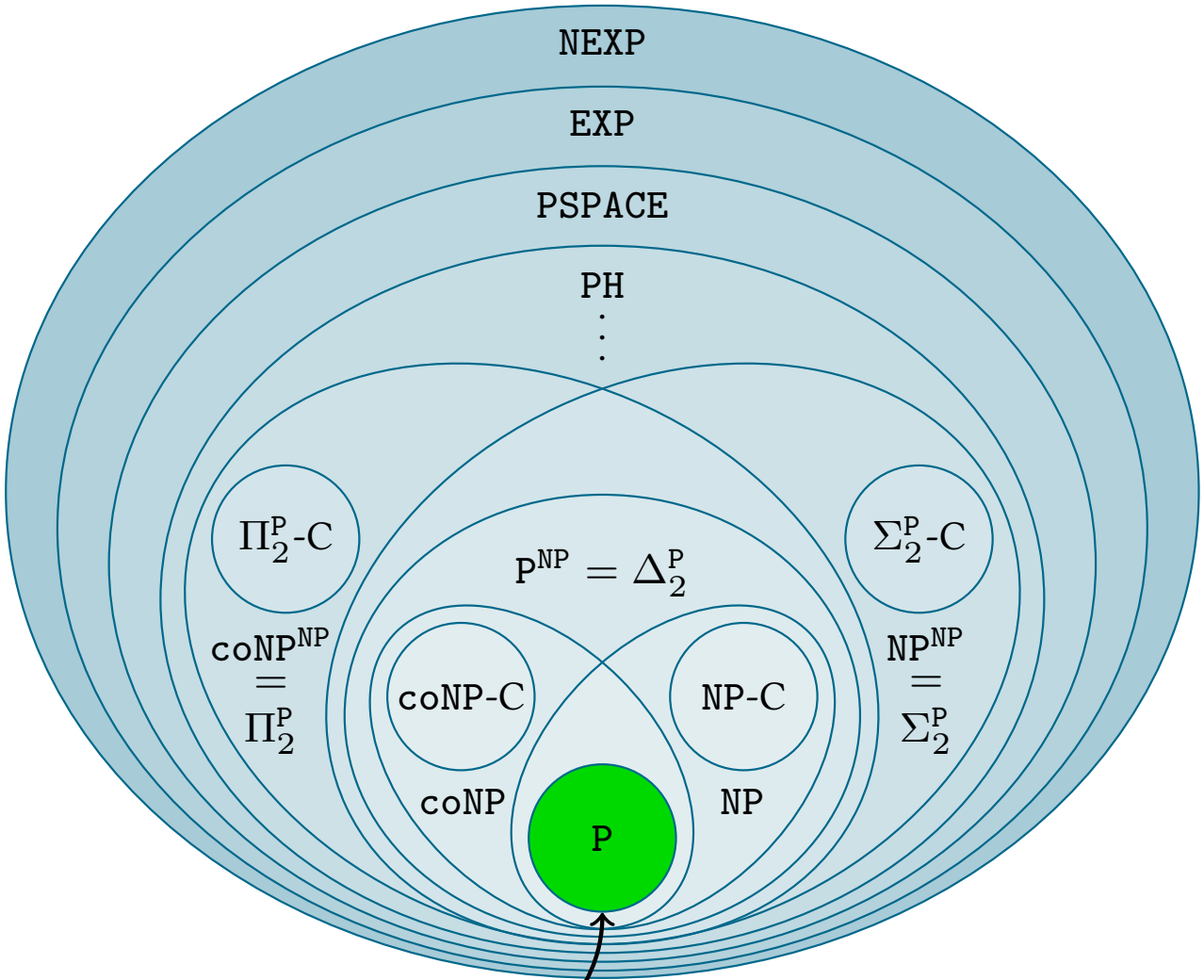

P = solvable in polynomial time

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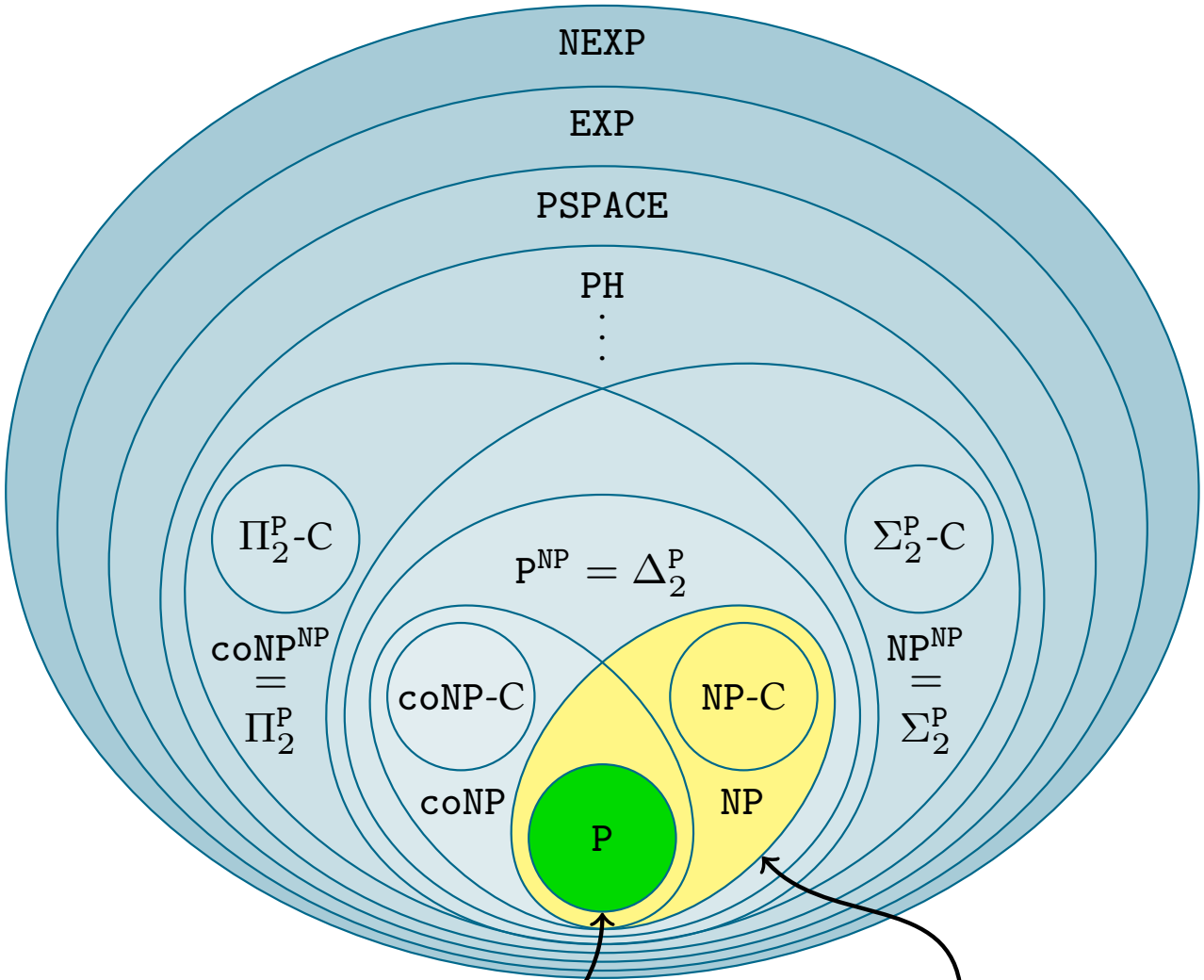

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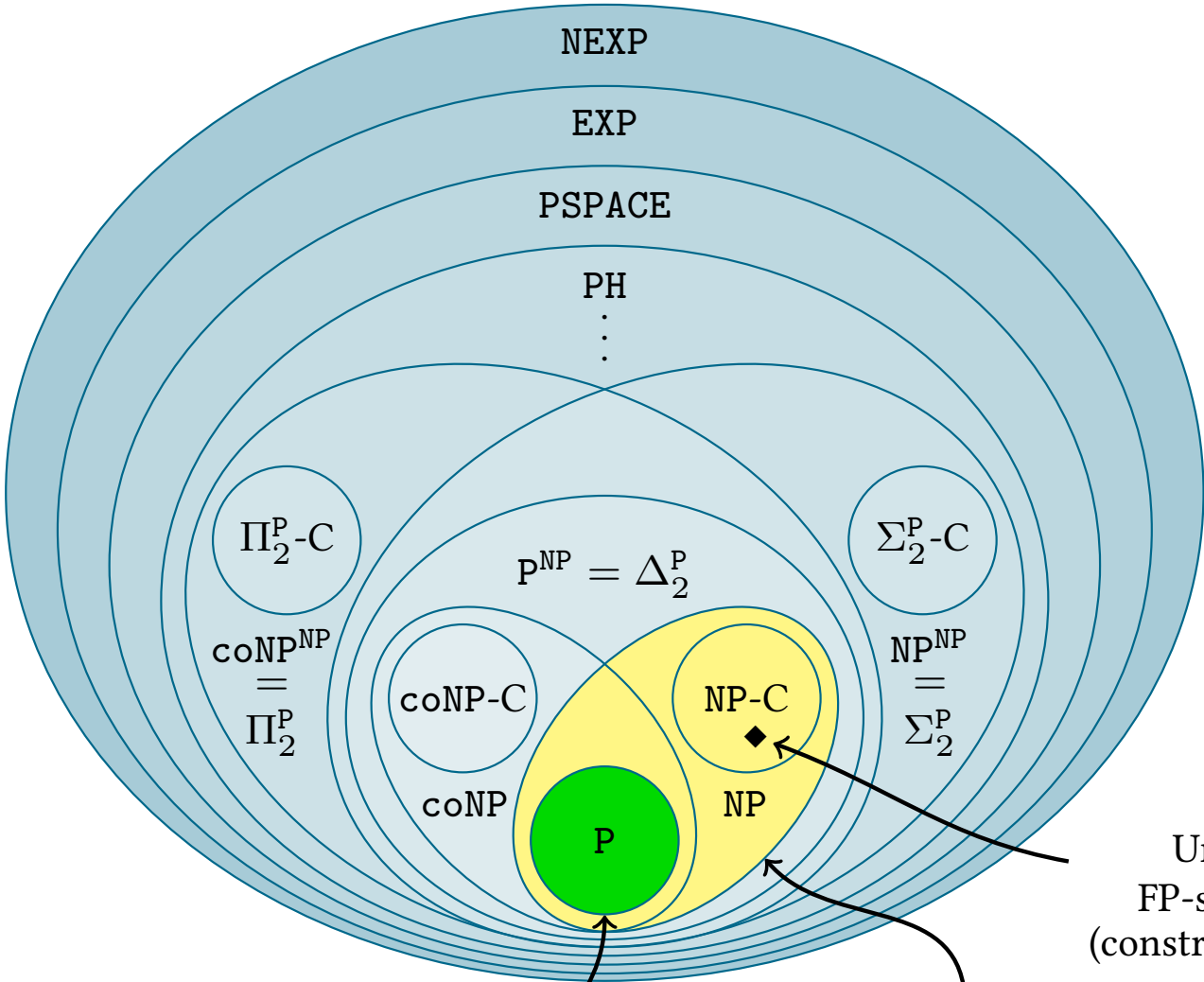
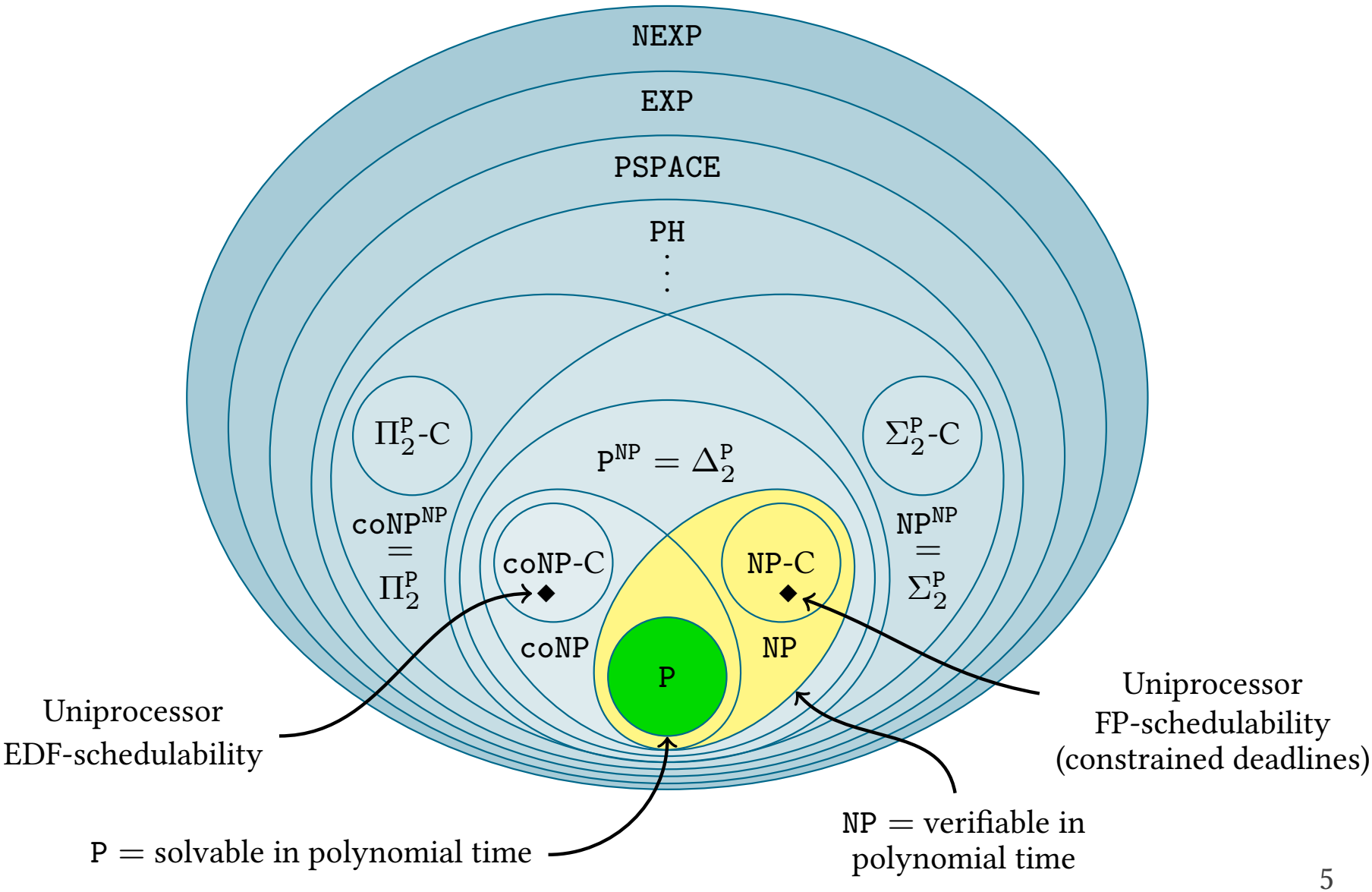


Uniprocessor FP-schedulability (constrained deadlines)

NP = verifiable in polynomial time

P = solvable in polynomial time

Does this relate to complexity?



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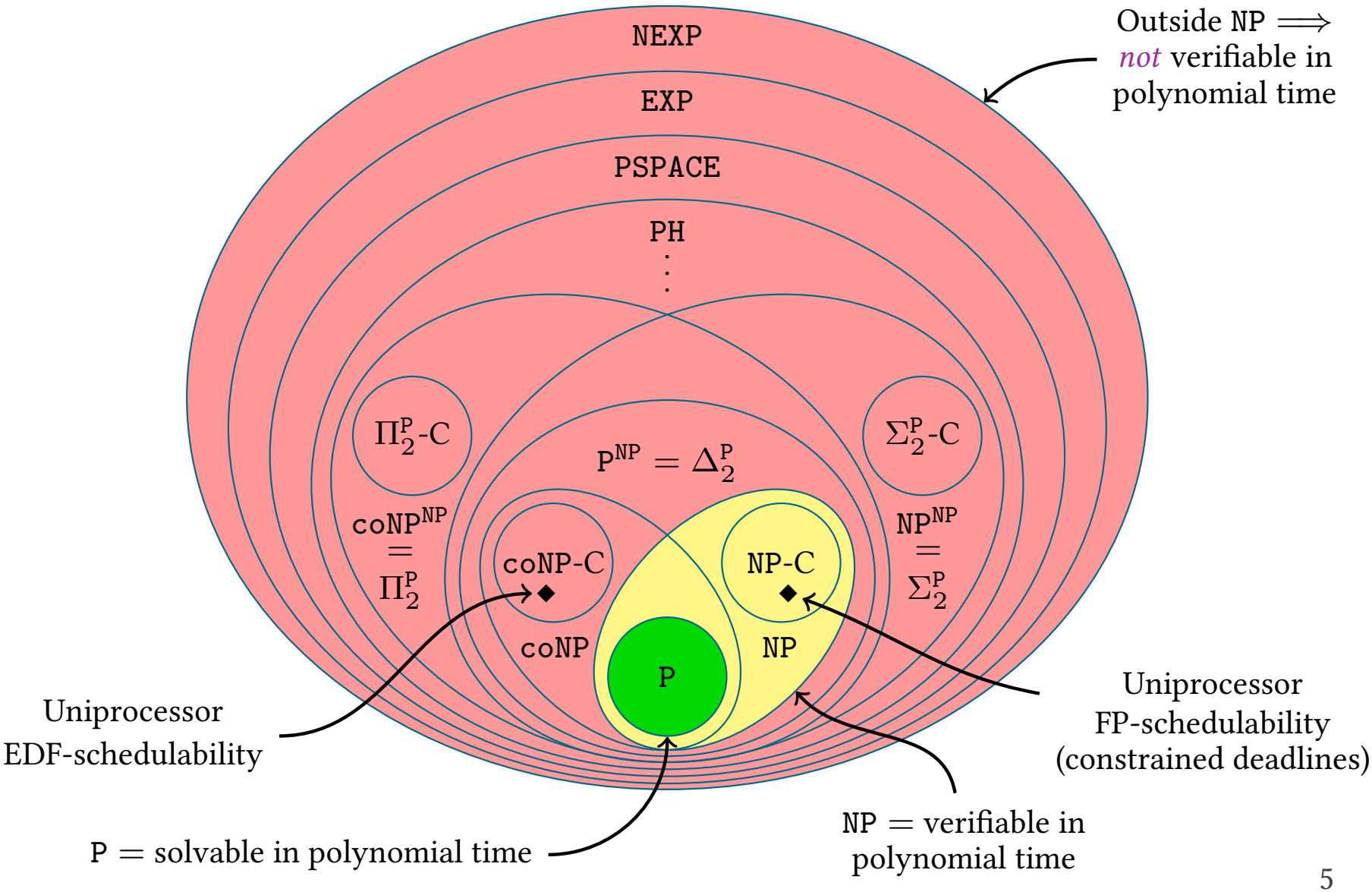

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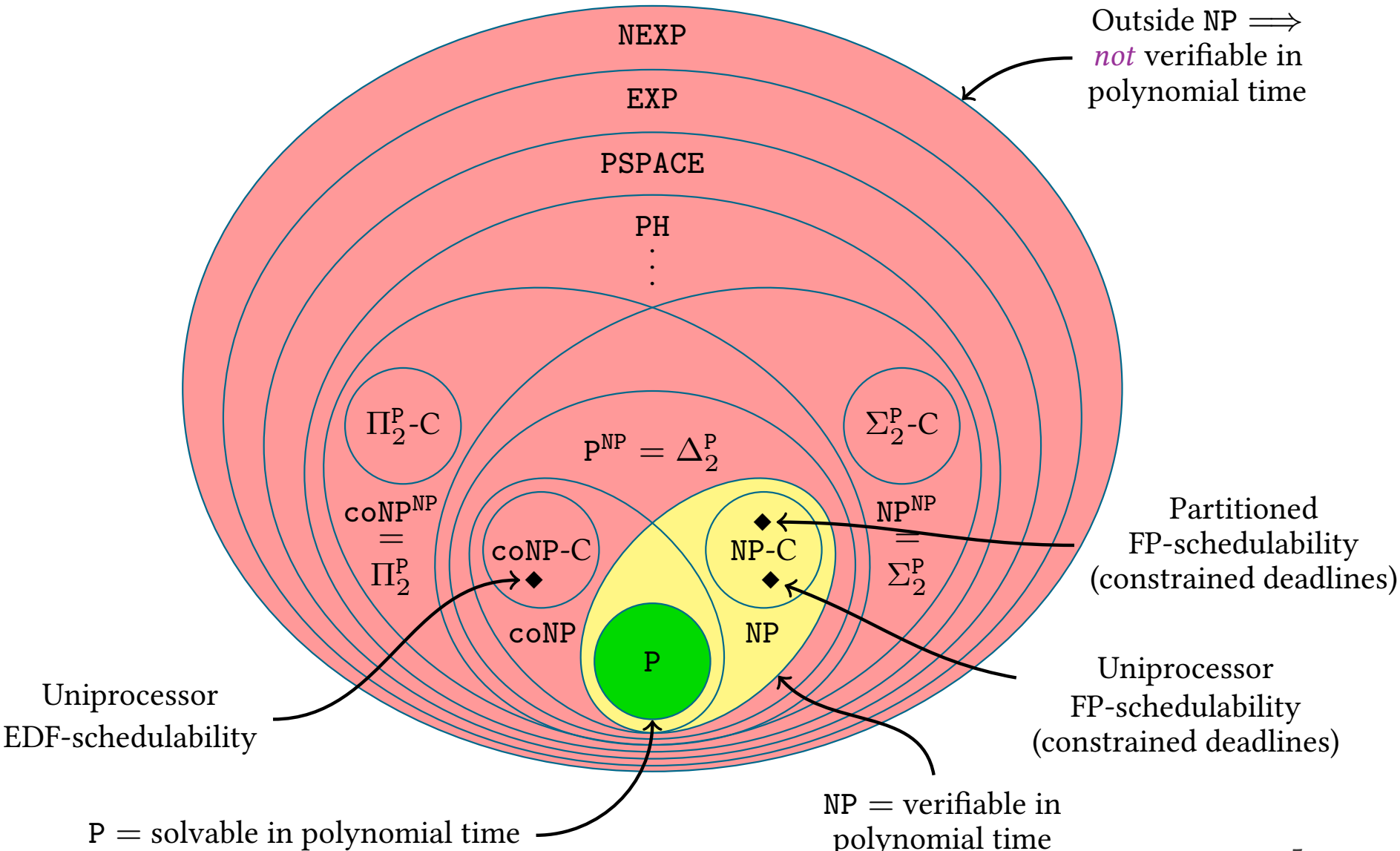

Does this relate to complexity?

