# Uniprocessor Feasibility of Sporadic Tasks with Constrained Deadlines is Strongly CONP-COMPLETE 

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## Context on the Uniprocessor Feasibility Problem



## Context on the Uniprocessor Feasibility Problem

|  | General case | Utilization bounded by a constant $c<1$ |
| :---: | :---: | :---: |
| Asynchronous periodic | $?$ | $?$ |
| Synchronous periodic (or sporadic) | $?$ | $?$ |



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| Asynchronous <br> periodic | Utilization <br> bounded by a <br> constant $c<1$ |
| (Weakly) <br> coNP-hard | (Weakly) <br> coNP-hard |
| conchronous <br> periodic <br> (or sporadic) | ? |



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| Asynchronous periodic | $\begin{aligned} & \text { (Weakly) } \\ & \text { coNP-hard } \end{aligned}$ | $\begin{aligned} & \text { (Weakly) } \\ & \text { coNP-hard } \end{aligned}$ |
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## How?

## SCP $\propto$ in-Feasibility

## How?



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## The Simultaneous Congruences Problem (SCP)

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$(A, 2) ? \rightarrow$ Yes

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\begin{equation*}
(A, 2) ? \rightarrow \text { Yes } \tag{A,3}
\end{equation*}
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$$
(A, 2) ? \rightarrow \text { Yes }
$$

$(A, 3) ? \rightarrow$ No

## Feasibility and Demand Bound Functions

$$
\mathbf{T}=\left\{\tau_{1}, \tau_{2}, \tau_{3}\right\}
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SCP instance $(A, k)$

## Feasibility and Demand Bound Functions



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## Conclusion

|  | General case | Utilization bounded by a constant $c<1$ |
| :---: | :---: | :---: |
| Asynchronous periodic | Strongly coNP-complete | Strongly coNP-complete |
| Synchronous periodic (or sporadic) | Strongly coNP-complete | Pseudo-poly. solution exists |

## $\forall$ Thank you!



## $\exists$ Questions?

