Uniprocessor Feasibility of Sporadic Tasks with Constrained Deadlines is Strongly coNP-complete

Pontus Ekberg & Wang Yi

Uppsala University

ECRTS 2015
Context on the Uniprocessor Feasibility Problem

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1980  1990  2000  2010

Leung & Merrill
Baruah et al.
Eisenbrand & Rothvoß

Sporadic Feasibility is Strongly coNP-complete
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- Leung & Merrill

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Leung & Merrill

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#### Timeline

- **1980**: Leung & Merrill
- **2000**: Strongly coNP-complete
- **2010**: Strongly coNP-complete

**Sporadic Feasibility is Strongly coNP-complete**
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Context on the Uniprocessor Feasibility Problem

General case
- Utilization bounded by a constant $c < 1$

Asynchronous periodic
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Synchronous periodic (or sporadic)
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Leung & Merrill
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Sporadic Feasibility is Strongly coNP-complete
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Weakly coNP-hard

Strongly coNP-complete

Pseudo-poly. solution exists

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1980: Leung & Merrill
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Pontus Ekberg

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**Timeline:**
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**Notes:**
- 1980: Sporadic Feasibility is Strongly coNP-complete

**References:**
- Pontus Ekberg

**Keywords:**
- Context on the Uniprocessor Feasibility Problem
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**Authors:**
- Pontus Ekberg
- Sporadic Feasibility is Strongly coNP-complete

**Page:** 2
How?

SCP $\propto \textit{in-Feasibility}$
How?

Strongly NP-complete
(Baruah et al., 1990)

SCP $\propto$ in-Feasibility
How?

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SCP $\propto$ \textit{in-Feasibility}

Pseudo-polynomial transformation
How?

- Strongly NP-complete (Baruah et al., 1990)
- Strongly NP-hard
- SCP $\propto$ in-FEASIBILITY
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How?

SCP

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(Baruah et al., 1990)

\(\propto\)

in-FEASIBILITY

Strongly NP-hard

Pseudo-polynomial transformation

Strongly coNP-hard
The Simultaneous Congruences Problem (SCP)

Example:

\[ A = (2; 4); (4; 6); (3; 8); (0; 3) \]

\[ k = 2 \]

Yes

\[ (A; 2) \]

?!

No

\[ (A; 3) \]
The Simultaneous Congruences Problem (SCP)

An SCP instance is given by a pair \((A, k)\).
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\[(A, 2)\] → Yes
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\((A, 2)\) \(\rightarrow\) Yes \quad \((A, 3)\)?
The Simultaneous Congruences Problem (SCP)

An SCP instance is given by a pair \((A, k)\).

Example: \( A = \{(2, 4), (4, 6), (3, 8), (0, 3)\} \quad k = 2 \)

\[(2, 4) \rightarrow \text{Yes} \quad (A, 2)\]
\[(4, 6) \rightarrow \text{No} \quad (A, 3)\]
Feasibility and Demand Bound Functions

\[ T = \{ \tau_1, \tau_2, \tau_3 \} \]
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Demand bound functions capture feasibility exactly! (Baruah et al., 1990)

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Feasible / Infeasible SCP instance \((A; k)\)

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SCP instance \((A, k)\)
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Slope = \[ \frac{1}{|A|} \]

Amount of shift depends on \( k \)
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\[ \tau_3 \]

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∀Thank you!

∃Questions?