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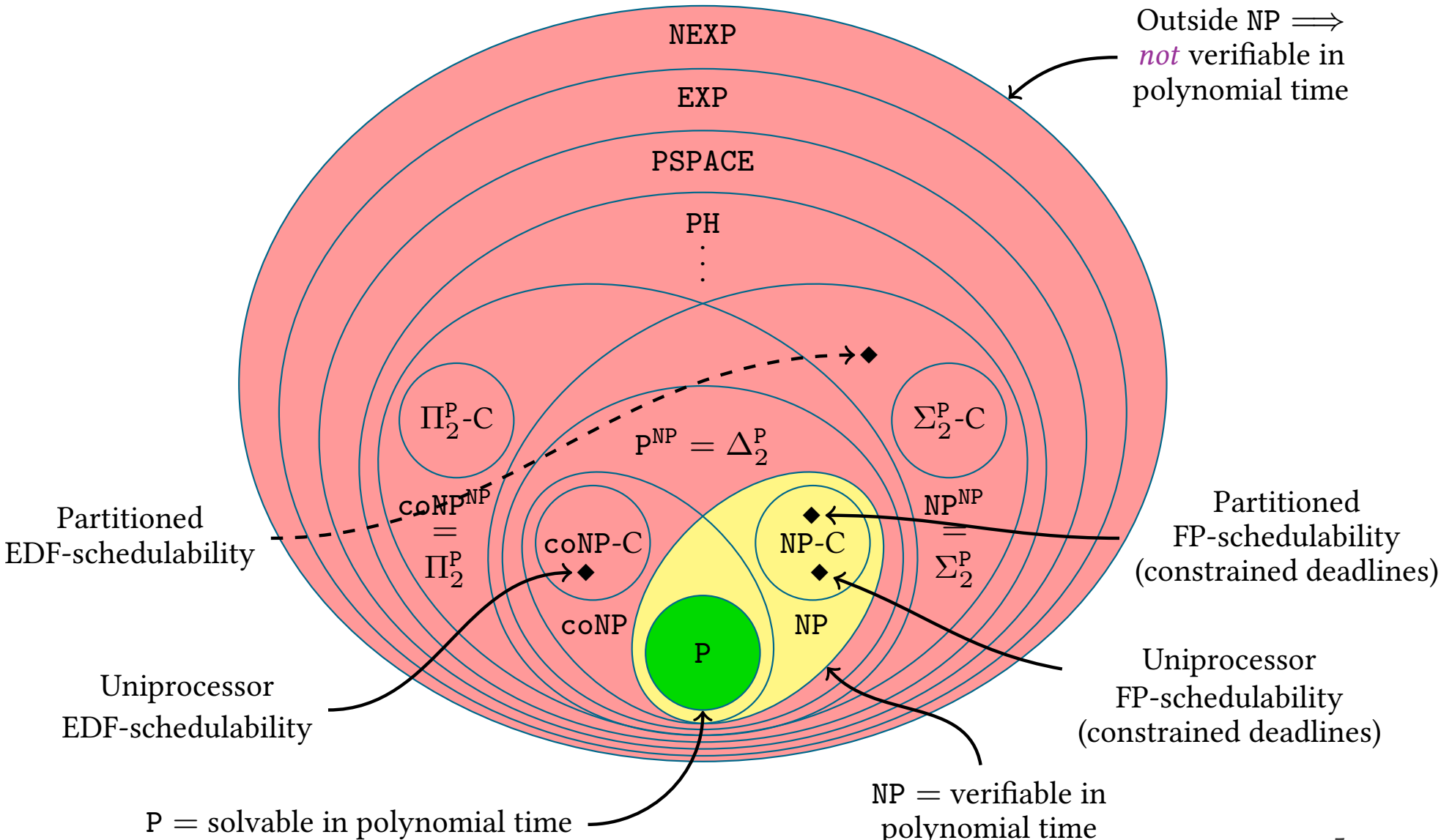

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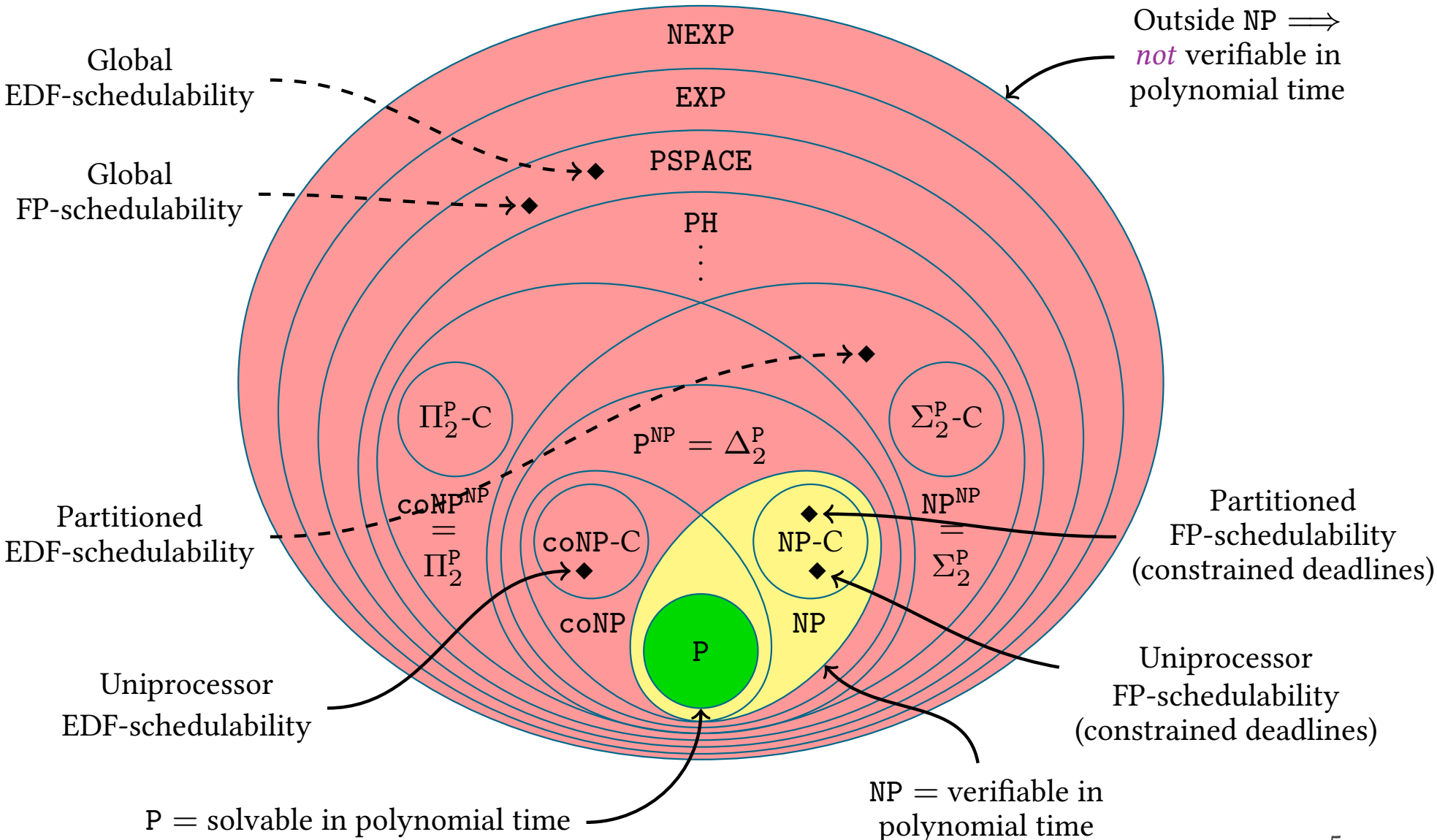

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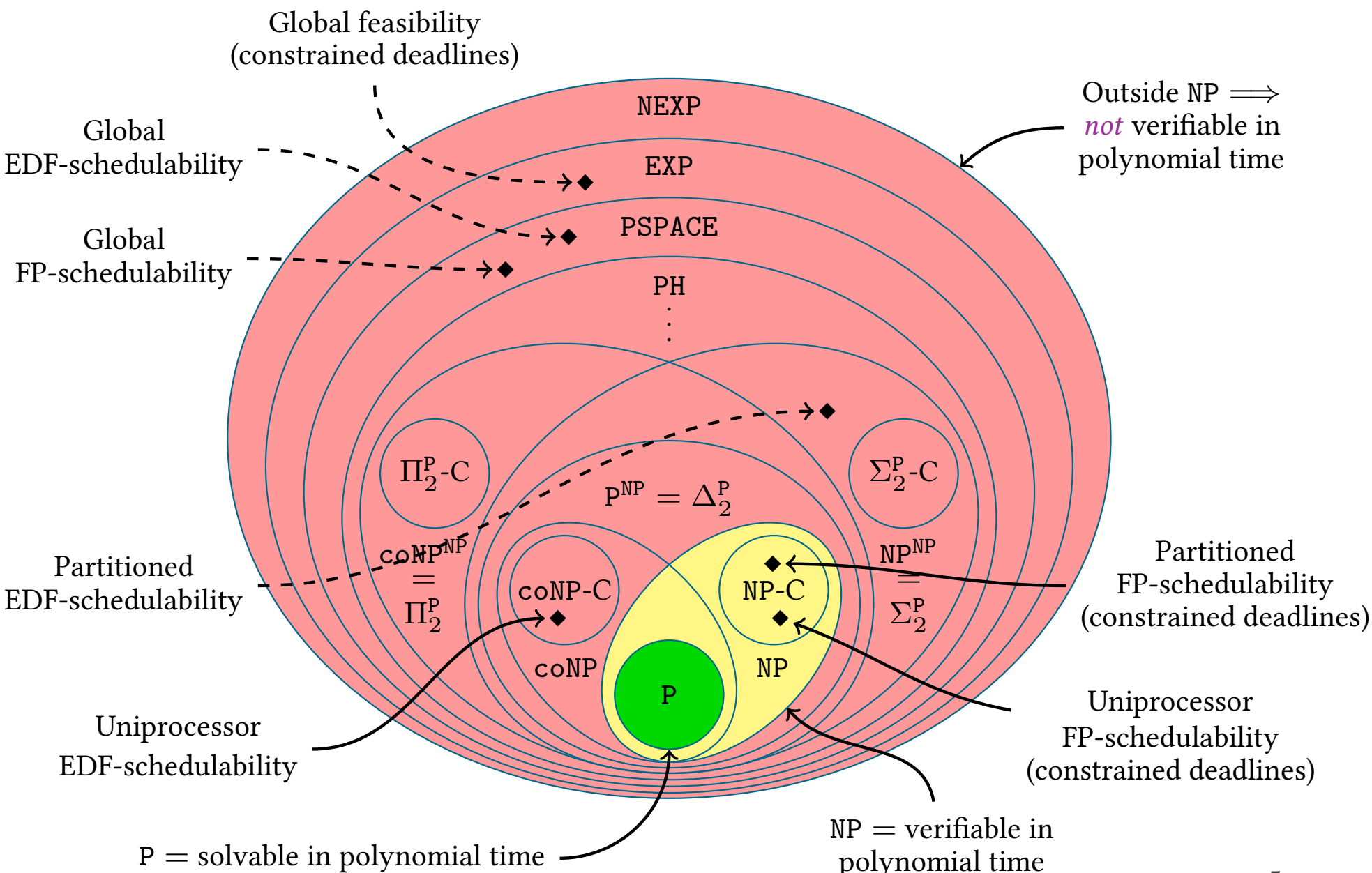

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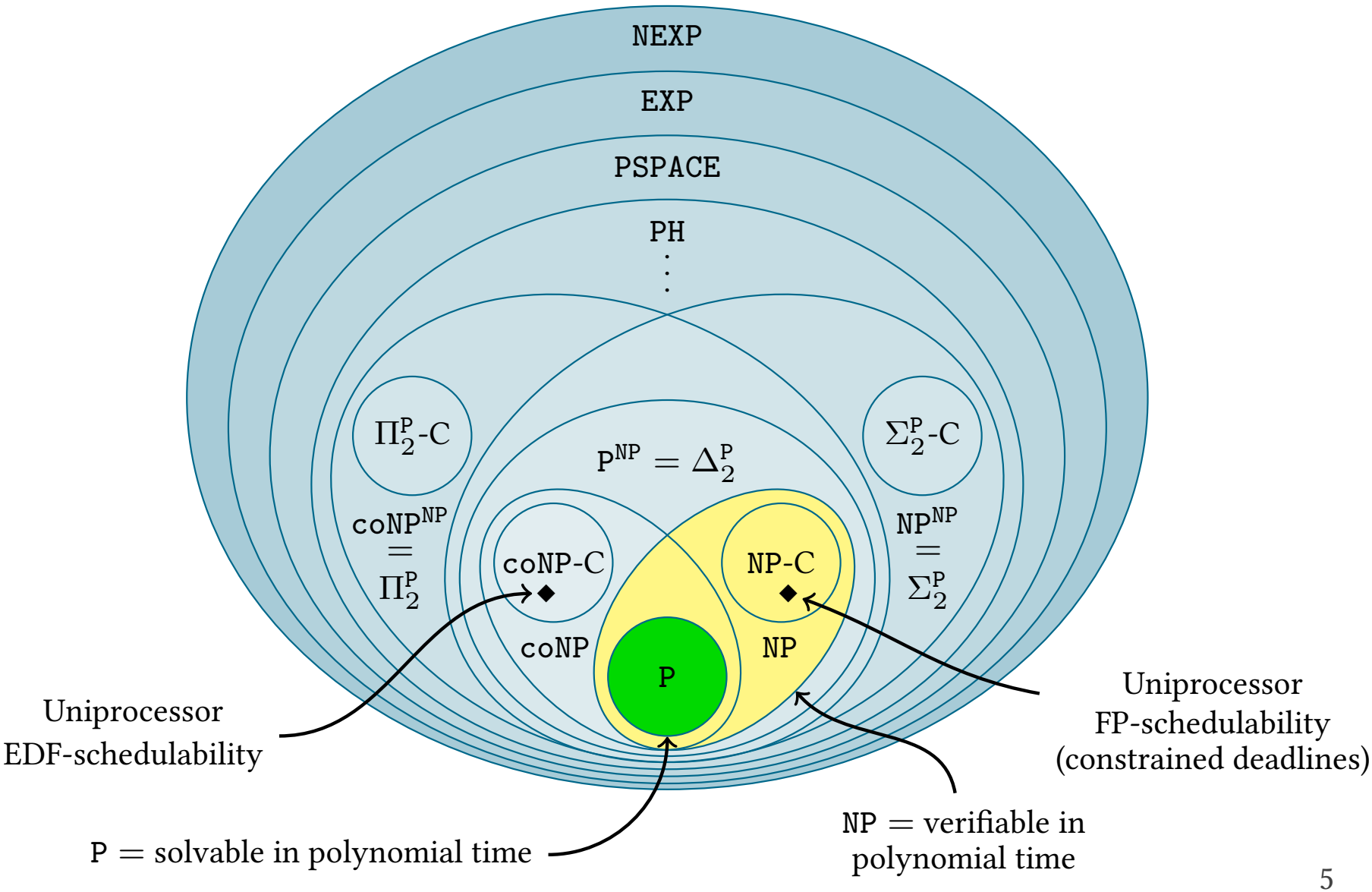

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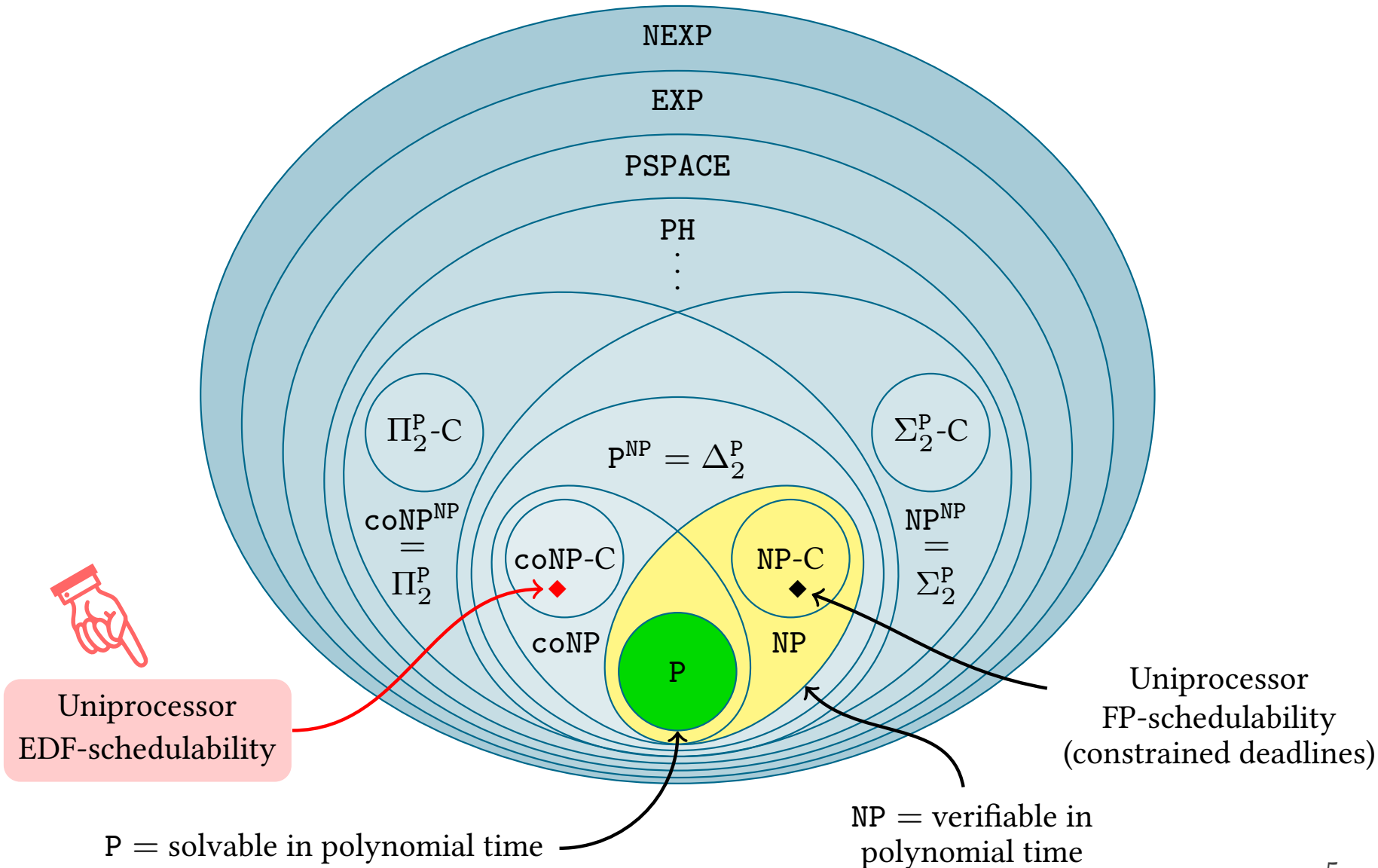
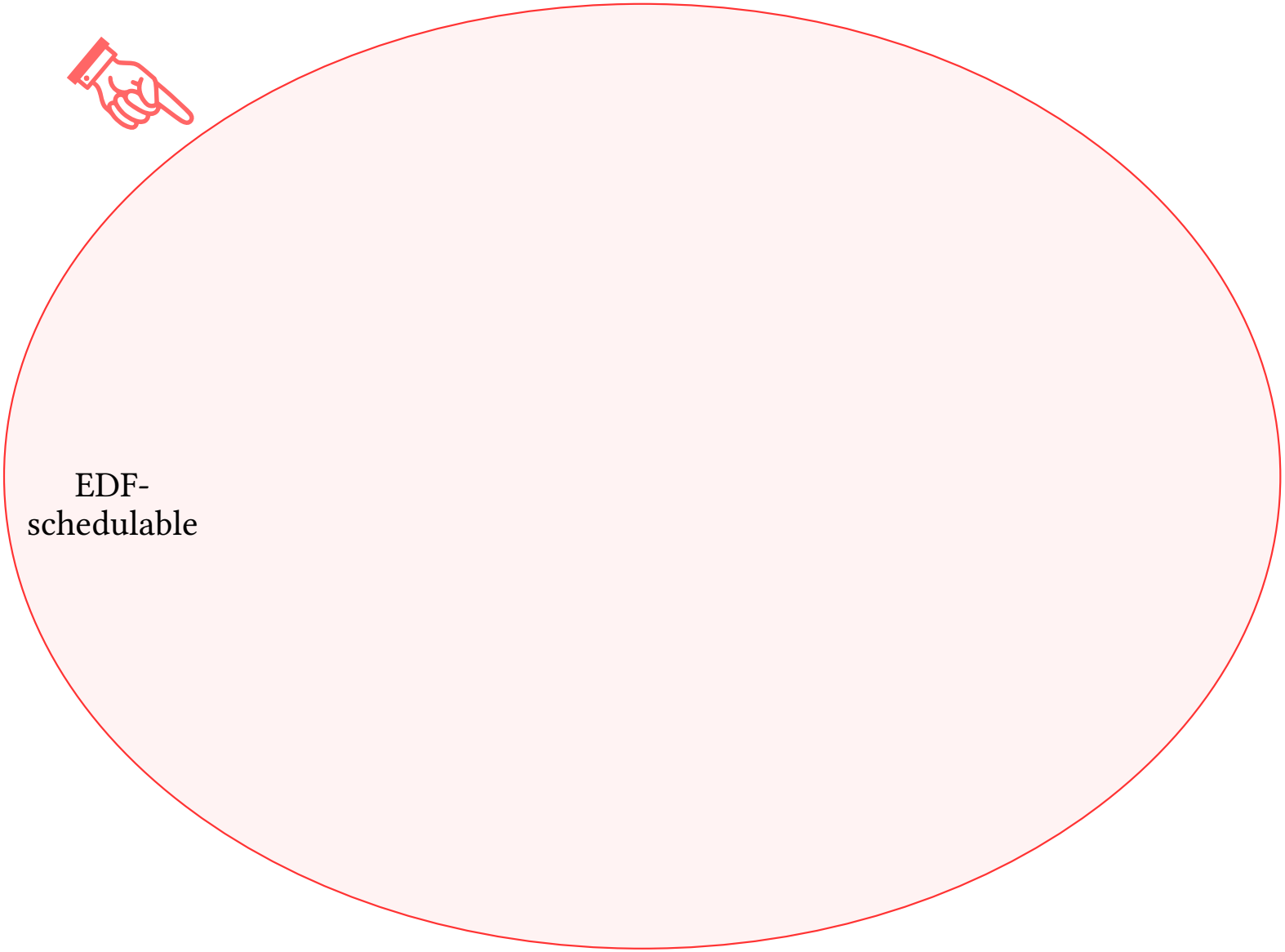
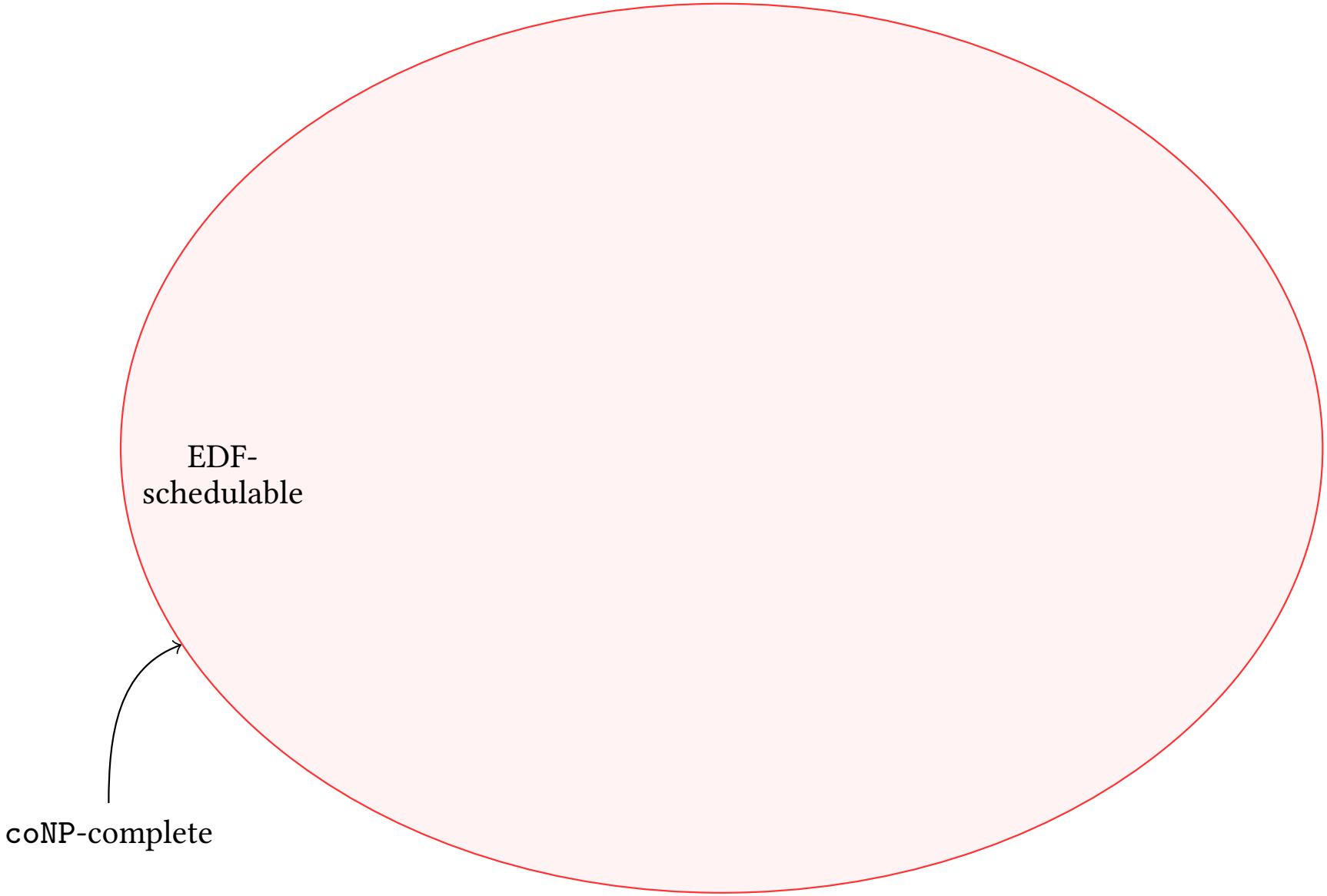


Dealing with EDF



(Sporadic tasks, preemptive uniprocessor)

Dealing with EDF



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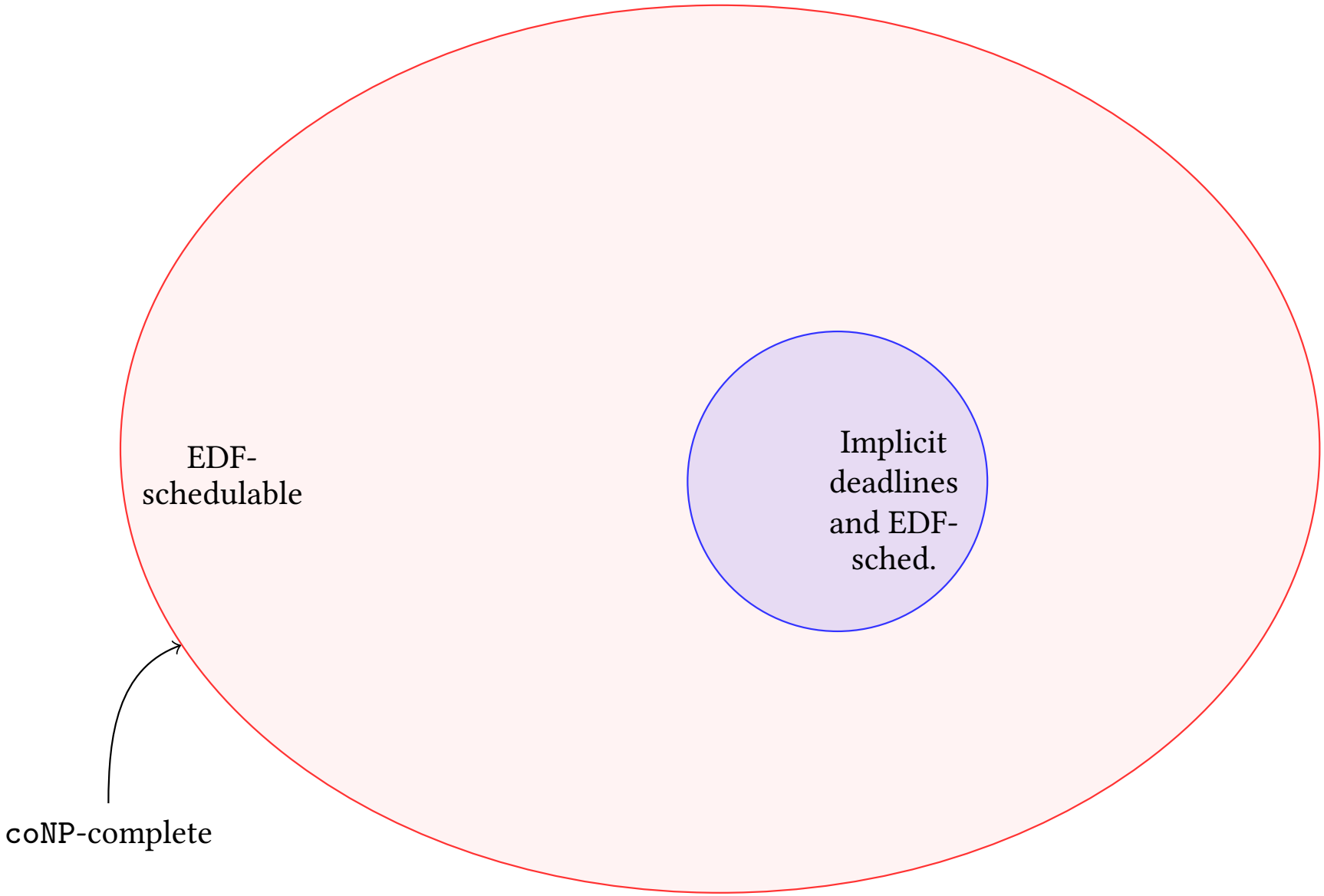

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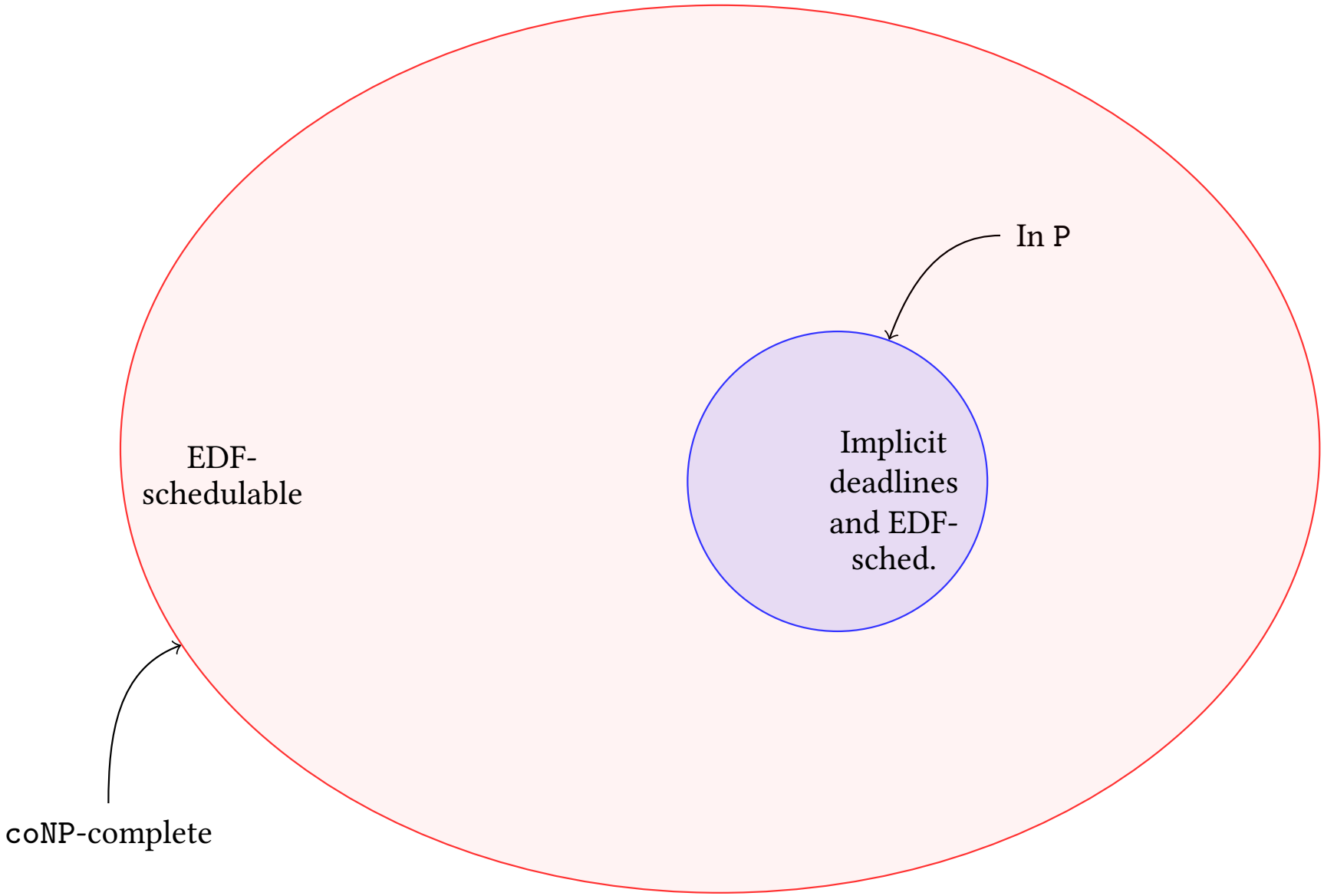

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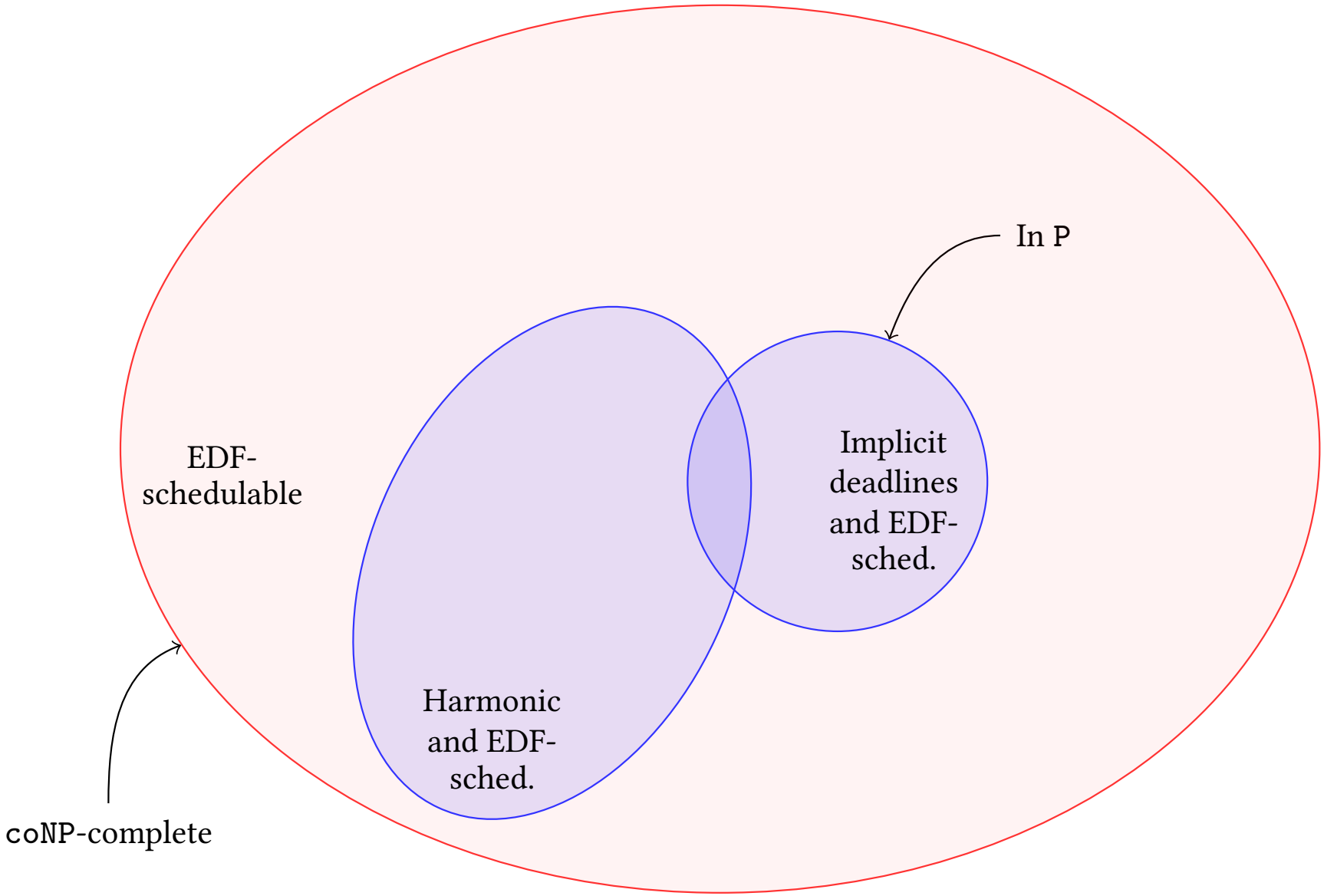

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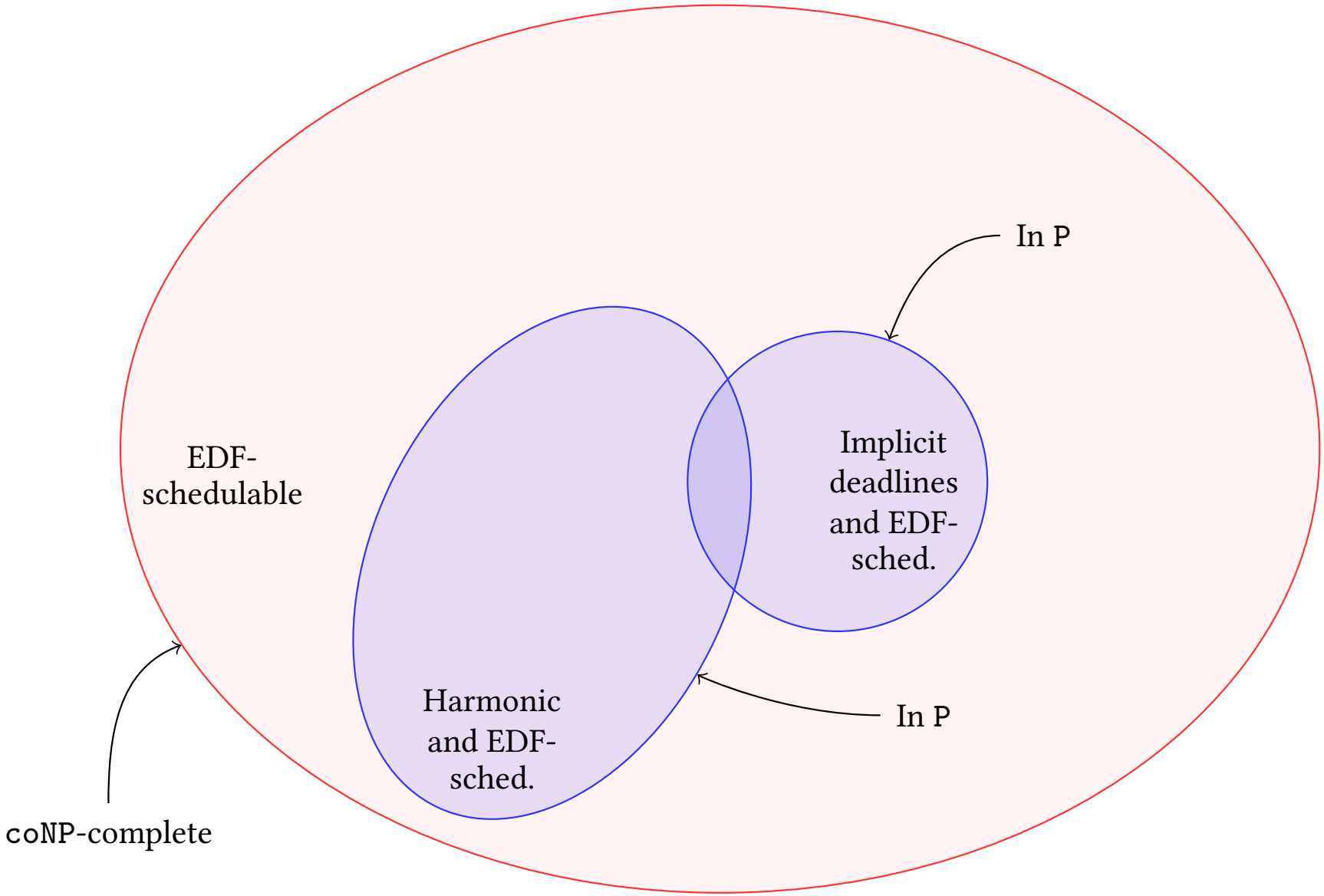

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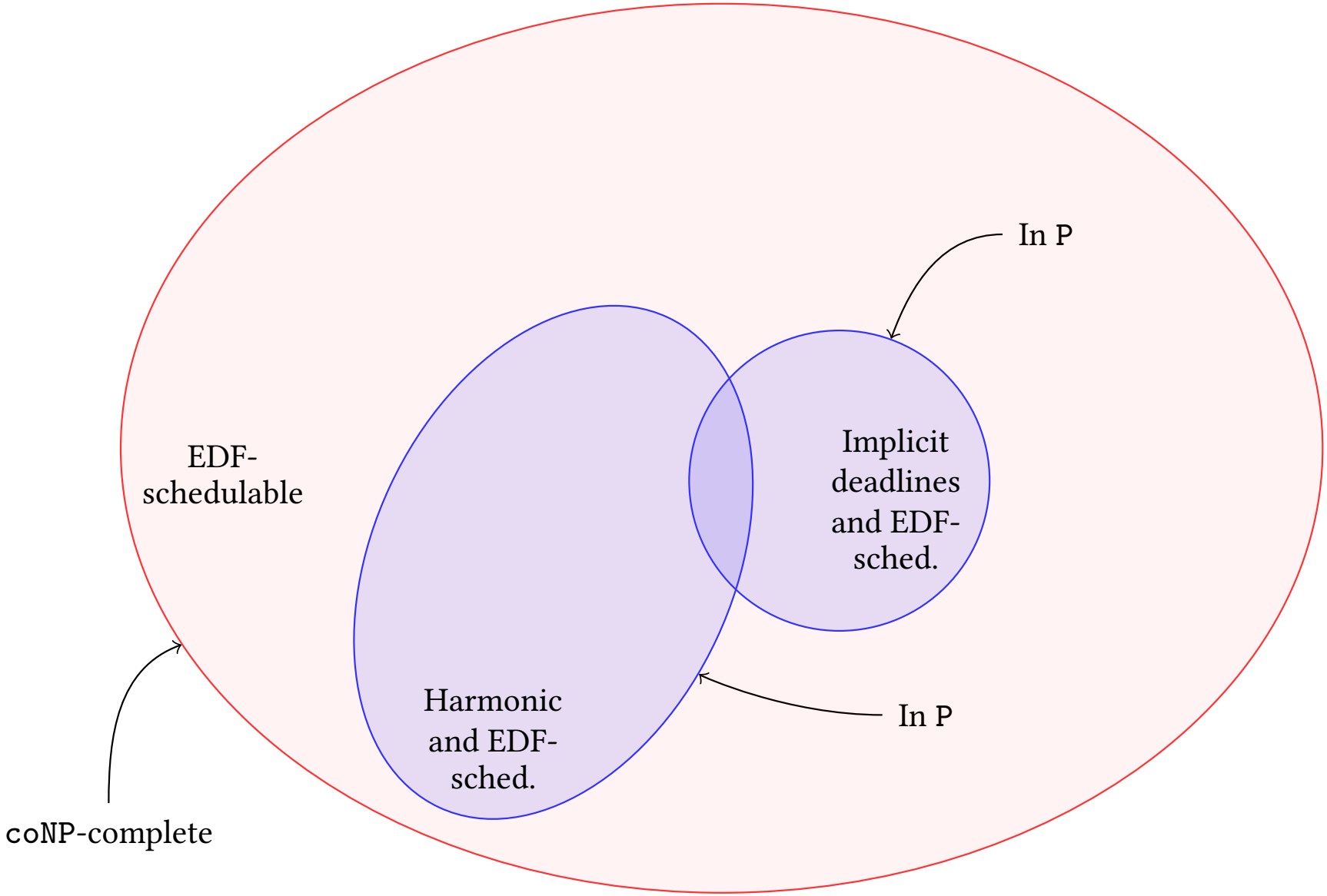
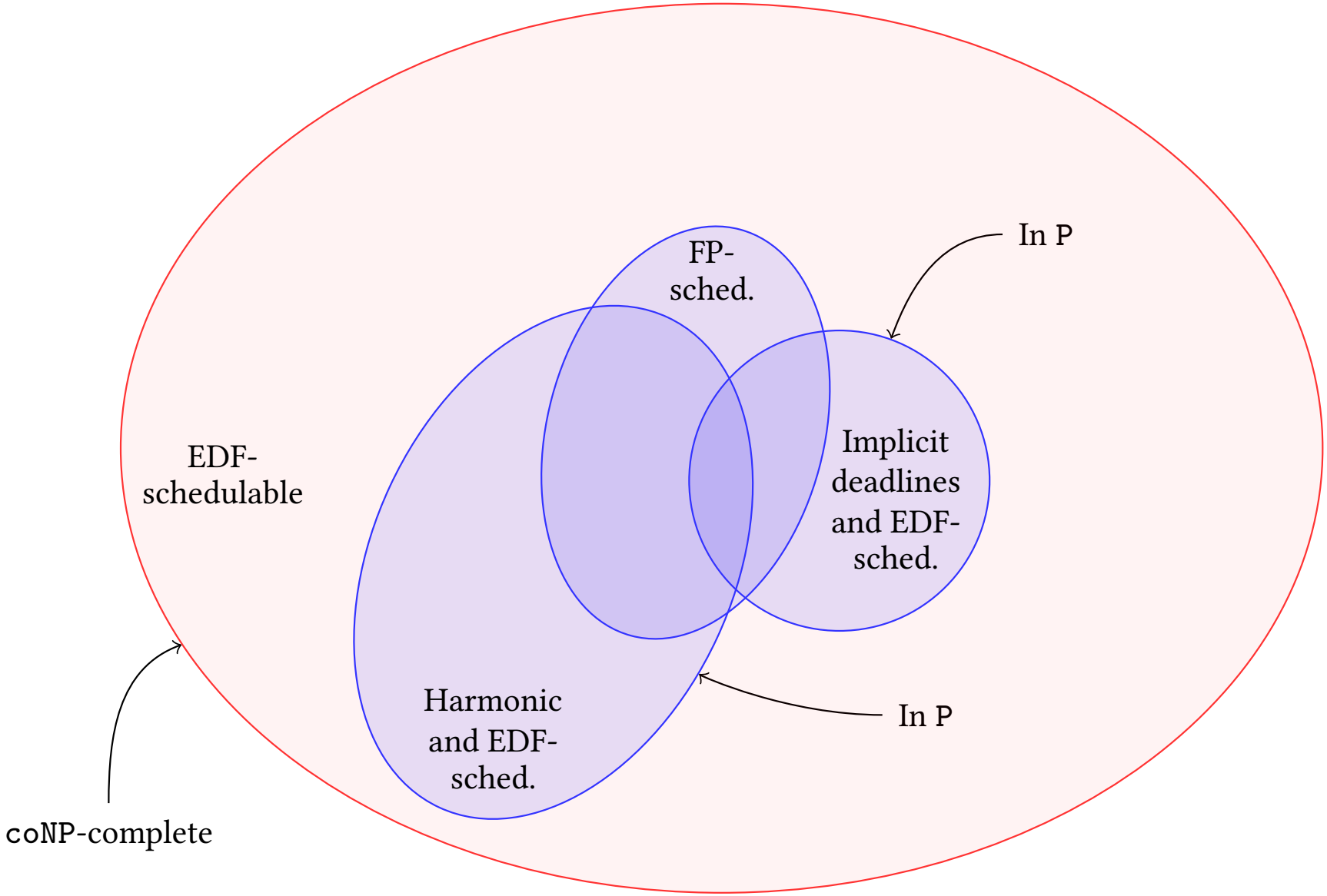


FP-schedulability $\Longrightarrow$ EDF-schedulability

Dealing with EDF



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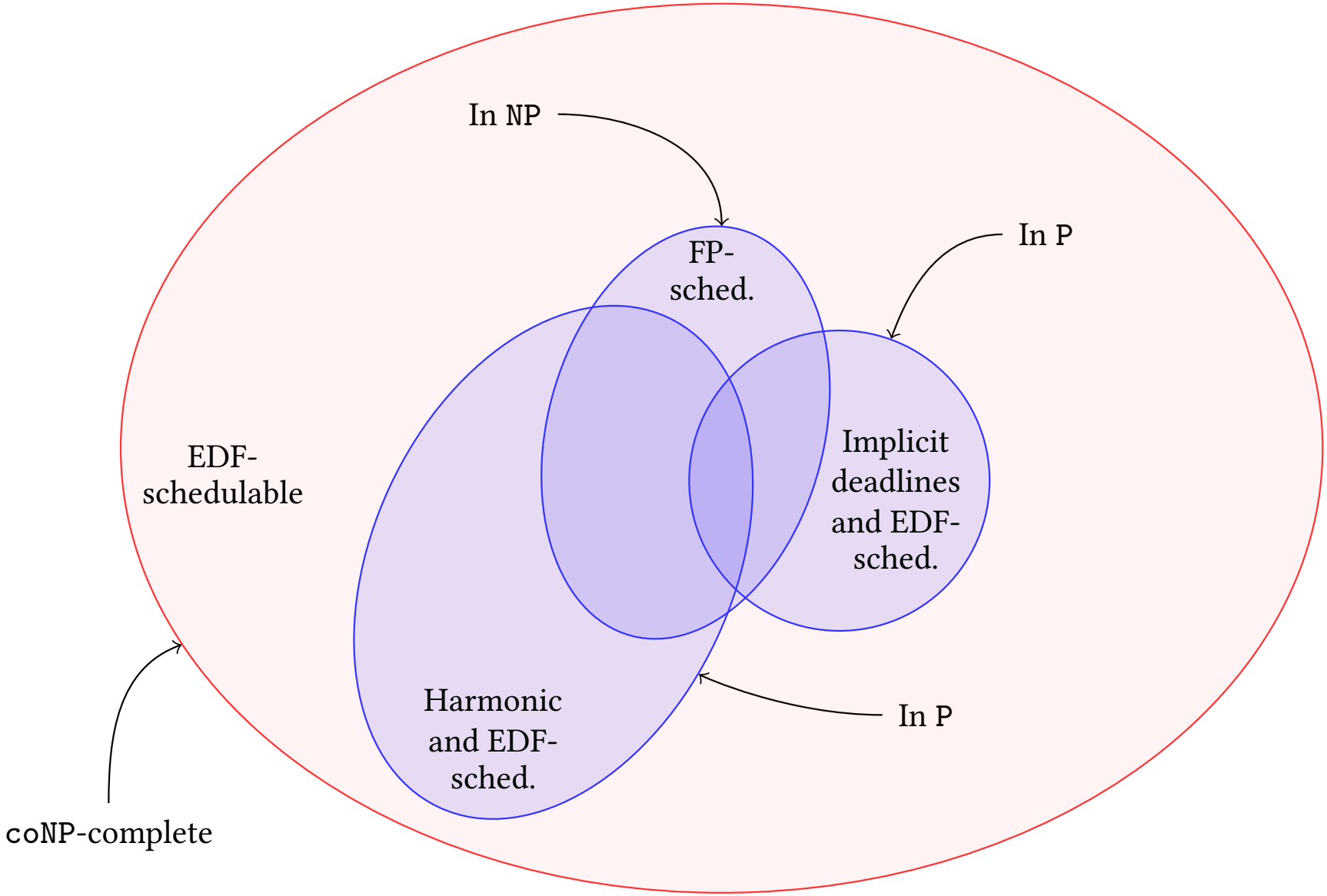

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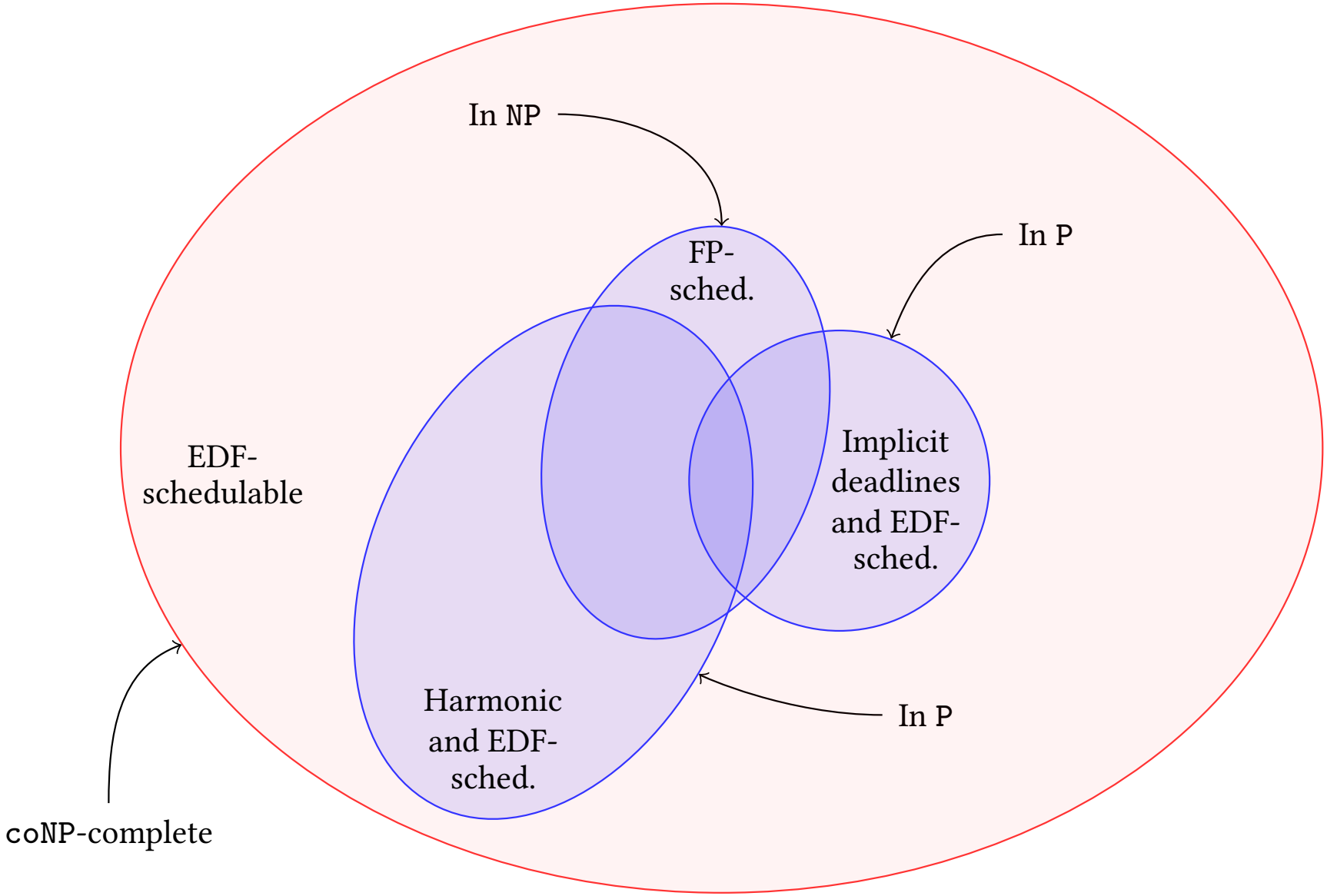

$$\forall \mathcal{A} : \quad \mathcal{A}\text{-schedulability} \Longrightarrow \text{EDF-schedulability}$$

Let's make up a scheduler: FP+fluid

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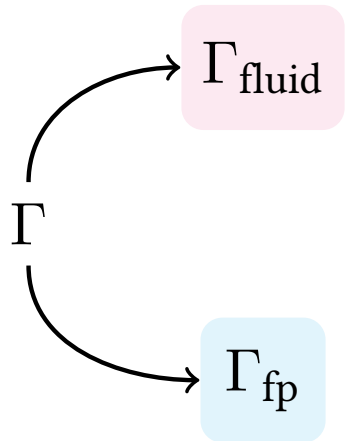

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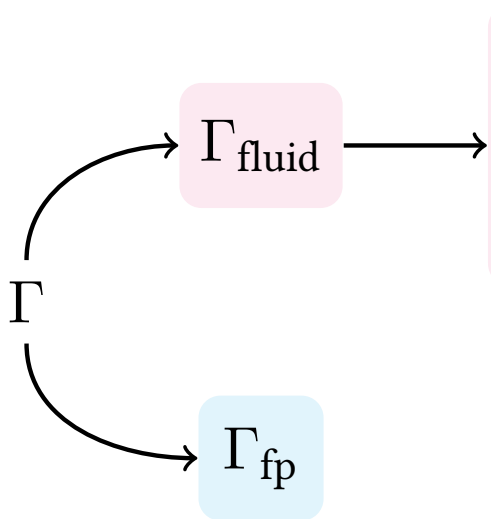

Runs fluidly on reserved processor capacity

$$\Delta = \sum_{\tau_i \in \Gamma_{\text{fluid}}} \frac{C_i}{\min(D_i, T_i)}$$

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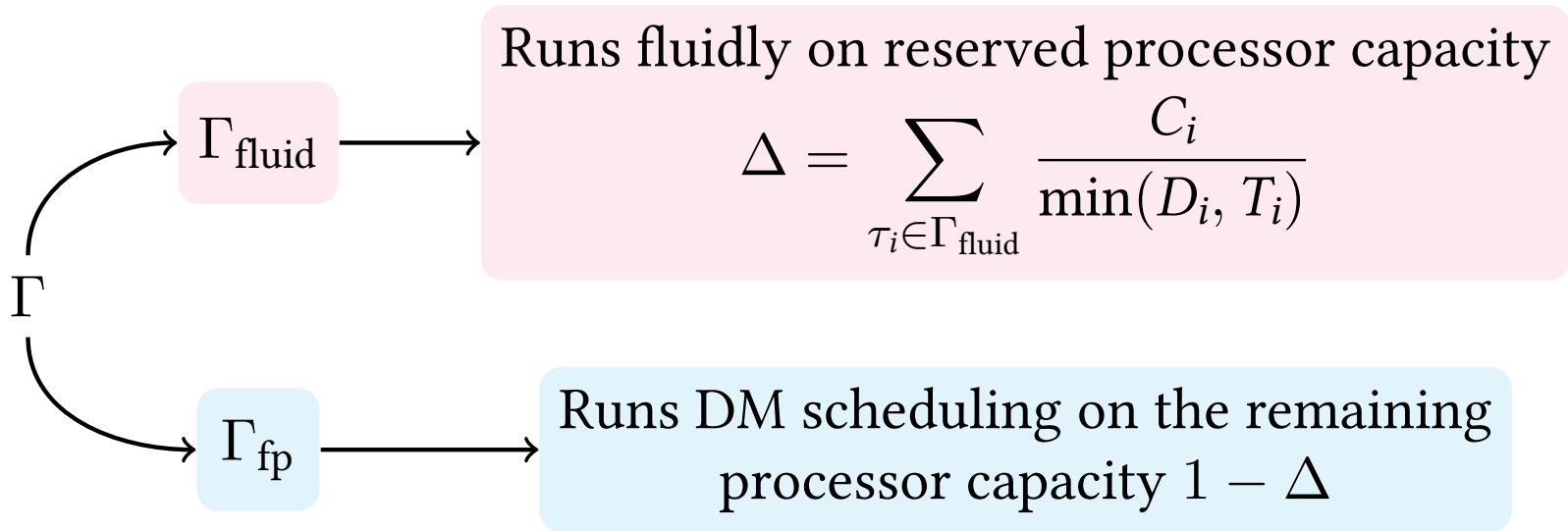

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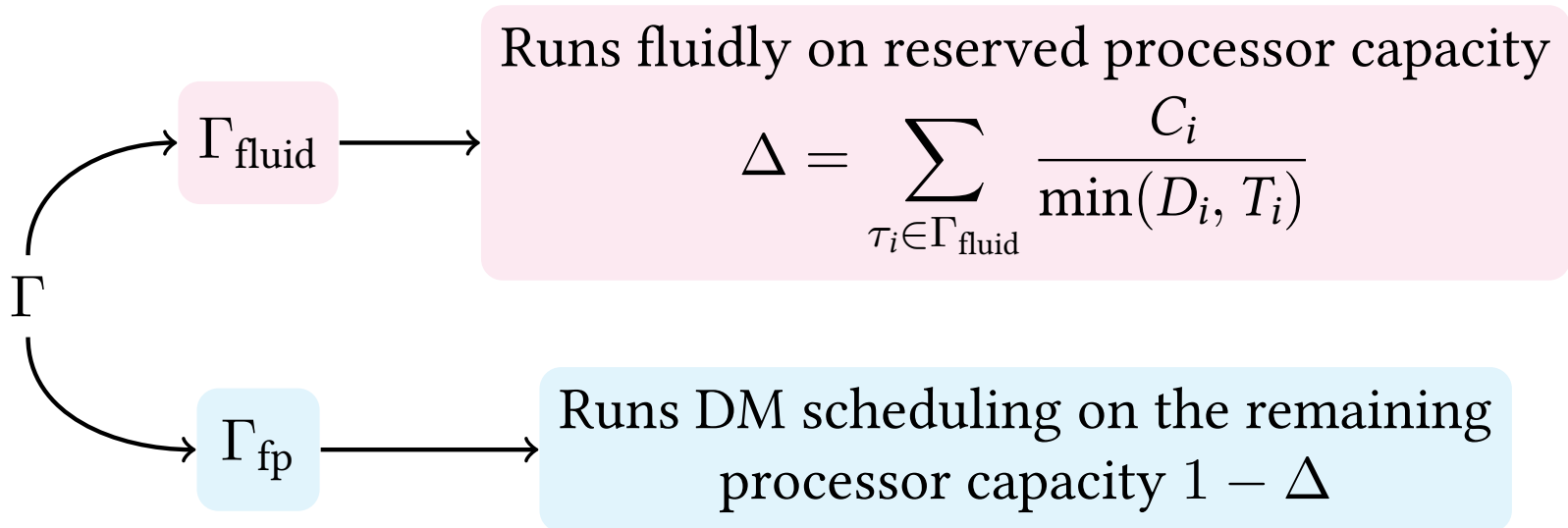

(1) The partitioning of Γ into Γ_{fluid} and Γ_{fp}
(2) RTA fixed-points for Γ_{fp} on a speed-$(1 - \Delta)$ processor

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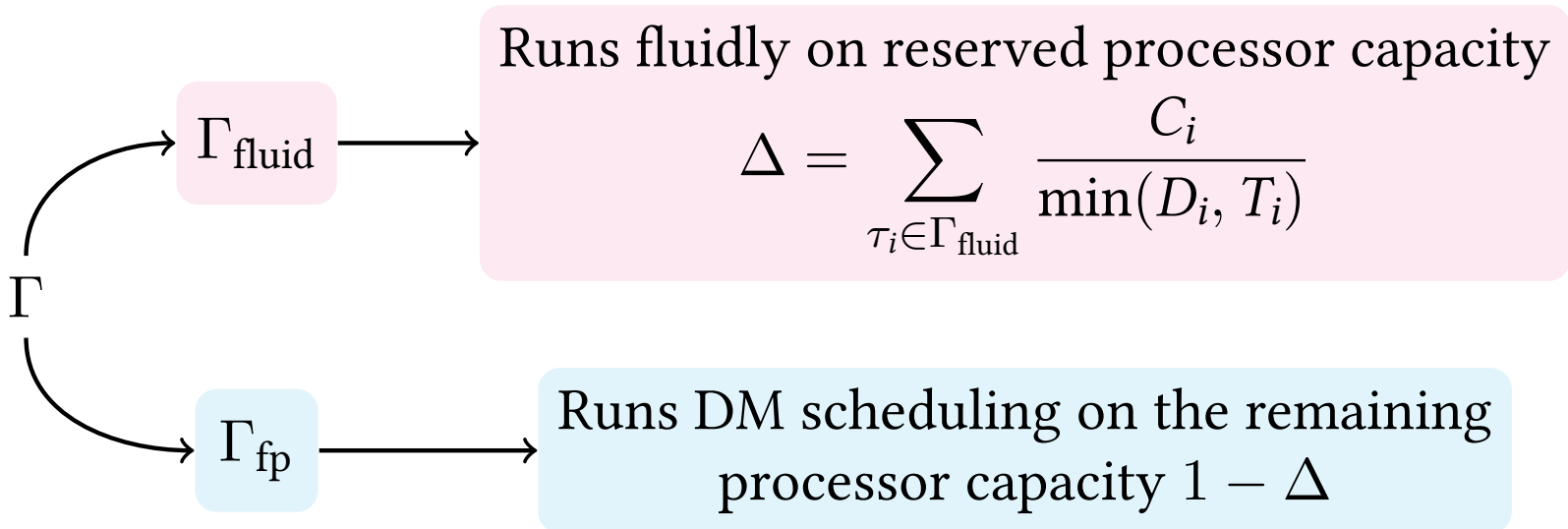

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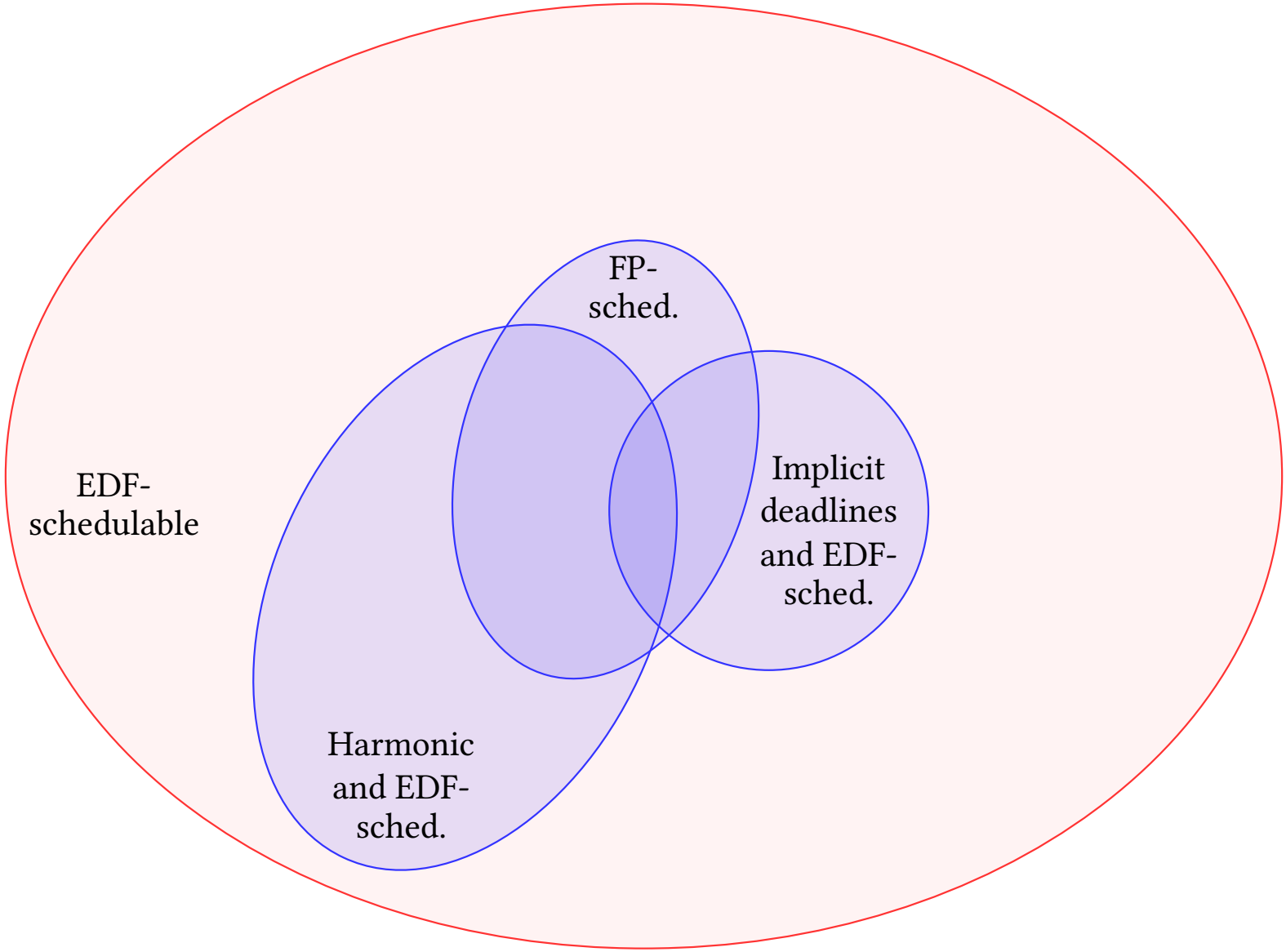
Verifiable in polynomial time $\iff$ FP+fluid-schedulability $\in$ NP

Dealing with EDF



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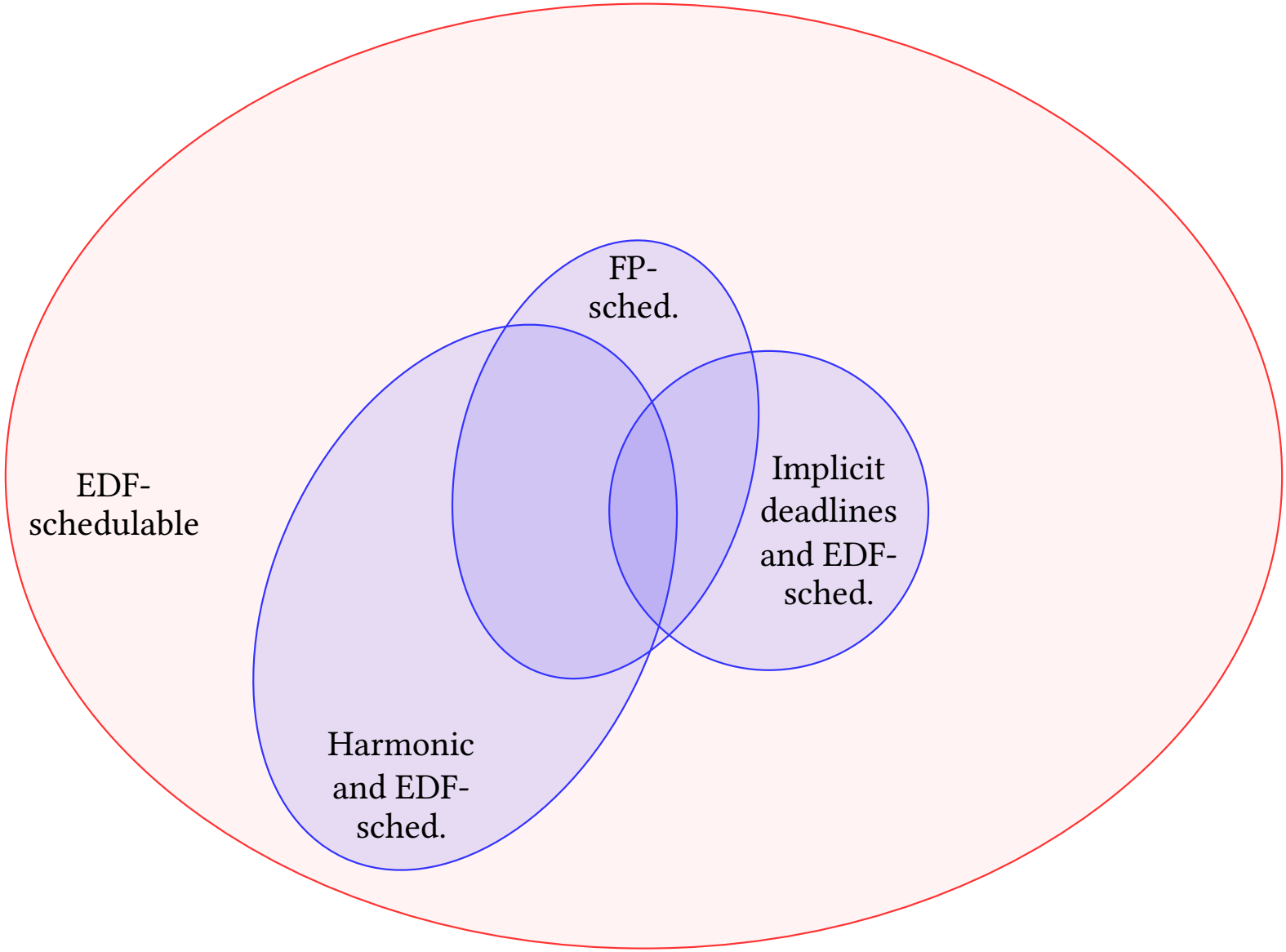

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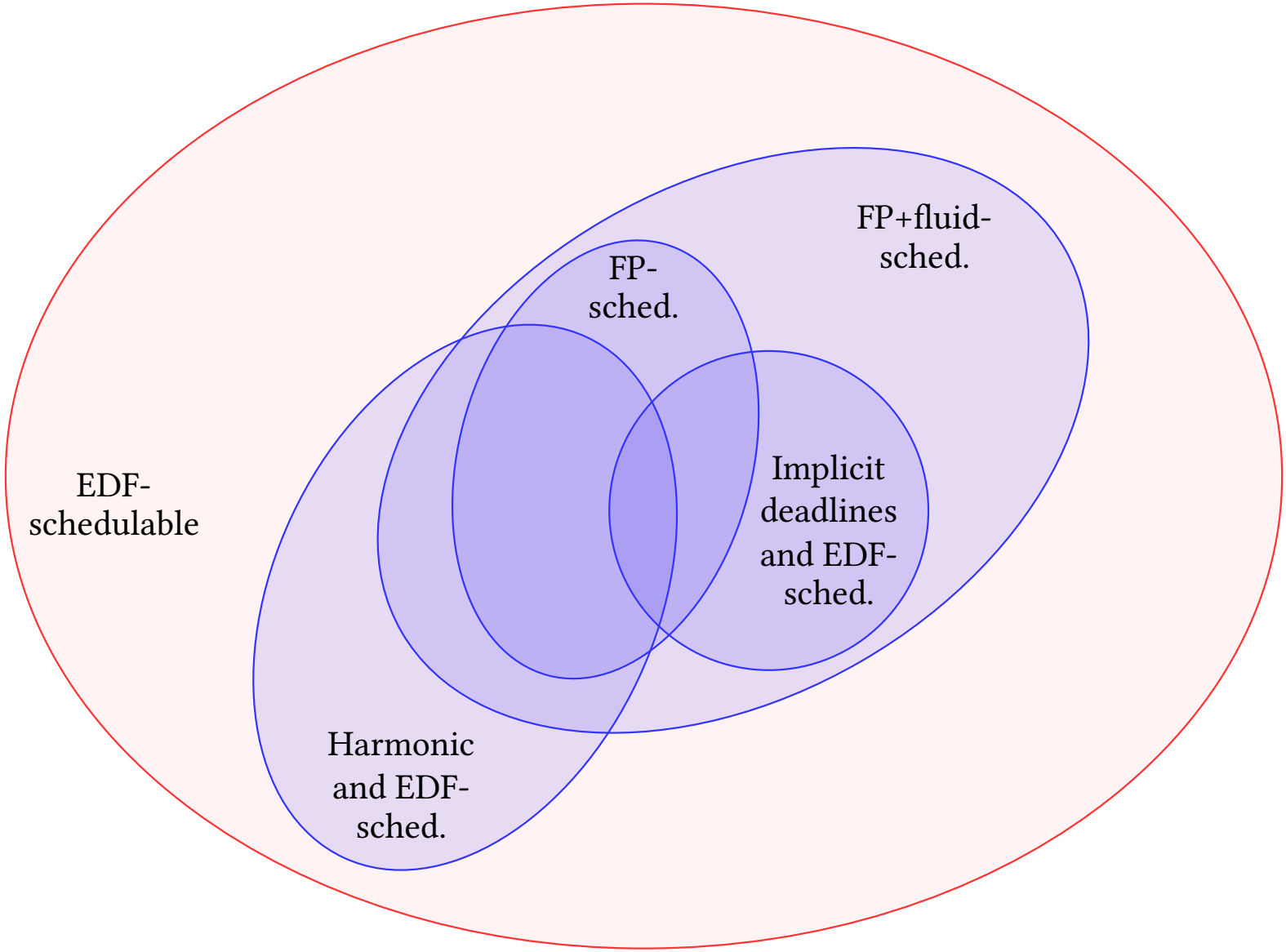

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Split each task τ_i into $k_i \in \mathbb{N}_+$ pieces, then schedule with DM.

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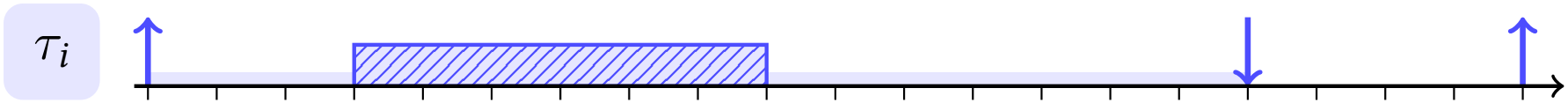

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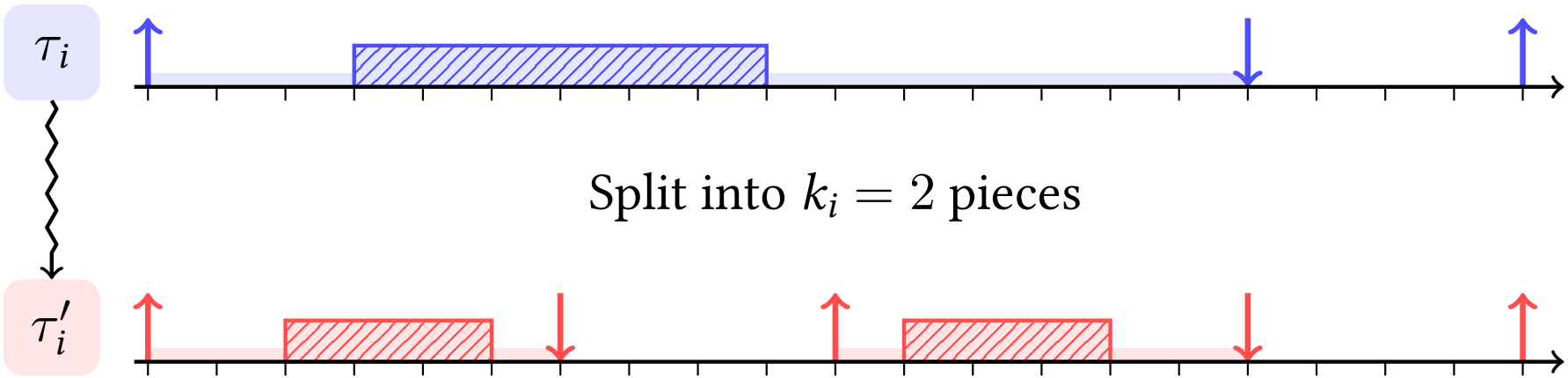

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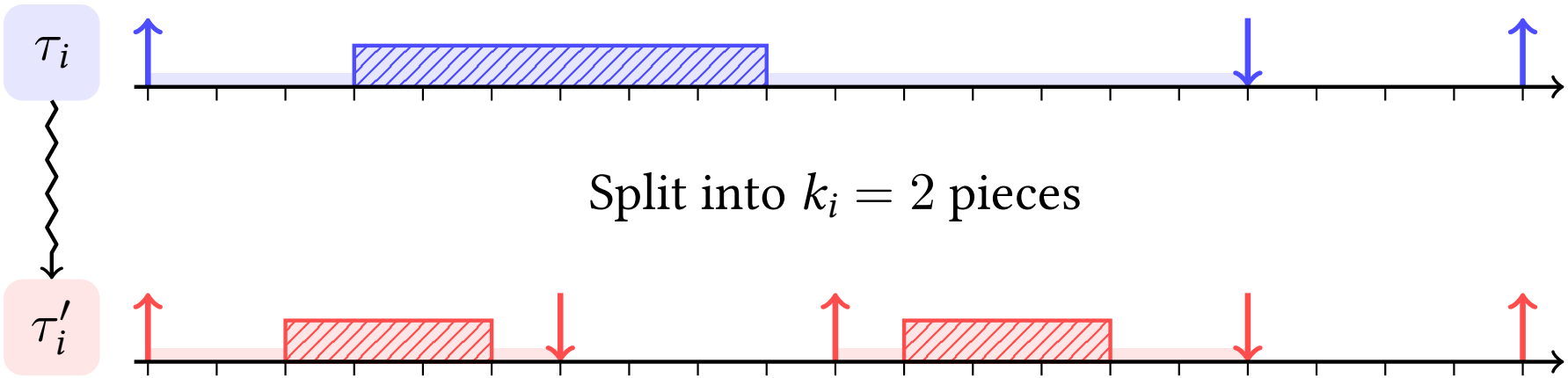

(1) The k_i for each $\tau_i \in \Gamma$
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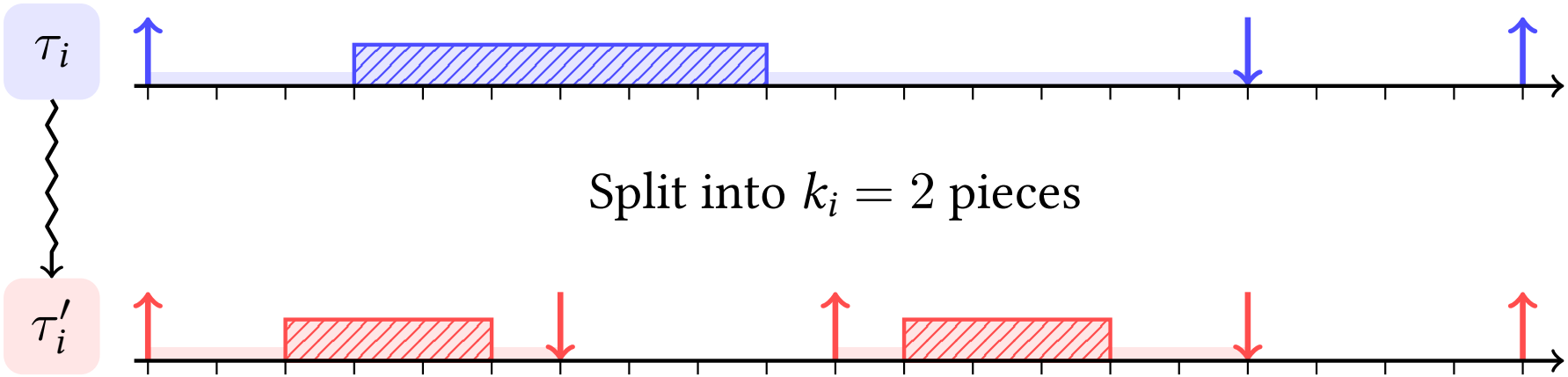

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Verifiable in polynomial time $\iff$ FP+split-schedulability $\in$ NP

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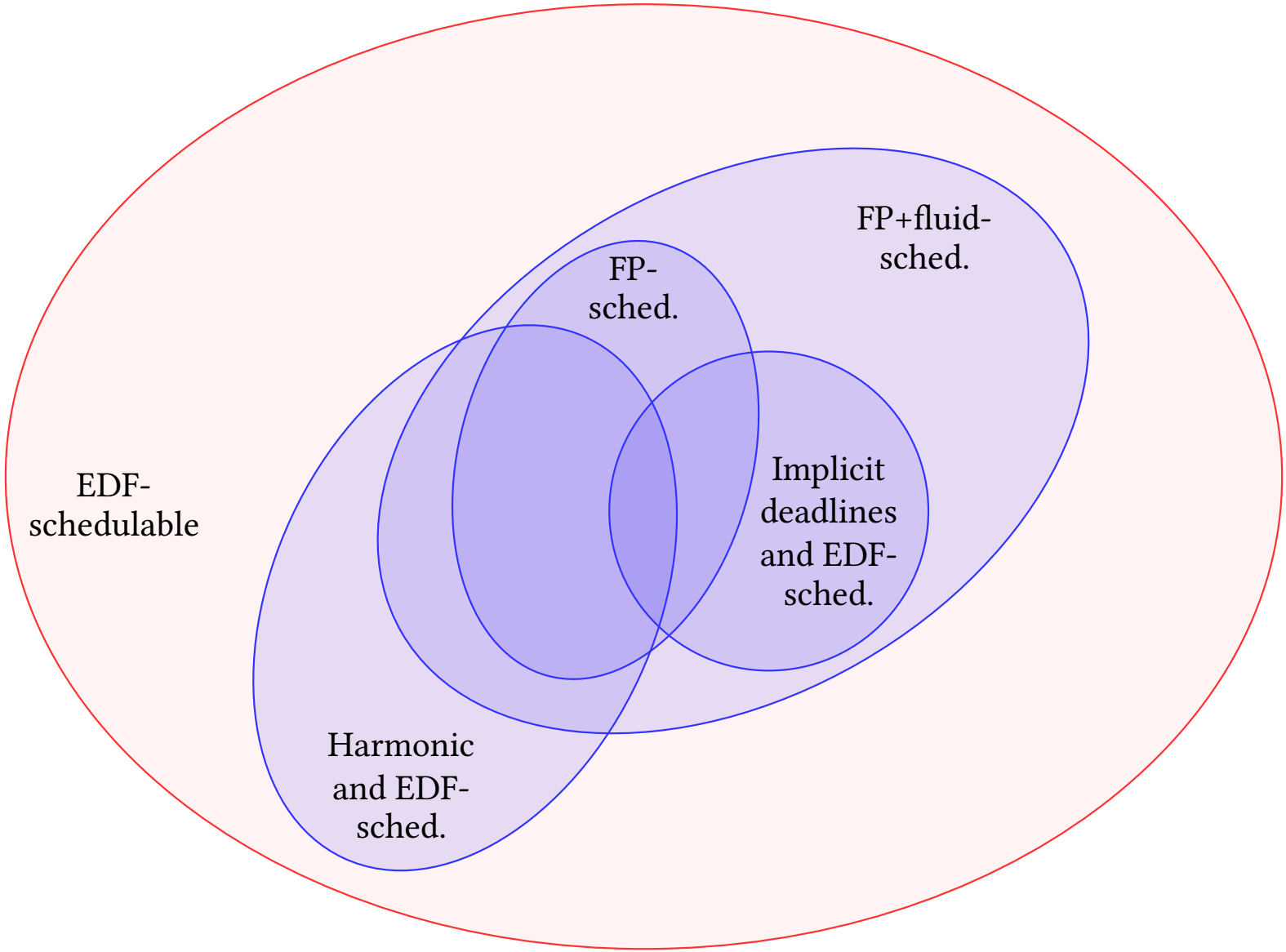

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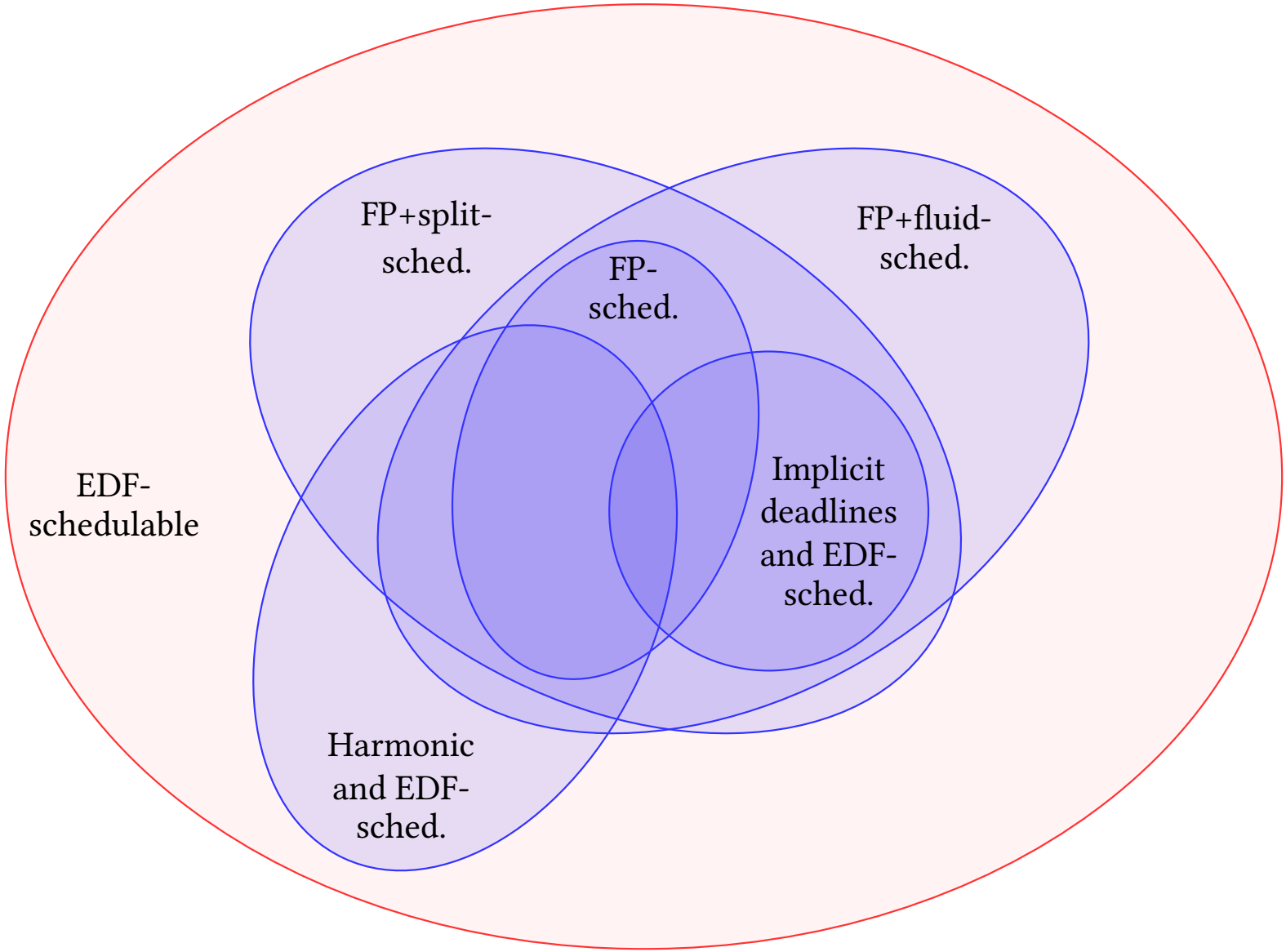
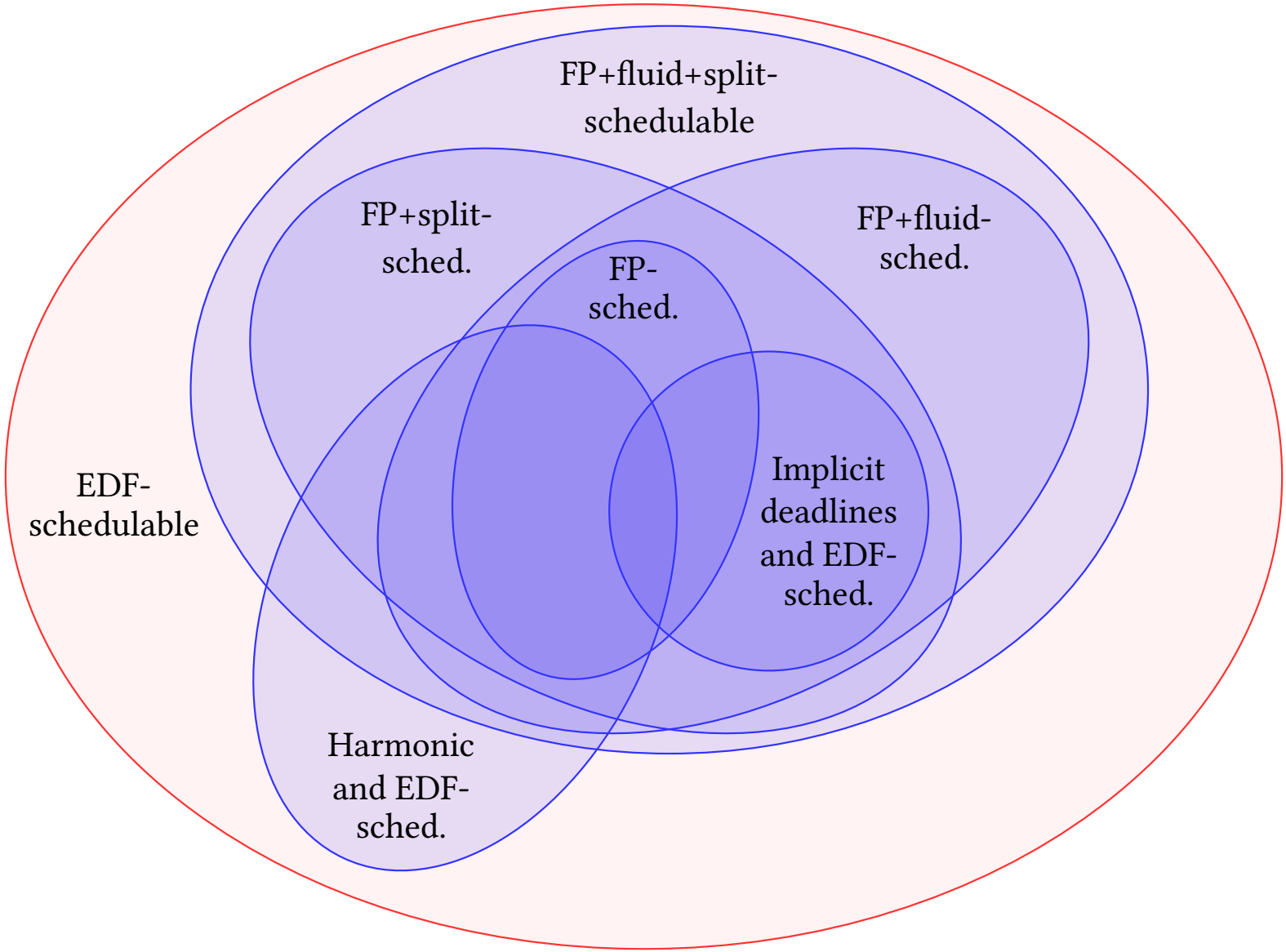


Dealing with EDF



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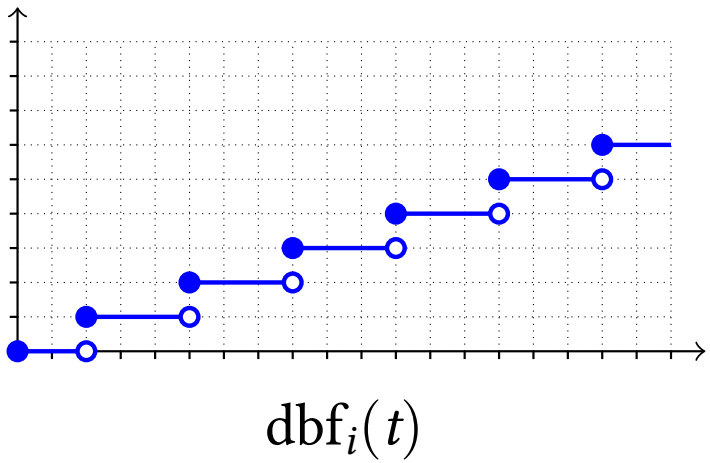

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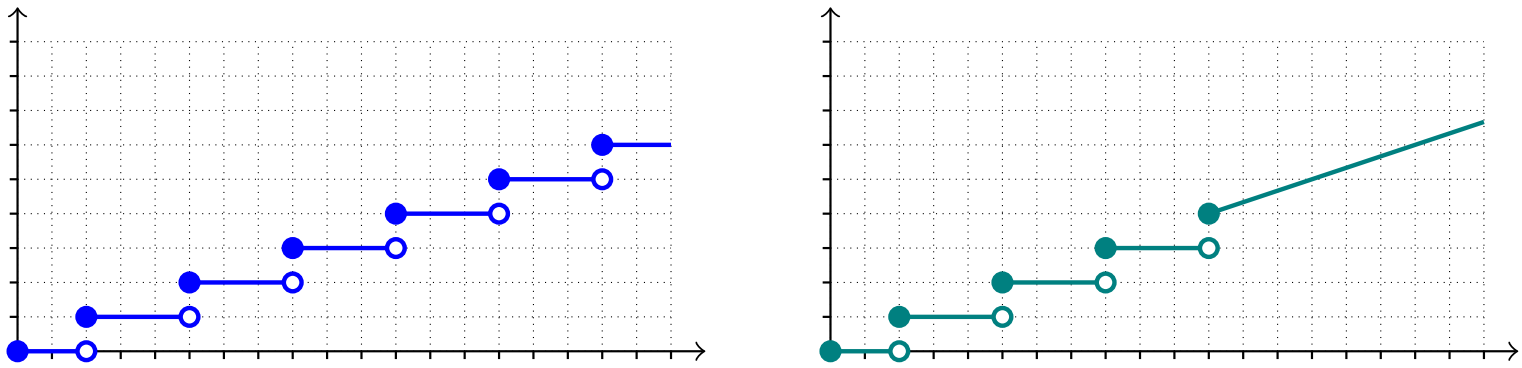

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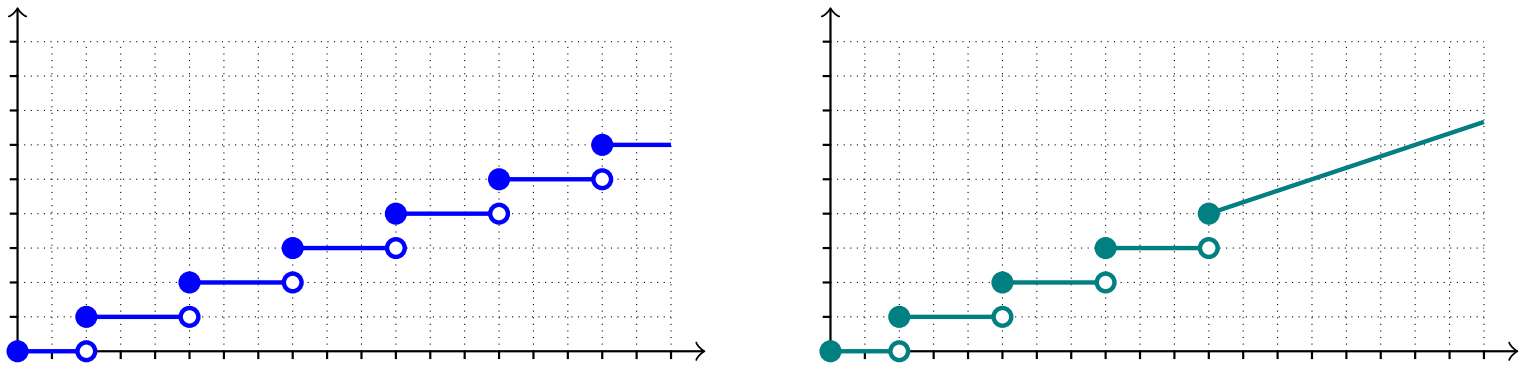

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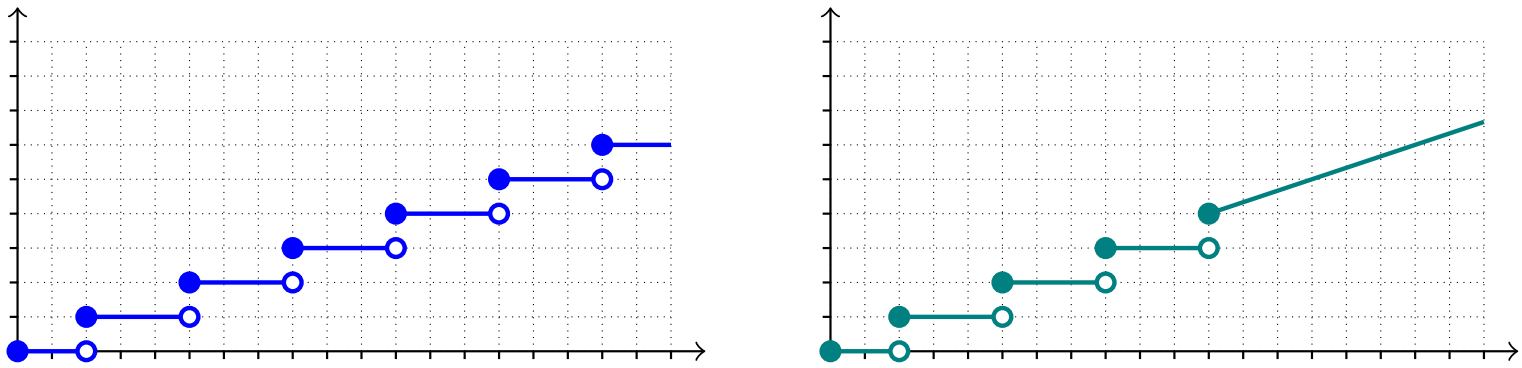

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(by Albers and Slomka, ECRTS 2004)

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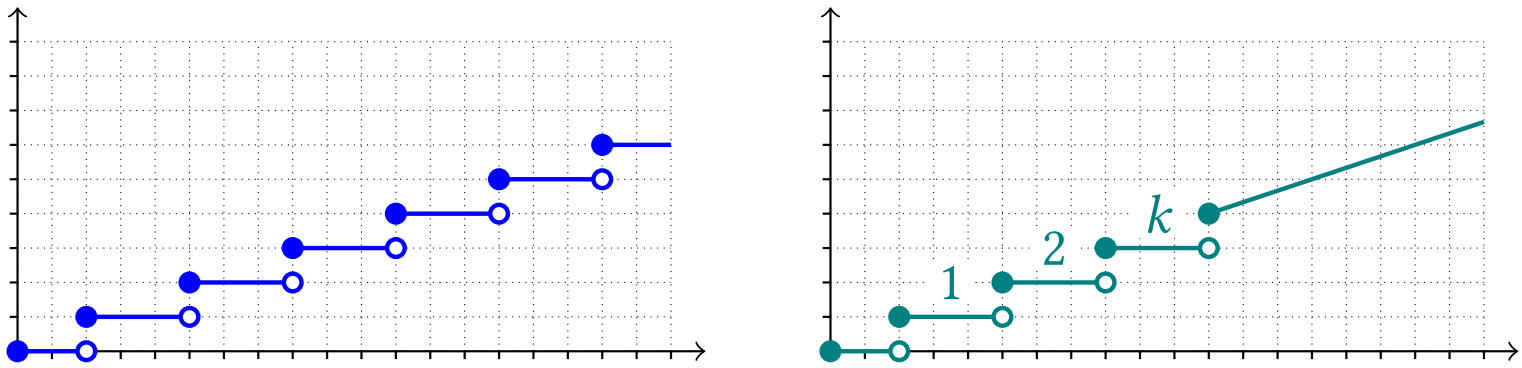

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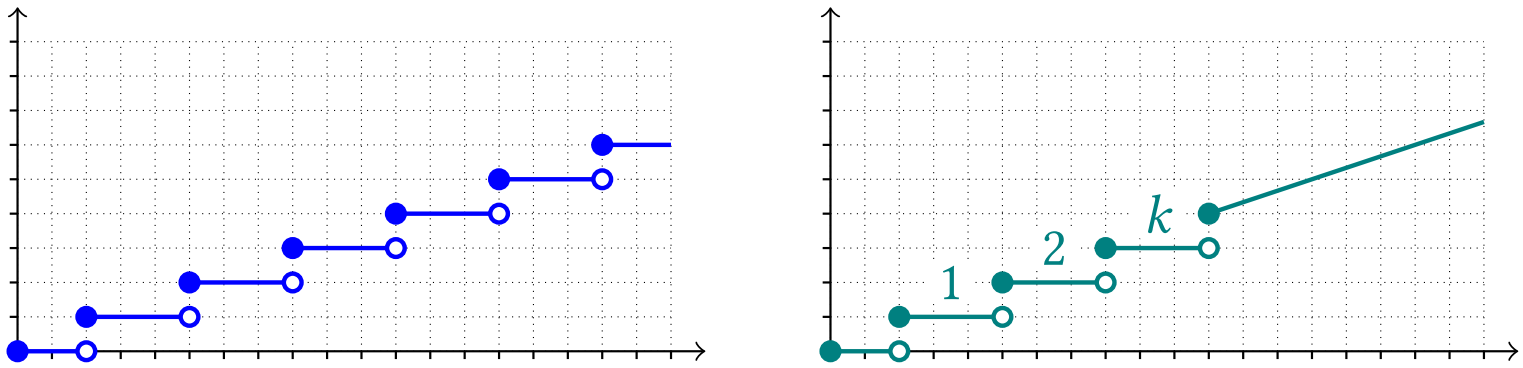

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$\Gamma, k \longrightarrow$ A&S FPTAS

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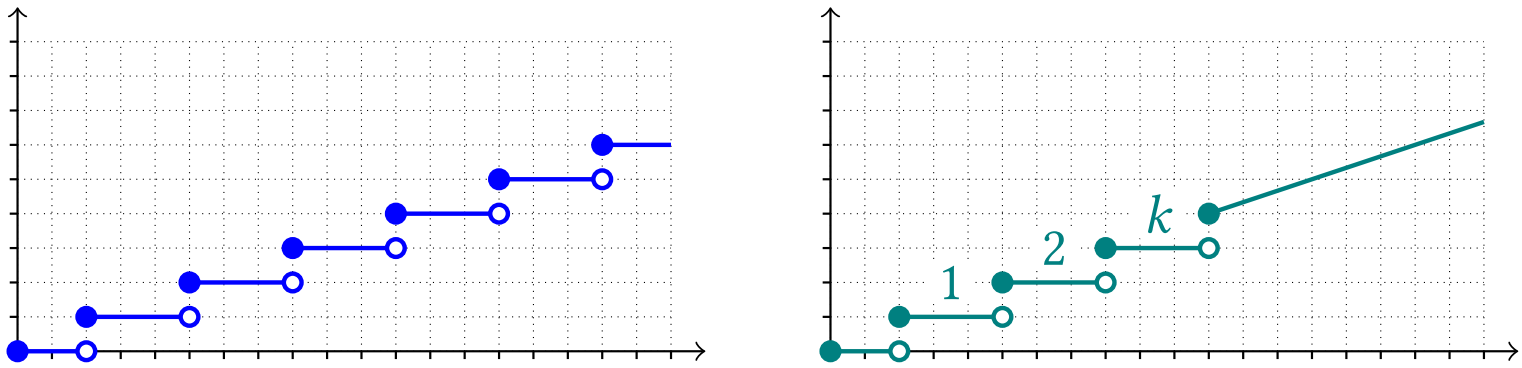

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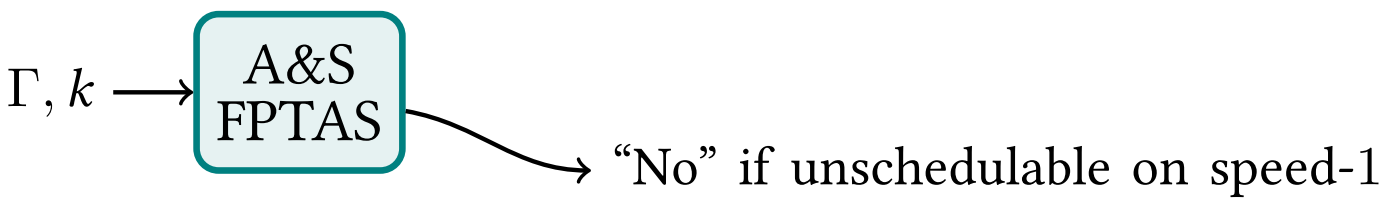

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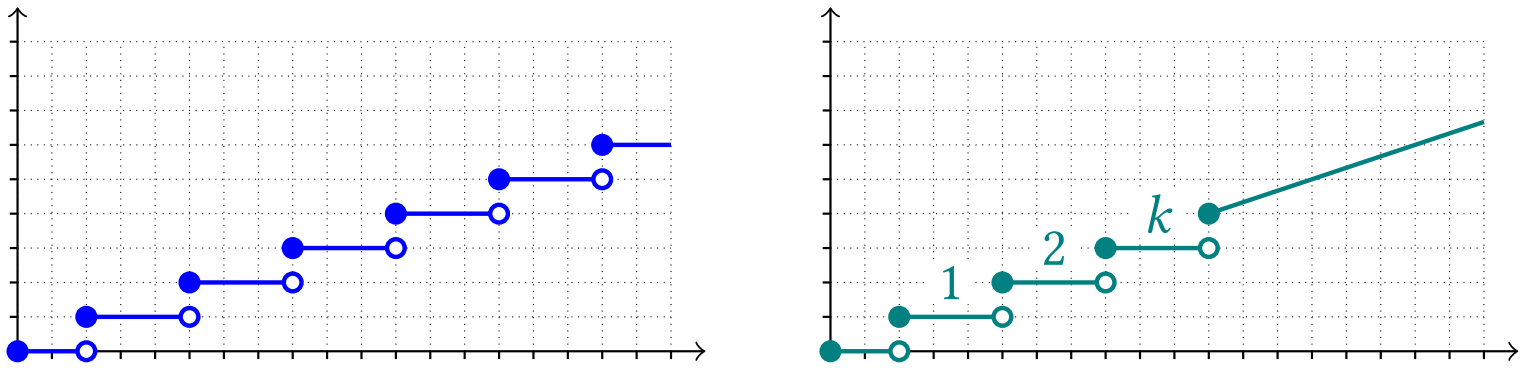

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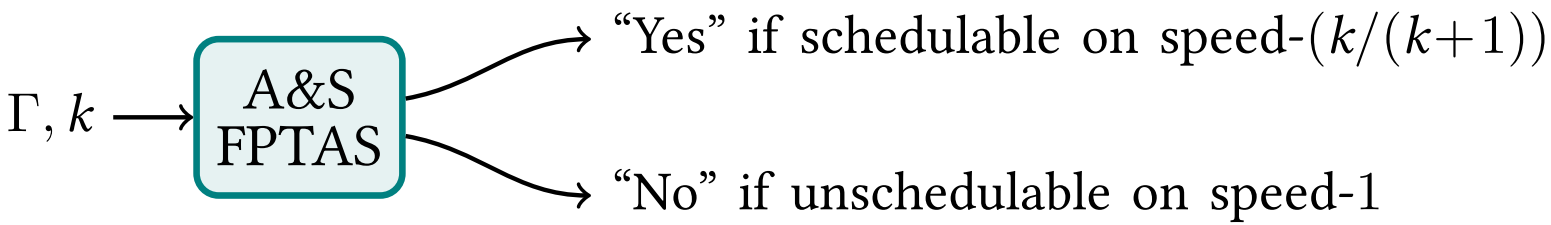

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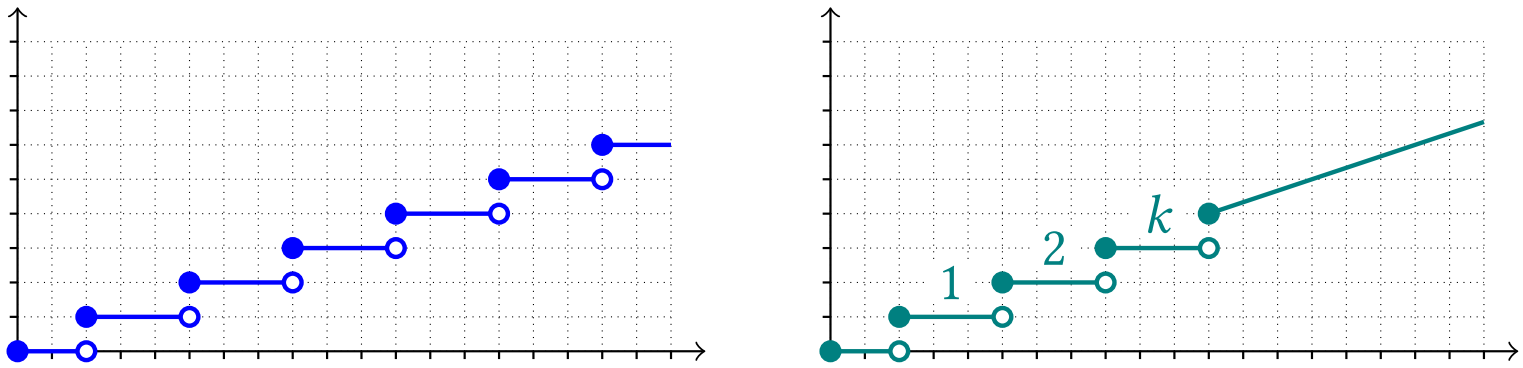

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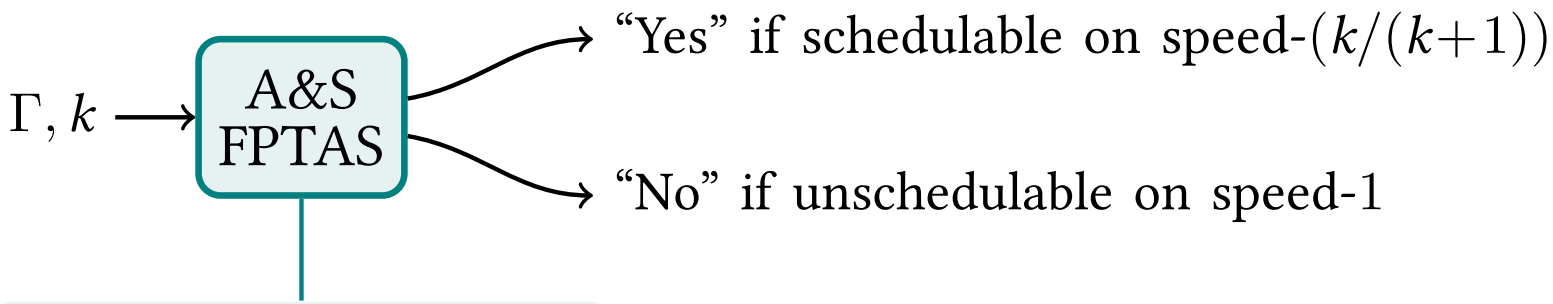

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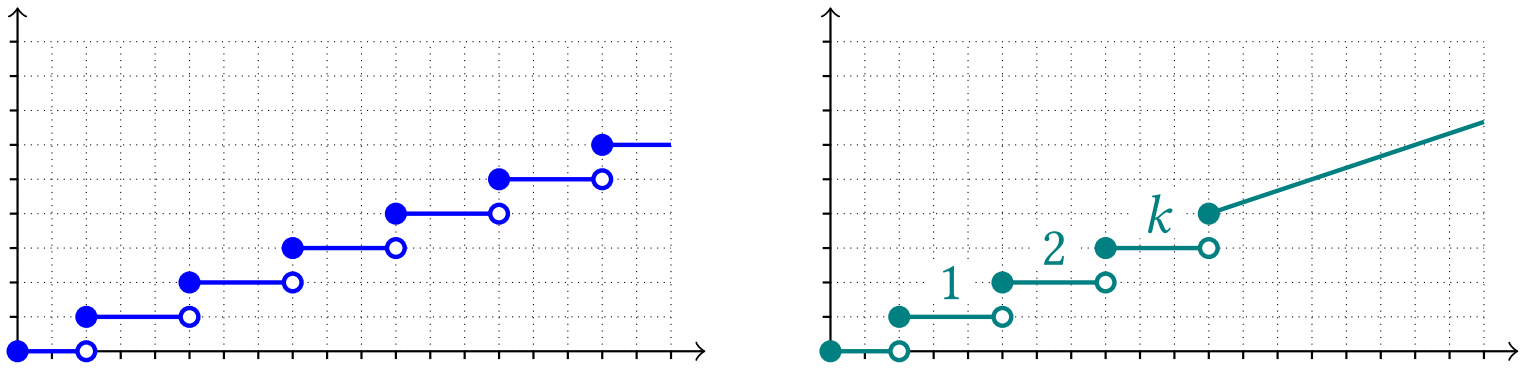

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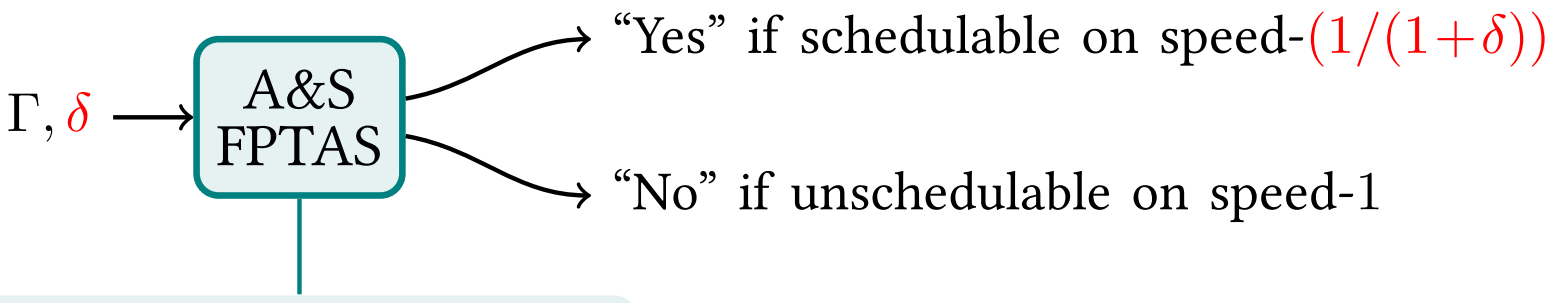

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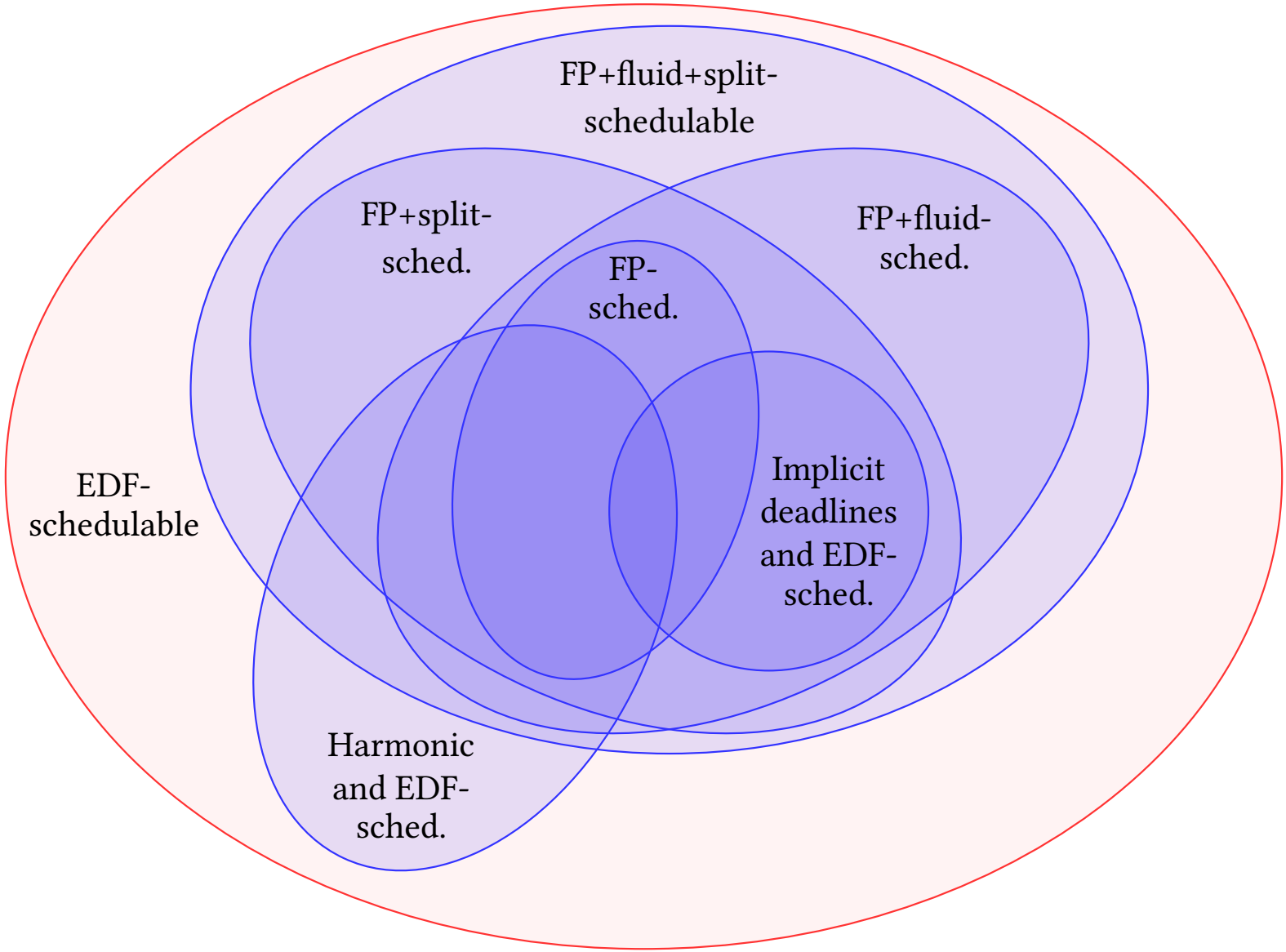

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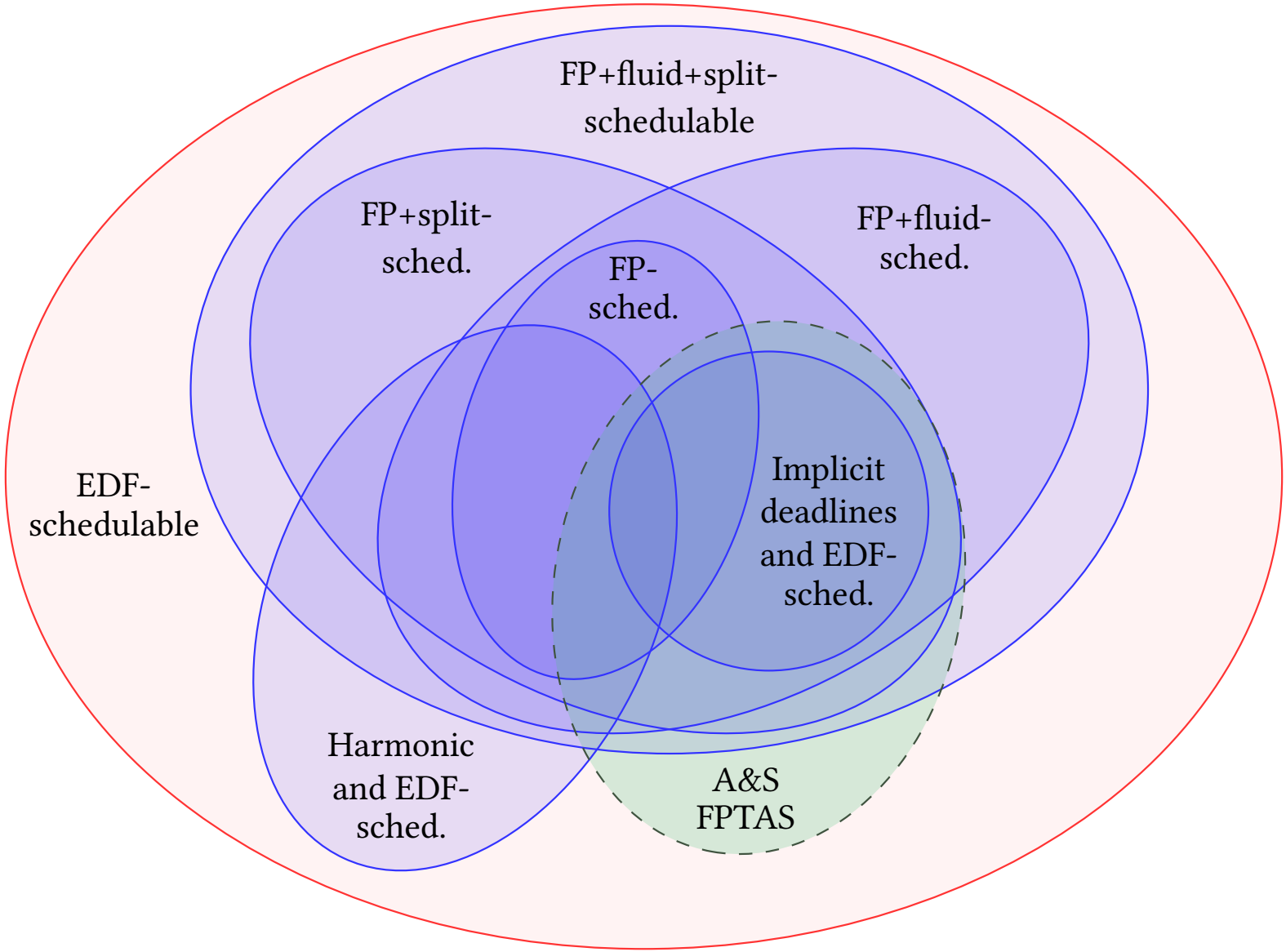

Approximation algorithms for explainability?

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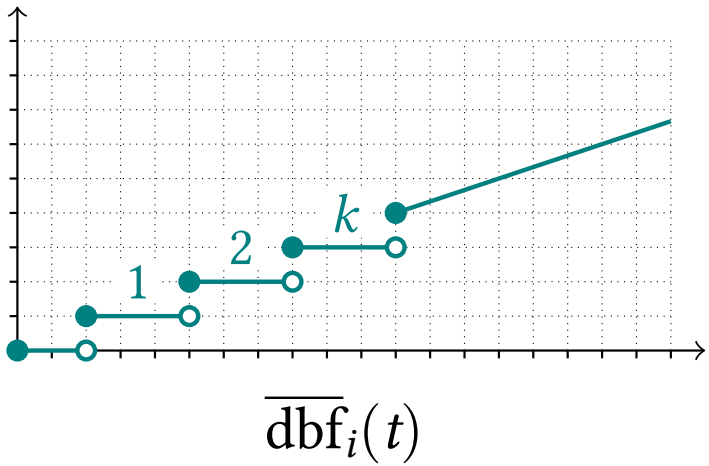

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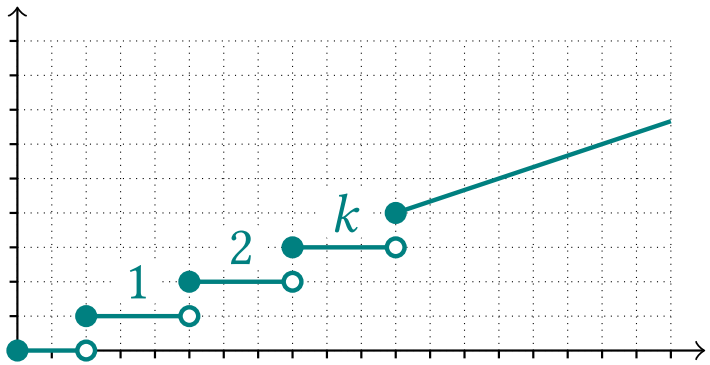

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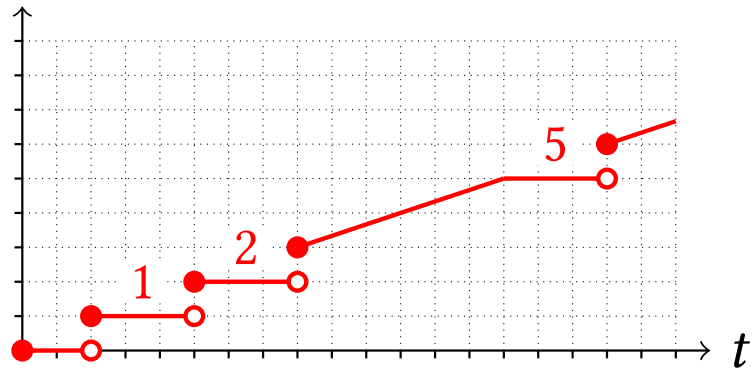

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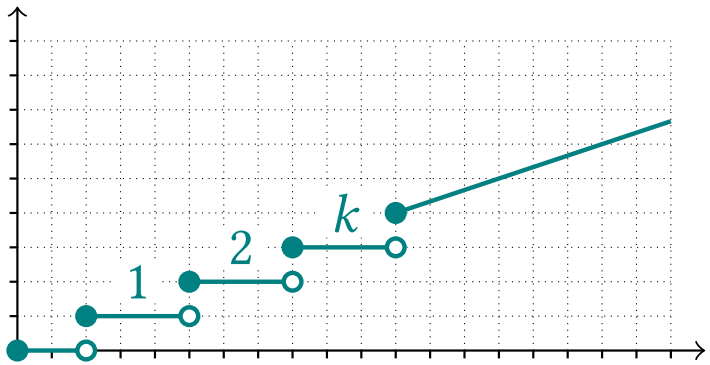

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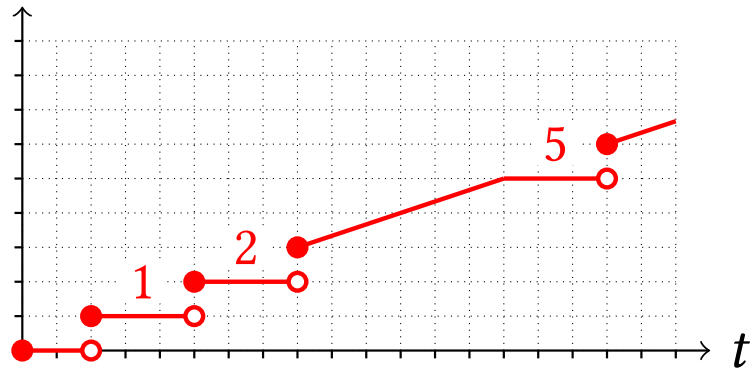

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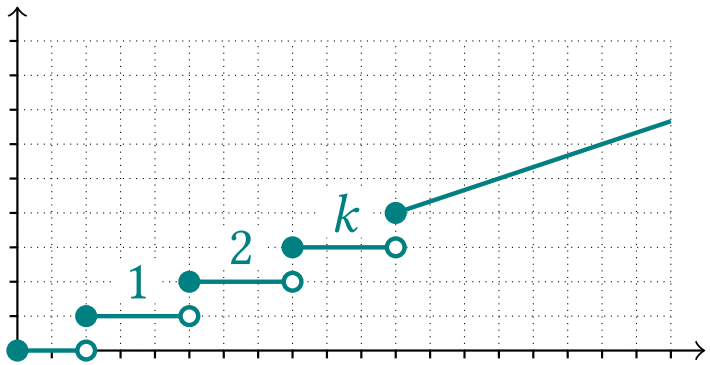

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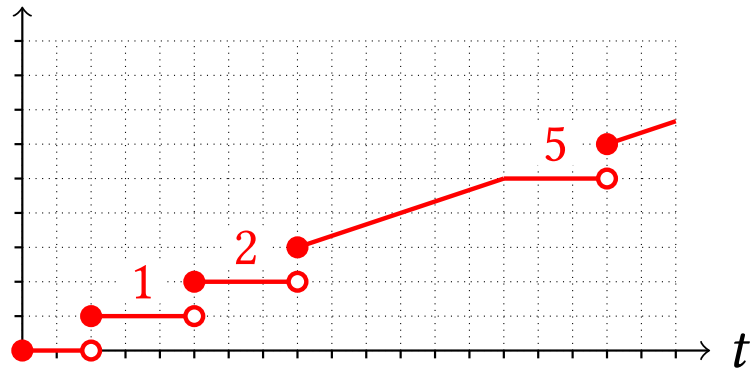

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FPTVAS =

Fully Polynomial-Time
Verification Approximation Scheme

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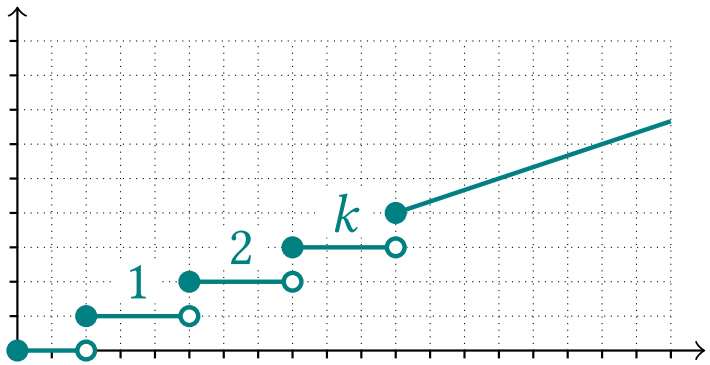

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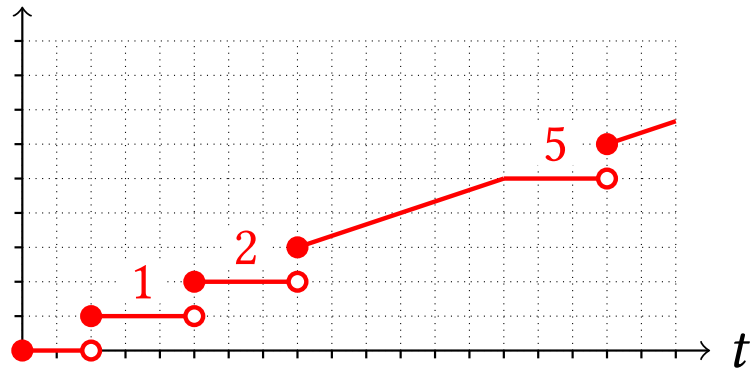

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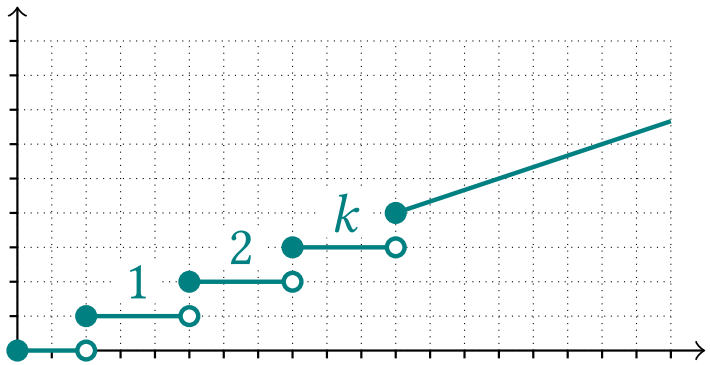

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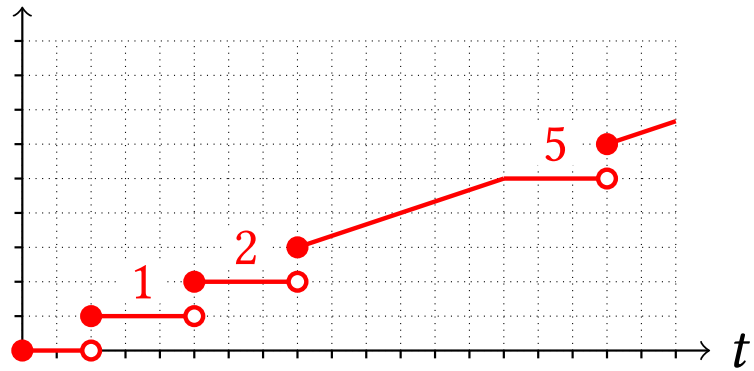

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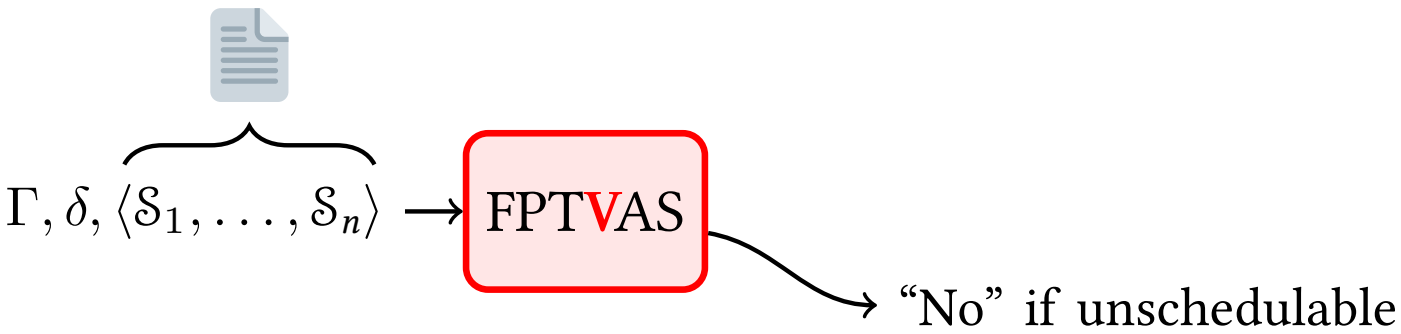

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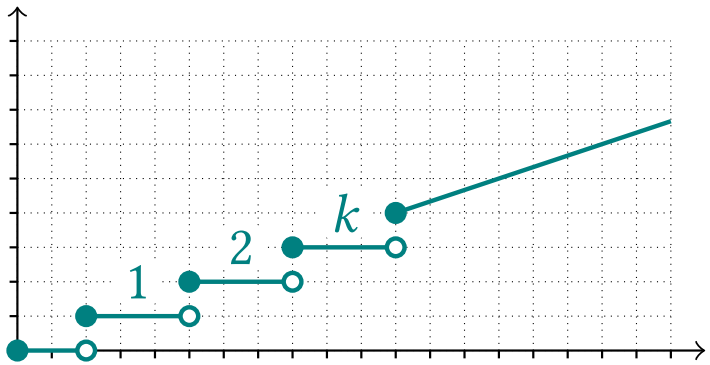

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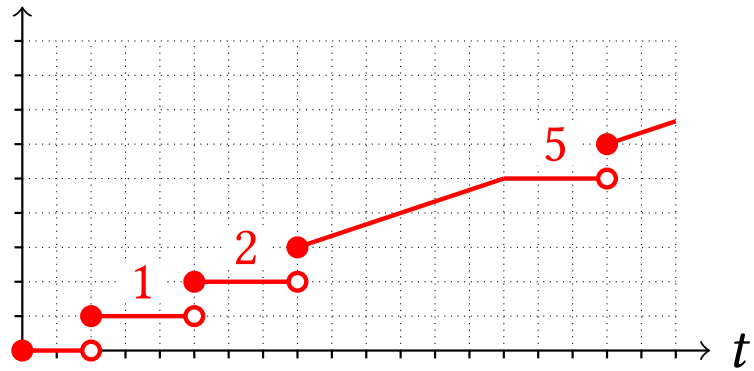

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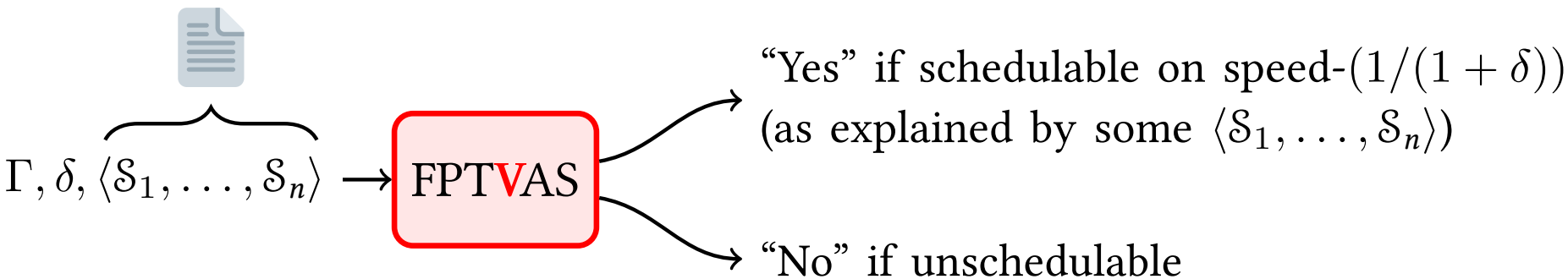

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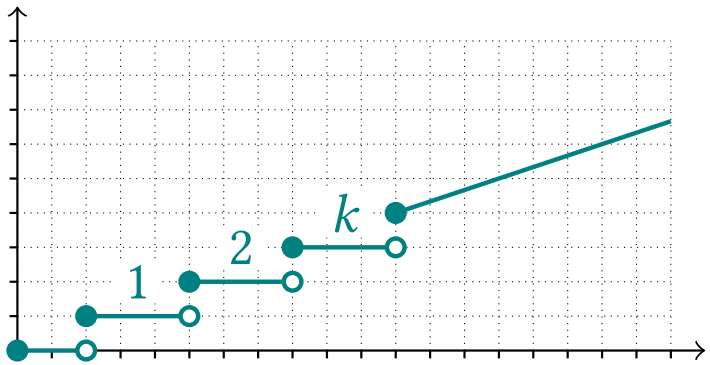

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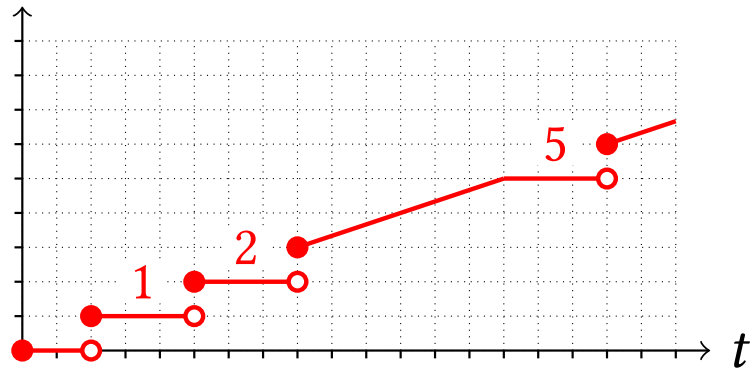

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"Yes" if schedulable on speed-$(1/(1+\delta))$ (as explained by some $\langle \mathcal{S}_1, \ldots, \mathcal{S}_n \rangle$)

"No" if unschedulable

Polynomial in $|\Gamma|$ and $(\frac{1}{\delta})$

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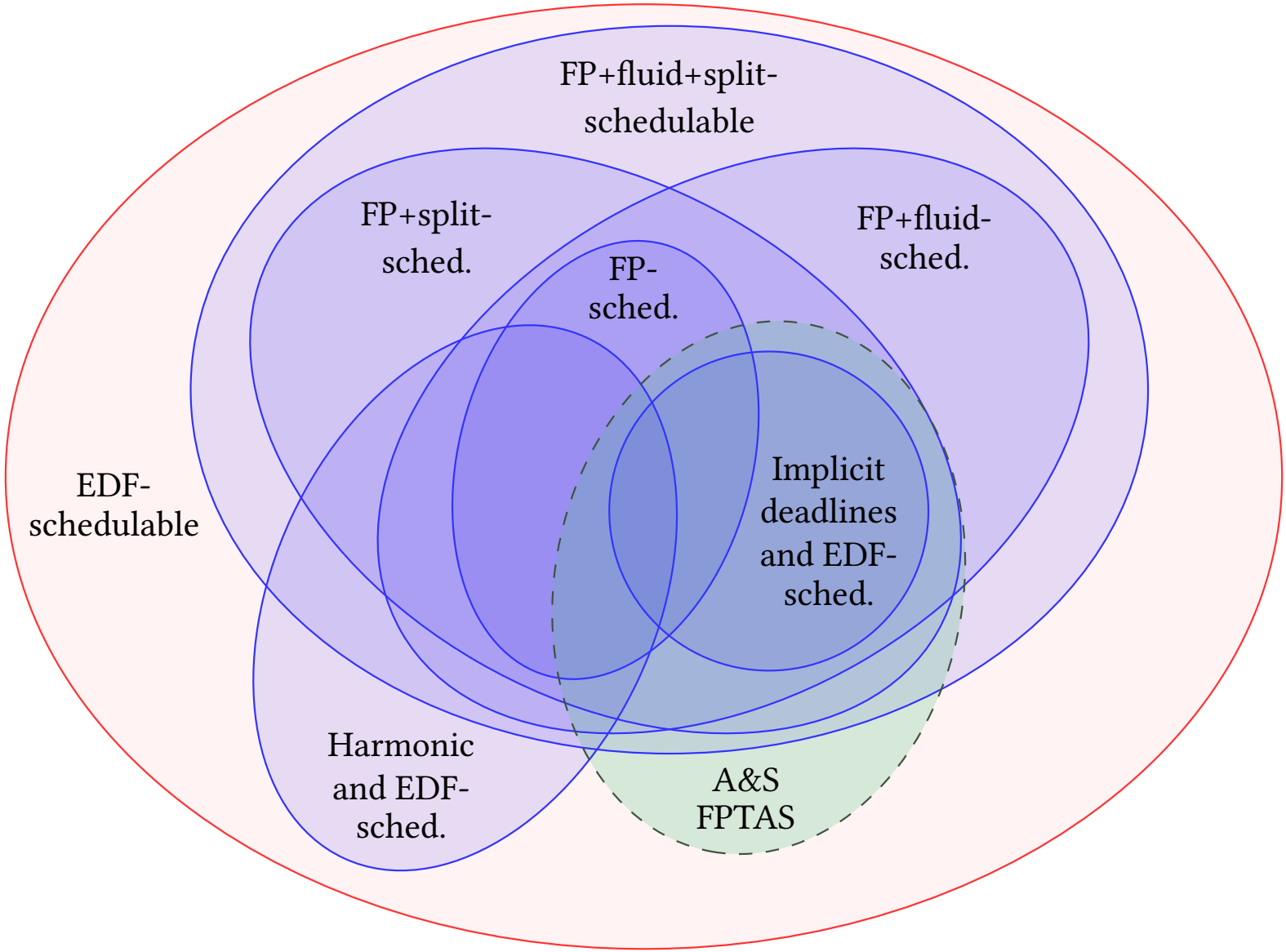

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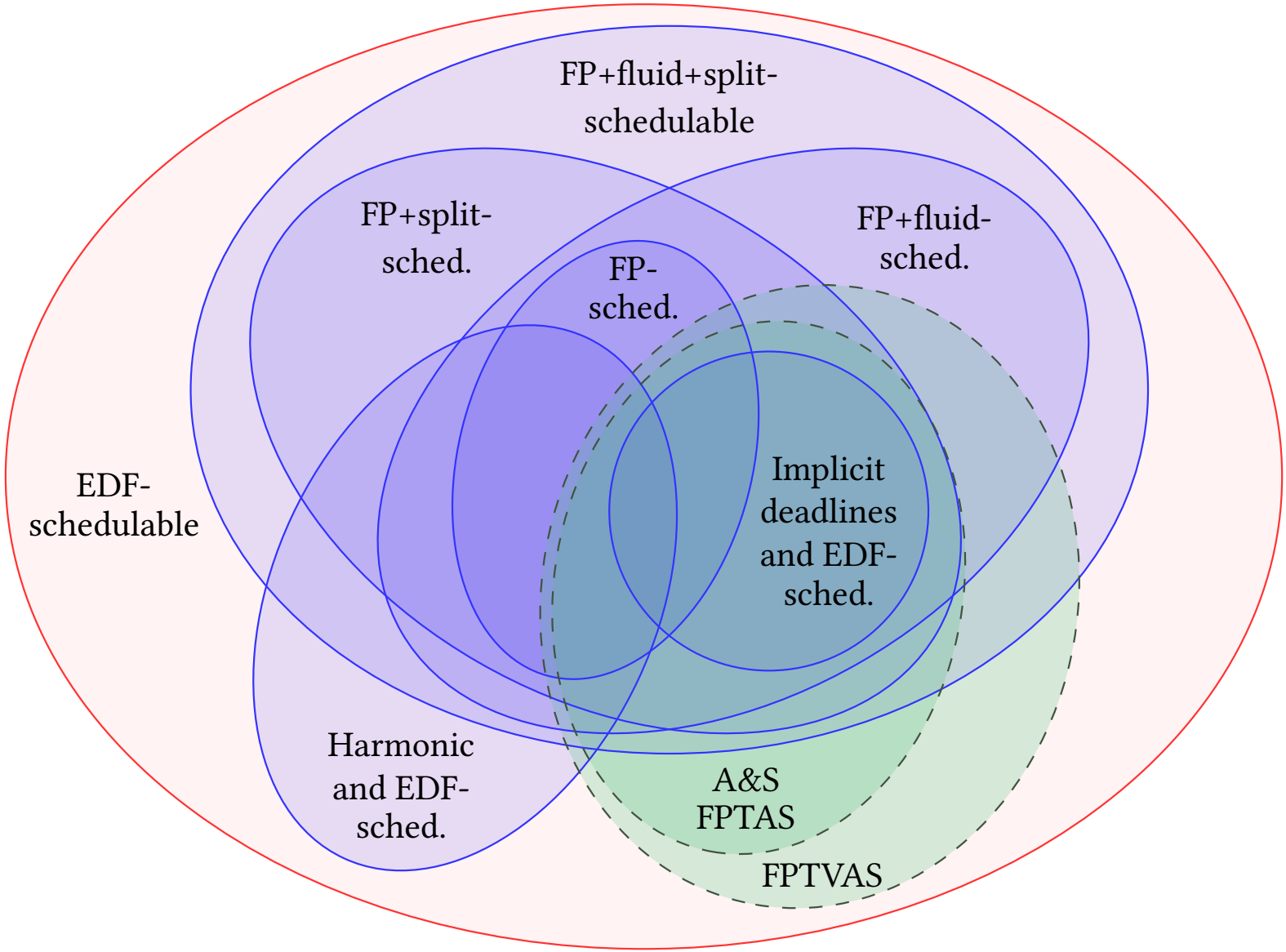

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  - A relevant complexity class: `pseudoNP`

$\forall$Thank you!

$\exists$Questions?